Thermodynamic Modelling, Simulation, and Techno-economic Analysis of a Cascaded Vapour Compression Refrigeration System

Department of Energy Science and Engineering, IIT Bombay
EN 317 Thermo-fluid Devices (Course Project)

Group 03

Atishay Singhai (20D170010), Rohit Gedam (20D170012), Shadab Ansari (20D170023), Pranav Limaye (20D170028)

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Abstract

In the 21st century, as the need for high-performance computers and data centres grows, the need for refrigeration systems that can achieve ultra-low evaporation temperatures is continuing to grow as well. The cascade vapour compression refrigeration system is such a system. The following paper provides a thermodynamic and techno-economic exploration of the cascade refrigeration system based on the R1234ze(E)-R23 pair of refrigerants, achieving an evaporation temperature of -65^{0}C. The thermodynamic model and a parametric study has been simulated using the Python library CoolProp, and key performance metrics have been reported. Detailed modelling has been attempted for the cascade heat exchanger. Commercial models of the system components have been proposed for purchase, and suitable cost functions have been applied (adjusted for the year 2022) to reach an estimate for their overall capital cost. All limitations have been reported, and suggestions have been made to potentially improve the system performance.

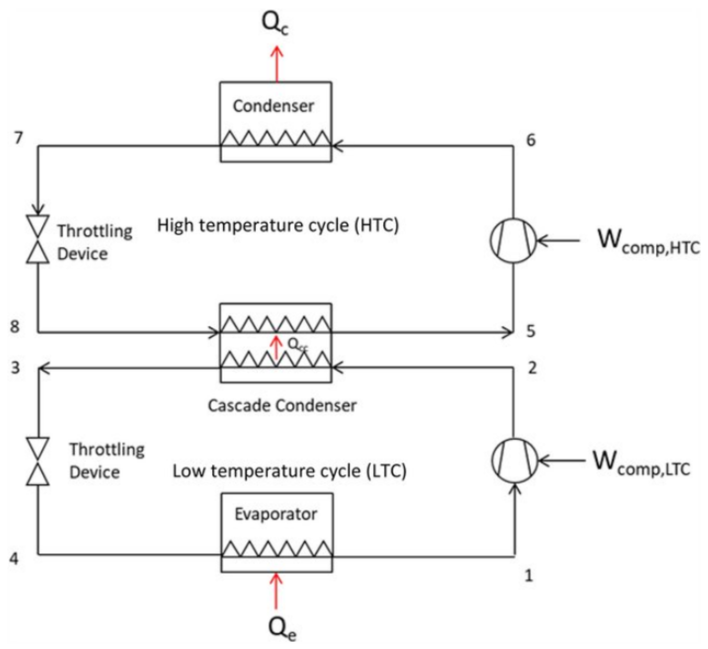
Keywords: cascade refrigeration system, CoolProp simulation, parametric study, heat exchanger modelling, techno-economic analysis

1 Introduction

Some industrial applications require very low temperatures, and the temperature range they involve may be too large for a single vapour compression refrigeration cycle to be practical. A large temperature range also means a large pressure range in the cycle and a poor performance for a reciprocating compressor. One way of overcoming this limitation is to perform the refrigeration process in stages, that is, to have two or more refrigeration cycles that operate in series. Such refrigeration cycles are called cascade refrigeration cycles, and thus one of the aims of the following paper is to explore the feasibility of a specific configuration with some well-defined simplifying assumptions.

The system (represented in Fig 1) employs two vapour compression refrigeration cycles (idealised, represented in Fig 2), with two different refrigerants, and operating within different temperature ranges. The two share a heat exchanger which acts as the condenser for the bottom cycle and the evaporator for the top cycle. In total, there are 8 different streams of refrigerant in the system. For continuous refrigeration to be possible, we ensure the following:

- stream 4 is colder than the refrigerated space; so that the LTC refrigerant can extract heat from the refrigerated space in the evaporator.
- streams 8 & 5 is colder than streams 2 & 3; so that heat transfer occurs from the LTC refrigerant to the HTC refrigerant in the heat exchanger.
- stream 6 is hotter than the surrounding atmosphere; so that the HTC refrigerant can expel its heat to the surroundings in the condenser.



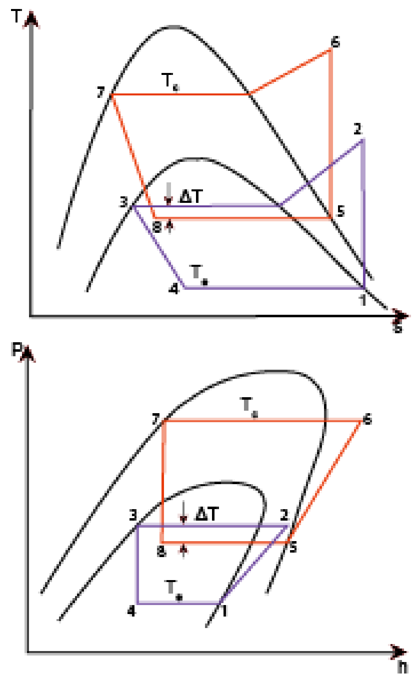


Fig 1: System Schematic [20] **Fig 2: Process curves**

(both figures generated using Adobe Illustrator, described in Section 2.7)

Streams	**1, 5:**	Saturated vapour	**Streams**	**2, 6:**	Superheated vapour
Streams	**3, 7:**	Saturated liquid	**Streams**	**4, 8:**	Low quality mixture

To begin, the behaviour of our system (defined in Section 2.1) has been formulated into equations (described in Section 2.2) and accordingly codified into a Python simulation (described in Section 2.3). An important parameter for the system performance is the ambient temperature at the installation site, and thus a parametric study has been performed accordingly (described in Section 2.3.4). The results obtained after the initial simulation have been utilised in Section 2.4, for the modelling of the CHX. Due to the presently niche nature of this refrigeration application, and the relative novelty of the chosen refrigerants, data corresponding to this system is sparse in the literature, and thermophysical correlations also produce significant errors. Regardless of this limitation, with a focus on the sizing methodology, suitable representative values for the heat transfer coefficients have been used to estimate the sizing parameters of the cascade heat exchanger. After considering all the system requirements, market models have been suggested for purchase (described in Section 2.5). Finally, using cost functions obtained from the literature and adjusted for the year 2022, the overall cost of all the components has been estimated in Section 2.6.

