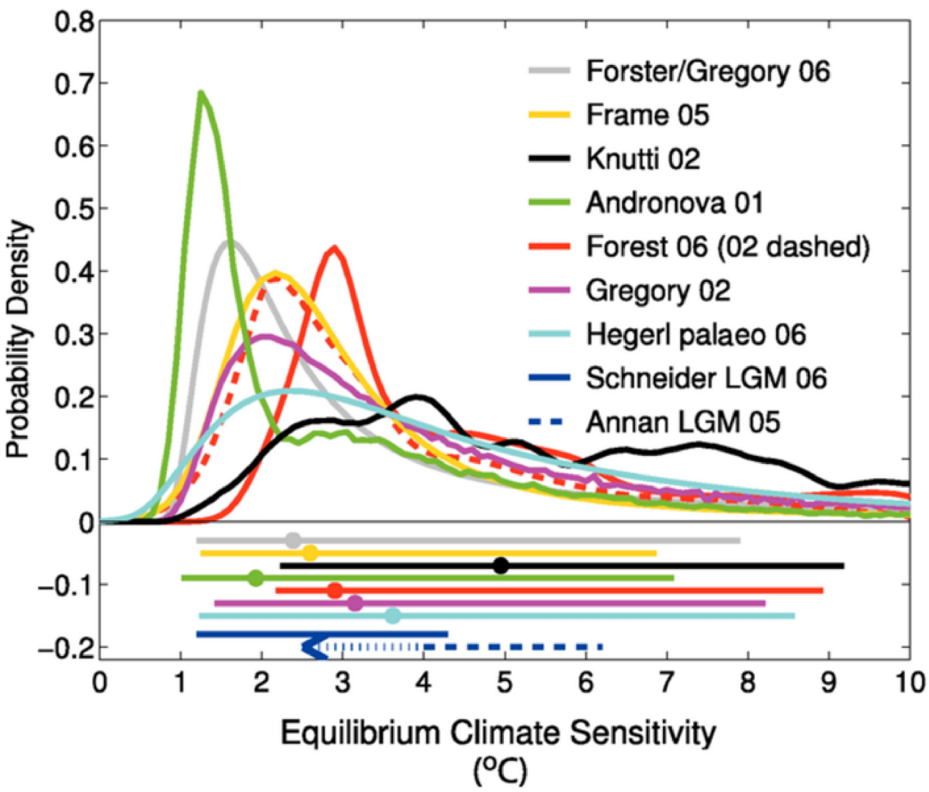
Climate Change

12. This summer (Jun 2018) the concentration of CO2 in the atmosphere was 411 parts per million (ppm) which is substantially higher than the pre-industrial concentration: 275 ppm. CO_2 is a greenhouse gas and as such increased CO_2 corresponds to a warmer planet.

    Absent some pretty significant policy changes we will reach a point within the next 50 years (eg well within your lifetime) where the CO_2 in the atmosphere will be double the pre-industrial level. In this problem we are going to explore the question: what will happen to the global temperature if atmospheric CO_2 doubles?

    The measure, in degrees Celsius, of how much the global average surface temperature will change (at the point of equilibrium) after a doubling of atmospheric CO_2 is called "Climate Sensitivity.âĂİ Since the earth is a complicated ecosystem climate scientists model S as a random variable. The IPPC Fourth Assessment Report had a summary of 10 scientific studies that estimated the PDF for Climate Sensitivity (S): In this problem we are going to treat S as



    part-discrete and part-continuous. For values of S less than 7.5, we are going to model sensitivity as a discrete random variable with PMF based on the average of estimates from the studies in the IPCC report. Here is the PMF for S in the range 0 through 7.5:

Sensitivity, S (degrees C)	0	1	2	3	4	5	6	7
Expert Probability	0.00	0.11	0.26	0.22	0.16	0.09	0.06	0.04

The IPCC fifth assessment report notes that there is a non-negligible chance of S being greater than 7.5 degrees but didn't go into detail about probabilities. In the paper "Fat-Tailed Uncertainty in the Economics of Catastrophic Climate Change" Martin Weitzman discusses how different models for the PDF of Climate Sensitivity (S) for large values of S have wildly different policy implications.

For values of S greater than 7.5 degrees Celsius, we are going to model S as a continuous random variable. Consider two different assumptions for S when it is greater than 7.5: a fat tailed distribution (f_1) and a thin tailed distribution (f_2):

$$f_1(x) = \frac{K}{x} \text{ s.t. } 7.5 < x < 30$$

$$f_2(x) = \frac{K}{x^3} \text{ s.t. } 7.5 < x < 30$$

For this problem assume that the probability that S is greater than 30 degrees Celsius is 0.

a. Estimate the probability that Climate Sensitivity is greater than 7.5 degrees Celsius.
b. Calculate the value of K for both f1 and f2.
c. It is estimated that if temperatures rise more than 10 degrees Celsius, all the ice on Greenland will melt. Estimate the probability that S is greater than 10 under both the f_1 and f_2 assumptions.
d. Calculate the expectation of S under both the f_1 and f_2 assumptions.
e. Let $R = S^2$ be a crude approximation of the cost to society that results from S. Calculate $E[R]$ under both thef_1 and f_2 assumptions.

Notes: (1) Both f1 and f2 are "power law distributions". (2) Calculating expectations for a variable that is part discrete and part continuous is as simple as: use the discrete formula for the discrete part and the continuous formula for the continuous part.