Modelling and Simulation of the Cascaded Vapour Compression Refrigeration System (CVCRS)

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Abstract

As the computer chip industry approaches the end of Moore's Law, they are faced with the challenge of increasing computational power without fabricating transistors of sub-molecular sizes. The tradeoff for simply adding more transistors is an increase in waste heat generation, which causes a considerable decline in component performance. Modern data centres thus deploy sophisticated cooling systems to provide optimum conditions to the chips. The following report examines the feasibility of a small-scale cascade vapour compression cooling system using thermodynamic modelling, computer simulation, and a parametric study with the ambient temperature using Python, CoolProp, and Engineering Equation Solver.

Keywords: cascaded vapour compression cooling, thermodynamic modelling, CoolProp, Python simulation, parametric study

1. Introduction

The system (represented in Figure 1) employs two vapour compression refrigeration cycles (idealised, represented in Figure 3) with two different refrigerants: R1234ze(E) in the HTC and R23 in the LTC. These operate within different temperature ranges. In total, there are 8 different streams of refrigerant in the system. The two share a heat exchanger, which acts as the condenser for the LTC and the evaporator for the HTC. For continuous refrigeration to be possible, we ensure the following:

- Stream 4 is at a lower temperature than the refrigerated space; so that the LTC refrigerant can extract heat from the refrigerated space in the evaporator and get vaporised.
- Streams 8 and 5 are at a lower temperature than Streams 2 and 3; so heat transfer occurs from the LTC refrigerant to the HTC refrigerant in the heat exchanger.
- Stream 6 is at a higher temperature than the ambient temperature; so that the HTC refrigerant can expel its heat to the surroundings in the condenser.

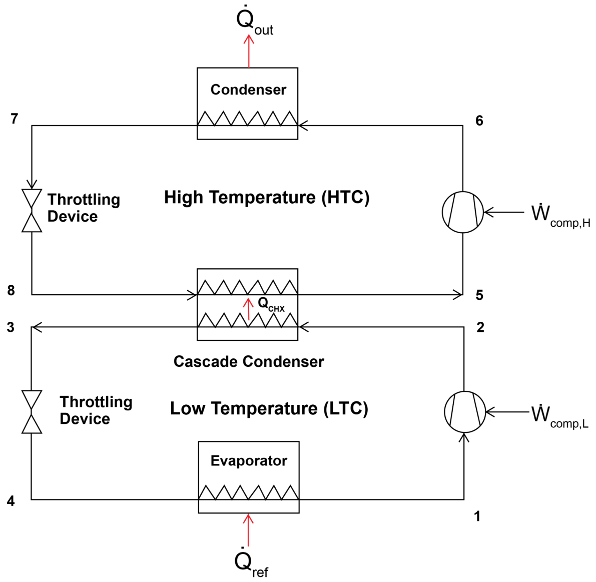


Figure 1: System Schematic [1]

Table 1: Process Description

Path	Process	Component
1 − 2	Roughly isentropic compression	Compressor (LTC)
2 − 3	Isobaric cooling	Cascaded Heat Exchanger (as LTC Condenser)
3 − 4	Isenthalpic expansion	Expansion Valve (LTC)
4 − 1	Isobaric heating (from cooling space)	Evaporator (LTC)
5 − 6	Roughly isentropic compression	Compressor (HTC)
6 − 7	Isobaric cooling (to the surroundings)	Condenser (HTC)
7 − 8	Isenthalpic expansion	Expansion Valve (HTC)
8 − 5	Isobaric heating	Cascaded Heat Exchanger (as HTC Evaporator)

