AML5103 | Applied Probability and Statistics | Problem Set-2

1. You have tracked the performance of the local meteorologist and complied the following data:

   P(forecast rain, and actual rain) = 0.4, P(forecast rain, and no rain) = 0.2,
   P(forecast no rain, and actual rain) = 0.15, P(forecast no rain, and no rain) = 0.25.

   (a) How often does she forecast rain?
   (b) How often does she make a mistake?
   (c) Given that she just forecast rain, what is the chance that it will actually rain?
   (d) Given that it rains today, what is the probability that she forecast rain in last night's broadcast?

2. An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker as it is when she is a nonsmoker. Suppose that 32 percent of women of childbearing age are smokers.

   (a) What is given in the problem statement?

   $$P(\text{ectopic pregnancy AND smoker}) = \boxed{?} \times P(\text{ectopic pregnancy AND nonsmoker})$$

   or

   $$P(\text{ectopic pregnancy} \mid \text{smoker}) = \boxed{?} \times P(\text{ectopic pregnancy} \mid \text{not smoker})$$

   Identify the correct one and fill in the missing value.
   (b) What is the probability that a randomly selected woman is a smoker?
   (c) What is the probability that a randomly selected woman is a nonsmoker?
   (d) What percentage of women having ectopic pregnancies are smokers?

3. Fifty-two percent of the students at a certain college are females. Five percent of the students in this college are majoring in computer science. Two percent of the students are women majoring in computer science. If a student is selected at random, find the conditional probability that

   (a) the student is female given that the student is majoring in computer science;
   (b) this student is majoring in computer science given that the student is female.

4. Suppose there are four failure modes for a single engine plane: *structural, engine, control system, and human error*. We assume that they will only occur one at a time if at all. The probabilities for these four failure modes are respectively 0.002, 0.002, 0.01, and 0.001. Given that it is a structural failure, there is a 25% chance the plane will crash. The crash probabilities given the other three failure modes are 30%, 90% and 10% respectively for engine, control system and human error failure modes. If a plane has crashed, what is the probability that it was due to a control system failure?

5. A robot, which only has a camera as a sensor, can either be in one of two locations: L_1 or L_2. The robot doesn't know exactly where it is but based on all past observations, the robot thinks that there is an 80% chance that it is in L_1 and a 20% chance that it is in L_2. Location L_2 is the only one that has a window. The robot's vision algorithm detects a window but its image recognition algorithm is not perfect; the probability of observing a window given there is no window at its location is 0.2 and the probability of observing a window given there is a window is 0.9. After incorporating the observation of a window, what is the robot's updated probability that it is in (1) L_1 (2) L_2?

6. Suppose a person needs a blood transfusion. We show the compatibility chart (where an X means compatibility) and population blood type percentage below (Source: Palo Alto Weekly, November 9, 1994):

Population Percentage	
O-	6.6%
O+	37.4%
A-	6.3%
A+	35.7%
B-	1.5%
B+	8.5%
AB-	0.6%
AB+	3.4%

Recipient ↓ / Donor →	O-	O+	A-	A+	B-	B+	AB-	AB+
O-	X							
O+	X	X						
A-	X		X					
A+	X	X	X	X				
B-	X				X			
B+	X	X			X	X		
AB-	X		X		X		X	
AB+	X	X	X	X	X	X	X	X

Compatibility Chart

Given the above information, examine the following questions. Also, pose other meaningful questions regarding the supply and demand of blood (types) given the above compatibility table and blood type distribution.

(a) What is the probability that a random person will be able to donate to another random person given no information about blood types of either the giver or the receiver?

(b) Given the above data, what can you say about blood transfusion policy in a hospital regarding blood drives target and blood transfusion priority?

(c) In a battle field hospital a soldier is brought in for immediate blood transfusion. Only blood type A+ is available in the supply for immediate use. We do not know the wounded soldier's blood type. There are two other soldiers present who are willing to donate their blood. We have time to do one blood typing before time becomes critical. What should we do?