MLPA.

Problem Set-1

1. $w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $b = -4$.

$$w^T x + b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + (-4) = 0.$$

$$= x_1 + 2x_2 + 3x_3 - 4 = 0.$$

$$\Rightarrow x_1 = -2x_2 - 3x_3 + 4.$$

$$x_2 = x_2$$

$$x_3 = x_3.$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} \quad | \quad x_2, x_3 \in R.$$

3. (10,10)
(100,)

5. Distance of sample $x^{(i)}$ from the hyperplane
= Sample projection of V_i onto w

$$= \frac{V_i^T w}{\|w\|} = \frac{(x^{(i)} - \text{vector to point on plane})^T w}{\|w\|}$$

$$= \frac{x^{(i)T} - w^T x}{\|w\|}$$

=> Distance of $x^{(i)}$ to the hyperplane $w^T x + b = 0$

$$= \frac{w^T x^{(i)} - (-b)}{\|w\|} = \frac{w^T x^{(i)} + b}{\|w\|}$$

6. maximize $\left(\text{minimum of } \frac{|w^T x^{(i)} + b|}{\|w\|} \right)$

$$= \text{maximize} \left(\frac{\text{minimum of } |w^T x^{(i)} + b| \; = \text{the distance of the sample } x^{(i)} \text{ from the plane } w^T x + b}{\|w\|} \right)$$

$$= \text{minimize } \frac{\|w\|^2}{2}.$$

$y^{(i)} (w^T x^{(i)} + b) \geq 1$. for $i = 1 \ldots n$.

7. $x_2 = -2x_1 + 4$. $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

=> $2x_1 + x_2 = 4$.

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 4.$$

Augmented matrix: $\left[\begin{array}{cc|c} 2 & 1 & 4 \end{array}\right]$

$x_1 \quad x_2$

Row operations: $\left[\begin{array}{cc|c} 1 & \frac{1}{2} & 2 \end{array}\right]$

$x_1 + \frac{1}{2} x_2 = 2$.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1/2\,x_2 + 2 \\ x_2 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$