2. Methodology

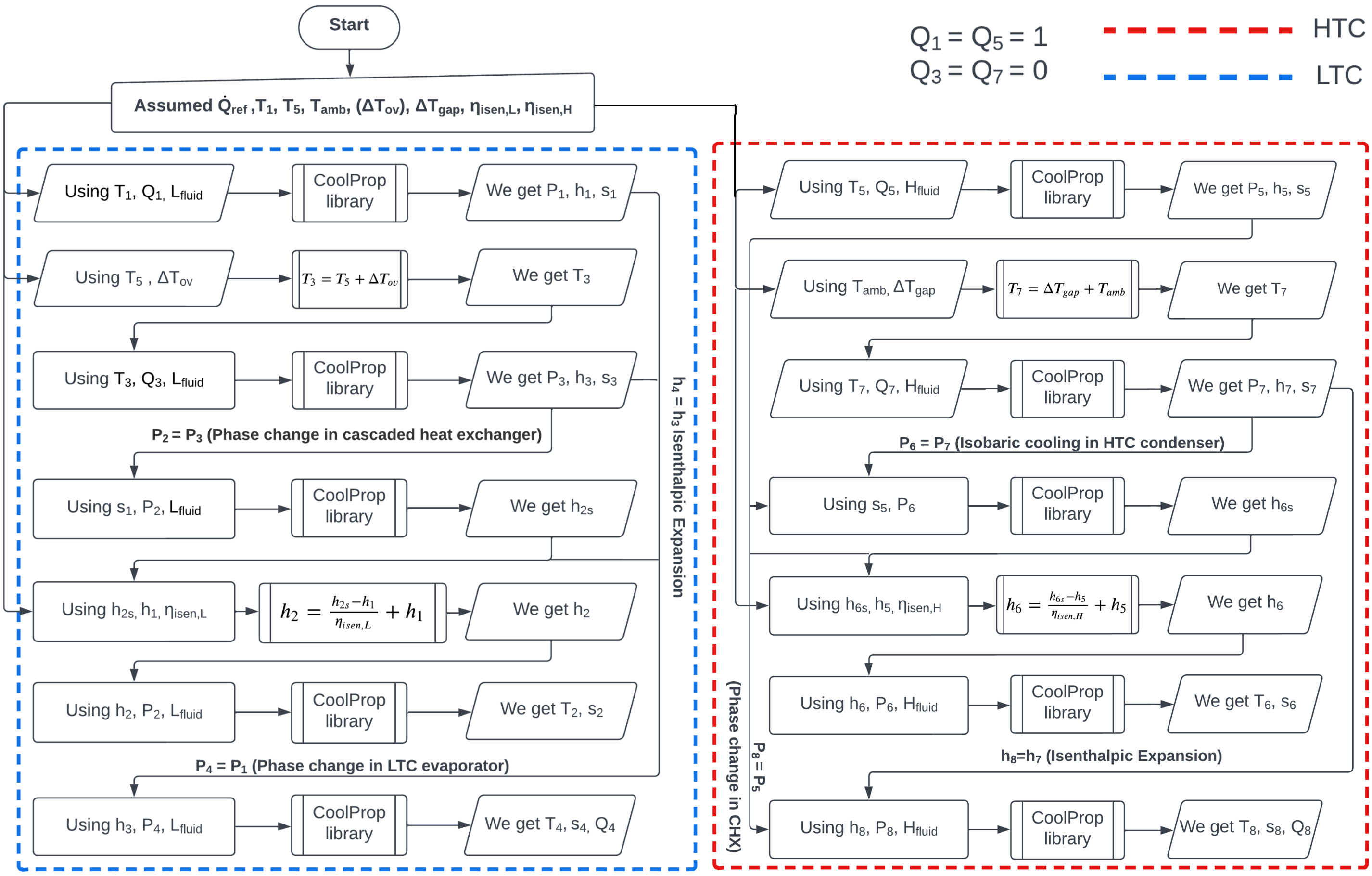
2.1 Assumptions[2]

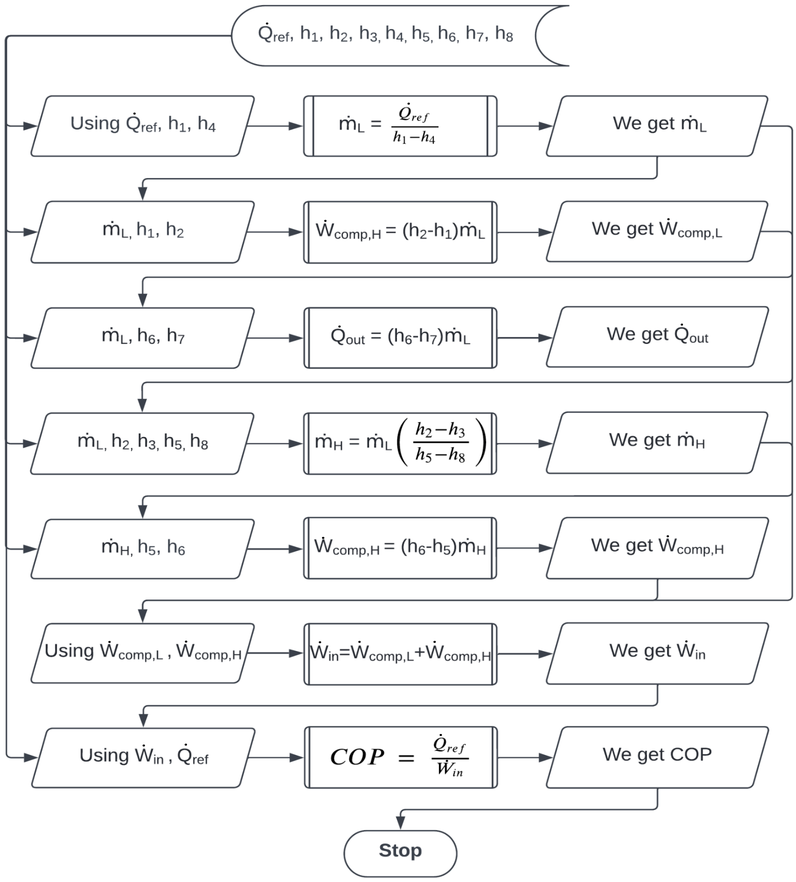
1. No leakage of refrigerants occurs in any of the pipes or components; both cycles are closed systems.
2. All 8 refrigerant streams are in a steady state and exhibit fully developed flow.
3. The pressure drops in all pipes have been neglected.
4. Heat transfer between the system and the surroundings occurs only in the HTC condenser and the LTC evaporator; i.e. all of the other components and all of the pipes are assumed to be insulated.
5. Streams 1 and 5 are assumed to be in a saturated vapour state; i.e. they are neither superheated nor partially condensed when they enter the compressor.
6. Streams 3 and 7 are assumed to be in a saturated liquid state; i.e. they are neither subcooled nor partially vaporised when they enter the expansion valve.
7. The expansion in the expansion valves is assumed to be isenthalpic.
8. The processes 4 − 1 and 8 − 5 are both phase changes at constant temperature and constant pressure.
9. The cascade heat exchanger completely transfers the heat released by the LTC refrigerant to the HTC refrigerant.

Table 2: Assumed Input Parameters

Parameter	Assumed value	Justification
$\dot{Q}_{ref}$	40 kW	Desired refrigeration effect
T_1	-65 ℃	Desired LTC evaporator stream temperature[3]
$\eta_{isen,L}$	0.6	Isentropic efficiency of the LTC compressor[4]
$\eta_{isen,H}$	0.75	Isentropic efficiency of the HTC compressor[4]
T_5	-33 ℃	Inlet temperature for the HTC compressor[5]
ΔT_{ov}	5 ℃	Minimum temperature difference in the CHX, i.e. T_3 - T_5[6]
T_{amb}	25 ℃	Approximate ambient temperature in Mumbai
ΔT_{gap}	10 ℃	Minimum temperature difference in the HTC condenser[5]

These assumed values are in accordance with existing literature on cascade refrigeration systems used in similar applications with similar refrigerants. For parameters relating to heat transfer, i.e. $\Delta T_{overlap}$ and ΔT_{gap}, it is assumed that their associated system components have suitable specifications and capabilities.





2.2 Compilation of all equations used:

(1) $T_3 = T_5 + \Delta T_{ov}$ **(2)** $P_2 = P_3$

(3) $h_2 = \frac{h_{2,s} - h_1}{\eta_{isen,L}} + h_1$ **(4)** $P_4 = P_1$

(5) $h_4 = h_3$ **(6)** $T_7 = T_{amb} + \Delta T_{gap}$

(7) $P_6 = P_7$ **(8)** $h_6 = \frac{h_{6,s} - h_5}{\eta_{isen,H}} + h_5$

(9) $P_8 = P_5$ **(10)** $h_8 = h_7$

(11) $\dot{m}_L = \frac{\dot{Q}_{ref}}{h_1 - h_4}$

(12) $\dot{W}_{comp,L} = (h_2 - h_1) \times \dot{m}_L$

(13) $\dot{Q}_{out} = (h_6 - h_7) \times \dot{m}_H$

(14) $\dot{Q}_{CHX} = \dot{m}_L(h_2 - h_3) = \dot{m}_H(h_5 - h_8)$

(15) $\dot{W}_{comp,H} = (h_6 - h_5) \times \dot{m}_H$

(16) $\dot{W}_{in} = \dot{W}_{comp,L} + \dot{W}_{comp,H}$

(17) $COP_{NET} = \frac{\dot{Q}_{ref}}{\dot{W}_{in}}$

Figure 2: Complete flowchart of the calculation algorithm

2.3 Step-by-step Calculations

All of the above equations and the flowchart in Figure 2 are in accordance with the model description (described process-by-process in Table 1) and our initial assumptions (described in Section 2.1). The step-by-step calculation procedure for all the unknown variables was performed as follows:

1. CoolProp was used to determine P_1, h_1, s_1 from T_1, Q_1.
2. T_3 was determined from T_5, ΔT_{ov} as per equation (1).
3. CoolProp was used to determine P_3, h_3, s_3 from T_3, Q_3.
4. P_2 was determined from P_3 as per equation (2).
5. CoolProp was used to determine h_{2s} from s_1, P_2.
   (enthalpy after perfectly isentropic compression)
6. h_2 was determined from h_{2s}, h_1, $\eta_{isen,L}$ as per equation (3).
7. CoolProp was used to determine T_2, s_2 from P_2, h_2.
8. P_4 was determined from P_1 as per equation (4).
9. h_4 was determined from h_3 as per equation (5).
10. CoolProp was used to determine s_4, T_4, Q_4 from h_4, P_4.
    This concludes the calculation of variables for the LTC.
11. CoolProp was used to determine P_5, h_5, s_5 from T_5, Q_5.
12. T_7 was determined from T_{amb}, ΔT_{gap} as per equation (6).
13. CoolProp was used to determine P_7, h_7, s_7 from T_7, Q_7.
14. P_6 was determined from P_7 as per equation (7).
15. CoolProp was used to determine h_{6s} from s_5, P_6.
    (enthalpy after perfectly isentropic compression)
16. h_6 was determined from h_{6s}, h_5, $\eta_{isen,H}$ as per equation (8).
17. CoolProp was used to determine T_6, s_6 from h_6, P_2.
18. P_8 was determined from P_5 as per equation (9).
19. h_8 was determined from h_7 as per equation (10).
20. CoolProp was used to determine T_8, s_8, Q_8, from h_8, P_8.
    This concludes the calculation of variables for the HTC.
21. All the performance and sizing parameters ($\dot{m}_L$, $\dot{W}_{comp,L}$, $\dot{Q}_{out}$, $\dot{m}_H$, $\dot{W}_{comp,H}$, $\dot{W}_{in}$, COP) were determined from $\dot{Q}_{ref}$, and h_1, h_2, … h_8; using the component equations, numbered (11) to (17).

2.4 Parametric Study: COP_{NET} vs. T_{amb}

As demonstrated in Figure 4, the performance of any refrigeration system depends on the ambient conditions at the location of installation (most importantly, on the ambient temperature). Thus, for a fixed value of T_{gap} = 10℃, a parametric study was conducted by varying T_{amb} from 15℃ to 35℃ (with a step temperature of 1℃). All the calculations were repeated accordingly. Finally, the corresponding values of COP_{NET} were graphically plotted against each of these values of T_{amb}.
In our cascade refrigeration cycle, a change in T_{amb} causes an equal change in T_7, as per equation (6).

2.5 Software Used

A. **Python (3.10.4 64-bit)**[7] was used in a multi-module program in order to conduct the modelling, simulation, and parametric study of the system. The entire code (alongwith the generated plots and a "README" file) has been made available here on Github. The following libraries were used in the code:
   a. CoolProp (6.4.1)[8] was used for acquiring thermodynamic data for all streams once their physical states had been determined analytically.
   b. matplotlib (3.5.2)[9] was used for generating the plot at the end of the parametric study.
   c. numpy (1.22.3)[10] was used for facilitating the variations of the parametric study via numpy arrays.
   d. pandas (1.4.3)[11] was used for data manipulation and restructuring (into DataFrame objects).

B. **Lucid**[12]was used to generate the flowchart in Figure 2.

C. **Engineering Equation Solver (Professional V9.478-3D)**[13] was used to generate Figure 3..

D. **Adobe Illustrator 2020**[14] was used to generate the system schematic in Figure 1, and to improve the readability of the process curves in Figure 3 and Figure 4.

3. Results and discussion

3.1 Observations based on the Refrigerant Streams

Table 3: Thermodynamic properties of all streams (and mass flow rates)

Stream	P (bar)	T (℃)	h (kJ/kg)	s (J/kgK)	Q	ṁ (kg/s)
1	2.4561	-65	330.594	1697.14	1	0.2287
2	10.7216	33	394.273	1784.65	–	0.2287
3	10.7216	-28	155.683	833.96	0	0.2287
4	2.4561	-65	155.683	856.22	0.2219	0.2287
5	0.5233	-33	360.843	1682.03	1	0.4819
6	6.6450	51	423.166	1731.27	–	0.4819
7	6.6450	35	247.629	1162.69	0	0.4819
8	0.5231	-33	247.629	1210.30	0.4438	0.4819

Assumed variables have been shaded light green. The rest have been calculated.

1. The highest pressure in each cycle occurs at its compressor outlet. While the HTC compressor has a higher compression ratio (12.7:1) than the LTC compressor (4.4:1), it has a lower inlet pressure (0.523 bar) than the LTC compressor (2.4561 bar). Both compressors should be chosen accordingly.
2. Maximum temperature in either of the cycles is obtained at the output of respective compressors (T_2 and T_6); this is because the large compressor work output Further T_6, T_7 are sufficiently above atmospheric which ensures well heat rejection from the condenser.
3. Also maximum enthalpy is obtained at the outlet of compressors because the input work is utilised to increase the enthalpy of refrigerants.
4. Entropy in either of the cycles is minimum at the out of condenser (T_3 and T_7) because refrigerants at these points are in saturated liquid, and entropy of a substance in the liquid phase is lower than that of vapour/mixture phase.
5. Quality of refrigerants in the LTC evaporator inlet is 0.2219 which is preferably good because the large portion of liquid in the mixture will absorb comparatively more CPU heat.

6. Required mass flow rates for LTC and HTC are 0.22 kg/s and 0.48 kg/s respectively, and the mass flow ratio of $\dot{m}_{HTC}/\dot{m}_{LTC} = 2.11$ which are fairly realistic[15].

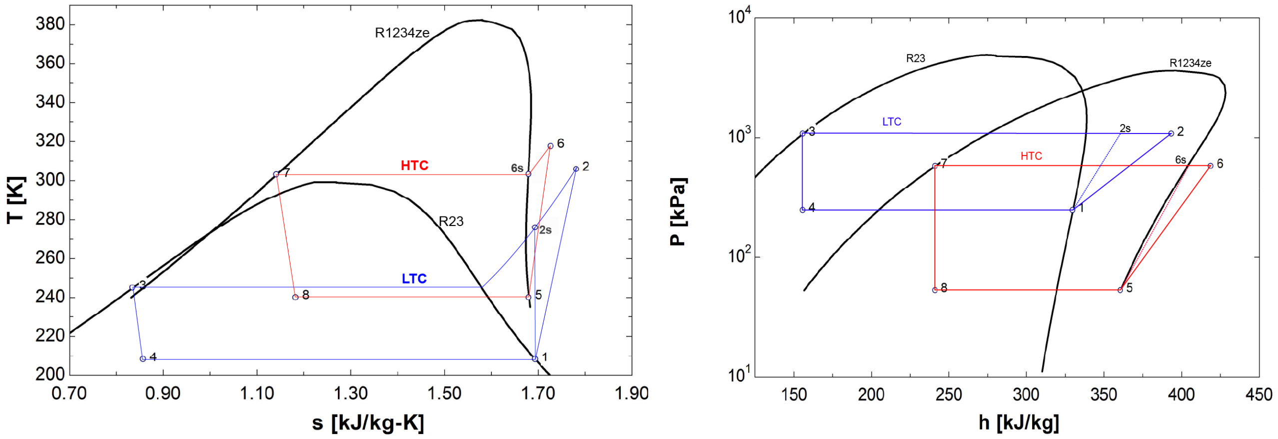


Figure 3: T-s and P-h curve (generated using EES)

3.2 Observations based on the Performance

Table 4: Table of results (performance and sizing parameters)

S. No.	Parameter	Values
1	LTC work input ($\dot{W}_{comp,L}$)	14.554 kW
2	HTC work input ($\dot{W}_{comp,H}$)	30.031 kW
3	Total work input ($\dot{W}_{in}$)	44.586 kW
4	Heat rejection ($\dot{Q}_{out}$)	84.586 kW
5	COP of LTC (COP_{LTC})	2.7483
6	COP of HTC (COP_{HTC})	1.8166
7	Overall COP (COP_{NET})	0.8971

1. As confirmed in Table 4, the entire system is in energy balance, i.e., $\dot{Q}_{ref} + \dot{W}_{comp,H} + \dot{W}_{comp,L} = \dot{Q}_{out}$.
2. One of the major advantages of the cascade refrigeration cycle is that over these operating temperatures (T_1 to T_6), it provides a higher refrigeration effect ($\dot{Q}_{ref}$) (and thus a higher COP_{NET} as per equation (17)) than a single vapour compression cycle within the same temperature range.
3. The obtained COP_{NET} was 0.897, similar to values observed for similar refrigerants in existing literature[6].
4. The high value of $\dot{Q}_{out}$ in the HTC condenser (for complete condensation of the HTC refrigerant) can be considered realistic because T_6=56℃ and T_7=35℃ are sufficiently above T_{amb}=25℃.

3.3 Observations based on the Parametric Study

Corresponding to the variation of T_{amb}, with 15℃ < T_{amb} < 35℃, the variation in T_7 was 25℃ < T_7 < 45℃. As a result, the variation in COP_{NET} was 1.050 > COP_{NET} > 0.758. This variation has been represented in Figure 4. This is indeed close to the variations observed in existing literature[5] for similar refrigerants (R507A-R23), which was as follows: 0.9274 > COP_{NET} > 0.5486 for 25℃ < T_7 < 50℃. In other words, as we increase T_{amb} over a typical range of ambient temperatures (or equivalently, as we increase T_7), we observe a decrease in COP_{NET} of our Cascade Refrigeration System. The reason for this is as follows: an increase in T_{amb} (or T_7) causes an increase in P_7 and P_6 (equation (7)), causing an increase in h_6, $W_{comp,H}$ and W_{in} (equations (15) and (16)), and finally causing a decrease in COP_{NET} (equation (17)). In other words, a higher/hotter T_{amb} requires a higher work input for the same refrigeration effect.

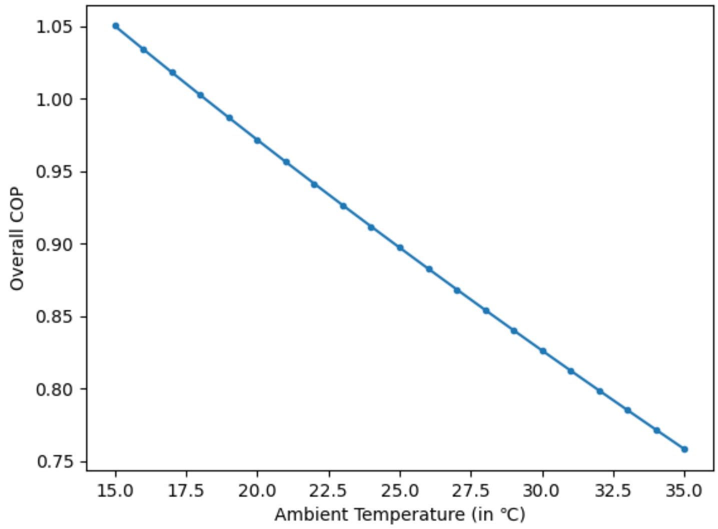


Figure 4: Parametric study with Ambient Temperature

4. Conclusion

Thus, the simulation demonstrates the feasibility of cascade vapour compression refrigeration systems, with respect to overall system performance, and also the availability of the required components. These results are in line with existing literature about this refrigeration system. For the chosen pair of refrigerants (R1234ze(E)-R23) and the chosen input parameters, a realistic COP_{NET} of 0.8971 was obtained. Moreover, after a parametric study, it was determined that COP_{NET} decreases with an increase in T_{amb} over typical ambient temperature ranges, as seen in Figure 4.

Nomenclature

Abbreviations

COP	Coefficient of performance
CHX	Cascade heat exchanger
CVCRS	Modelling and Simulation of the Cascaded Vapour Compression Refrigeration System
HTC	Higher temperature circuit
LTC	Lower temperature circuit

Symbols

h	Specific enthalpy (kJ/kg)
$\dot{m}$	Mass flow rate (kg/s)
P	Pressure (bar)
Q	Quality (unitless)
$\dot{Q}$	Rate of heat transfer (kW)
s	Specific entropy (kJ/kg K)
T	Temperature (℃)
$\dot{W}$	Work input of a compressor (kW)

Subscripts and superscripts

1, 2…	Stream numbers
amb	Ambient/surrounding conditions
C	Condenser
comp	Compressor
E	Evaporator
gap	Temperature difference between T_7 and T_{amb}
H	relating to the HTC

in	Energy coming into the system
isen	isentropic (for efficiency)
L	relating to the LTC
NET	Net/Overall/Total
out	Energy going out of the system

Greek symbols

Δ	Difference between 2 quantities
η	Efficiency (as a fraction between 0 and 1)

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