Thus, perhaps a more important goal of this paper is to lay down the general methodology for the modelling, simulation, and techno-economic analysis of cascade refrigeration systems.

All of the results and observations of the modelling, simulation, and analysis have been listed in Section 3.

2 Methodology

2.1 System Description

The exact processes involved in the system have been described in Table 1.

Table 1 : Process Description

#	Process	Component
1-2	Roughly Isentropic Compression	Compressor (LTC)
2-3	Isobaric Cooling	CHX (as LTC Condenser)
3-4	Isenthalpic Expansion	Throttle Valve (LTC)
4-1	Isobaric heating	Evaporator (LTC)
5-6	Roughly Isentropic Compression	Compressor (HTC)
6-7	Isobaric Cooling	Condenser (HTC)
7-8	Isenthalpic Expansion	Throttle Valve (HTC)
8-5	Isobaric Heating	CHX (as HTC evaporator)

The components required for such a system have been described in Table 2, and described in detail below.

Table 2 : Component Type

Component	Type
Compressors (x2)	Hermetic Sealed Reciprocating[1]
Condenser (HTC)	Forced Air-Cooled[1]
Evaporator (LTC)	Fin-type Microchannel[1]
Cascade HX	Brazed Plate, Counter flow[2]
Throttle Valves (x2)	Thermostatic[3]
LTC Refrigerant	R23[4]
HTC Refrigerant	R1234ze(E)[5]

1) **Hermetic Sealed Reciprocating Compressor**

The compressor and the motor operate on the same shaft. Both are enclosed in a common casing. The refrigerant itself is used to cool the motor before it enters the compressor cylinder. It is preferable because of its compact size, reduced vibrations, long life (due to protected enclosed structure), reduced maintenance costs (due to absence of a crankshaft seal), and smaller power requirement than an open compressor.

2) **Forced Air-cooled Condenser**

The fan forcefully blows air into the wind coils to improve heat removal. Its advantages include its simple operability, small size, low installation & maintenance costs, and ease of cleaning. Besides, as no water is used, all of its associated problems (such as corrosion) are prevented.

3) **Fin-type Microchannel Evaporator**

Fins are added to the bare tubes to increase its capability to collect heat from the refrigerated space and transfer it to the colder refrigerant inside the tubes. The same function is obtained at a reduced size, reduced costs, and reduced weight. Moreover, the fins prevent frosting that plagues regular microchannel evaporators.

4) **Brazed Plate Heat Exchanger**

Its stainless steel plates are brazed together by a layer of thermally conductive copper; benefitting both the evaporator and the condenser sides. Moreover, its small fluid passages induce turbulent flow; further improving heat transfer and reducing the required plate area. It's also preferable because of its compact size, ease of repair, and low costs.

5) **Thermostatic Expansion Valve**

These are preferred because of their ability to sense the temperature & pressure of the incoming fluid, and control the amount of fluid that enters the evaporator. This provides better overall performance than capillary-tube expansion valves, and prevents damage to the compressor due to flooding.

6) **Refrigerants**

R508b (LTC) and R134a (HTC) are widely used in vapour compression refrigerant systems. In this analysis, we are using their respective substitutes R23 (LTC) and R1234ze(E) (HTC), which have similar thermodynamic properties, but better GWP & better data availability on CoolProp (the thermodynamic data library described in Section 2.7).

2.2 Model Formulation

Table 3: Mass and Energy Balance equation

	Component	**Mass**	**Energy**
Higher Temperature Circuit (HTC)	Compressor	$\dot{m}_6 = \dot{m}_5$	$\dot{W}_H = \dot{m}_5(h_{6,s} - h_5)$
	Condenser	$\dot{m}_7 = \dot{m}_6$	$h_6 = \frac{h_{6,s} - h_5}{\eta_{isen,H}} + h_5$
	Expansion Valve	$\dot{m}_8 = \dot{m}_7$	$\dot{Q}_C = \dot{m}_7(h_{7,s} - h_6)$ & $h_8 = h_7$

	Cascade Heat Exchange	$\dot{m}_5 = \dot{m}_8$, $\dot{m}_3 = \dot{m}_2$	$\dot{Q}_{cas} = \dot{m}_5(h_5 - h_8) = \dot{m}_3(h_3 - h_2)$
Lower Temperature Circuit (LTC)	Compressor	$\dot{m}_2 = \dot{m}_1$	$\dot{W}_L = \dot{m}_1(h_{2,s} - h_1)$ & $h_2 = \frac{h_{2,s} - h_1}{\eta_{isen,L}} + h_1$
	Expansion Valve	$\dot{m}_3 = \dot{m}_4$	$h_3 = h_4$
	Evaporator	$\dot{m}_4 = \dot{m}_1$	$\dot{Q}_E = \dot{m}_1(h_1 - h_4)$

2.3 Thermodynamic Simulation

2.3.1 Assumptions

1. No leakage of refrigerants occurs in any of the pipes or components; both cycles are closed systems.
2. All 8 streams are in steady state and exhibit fully developed flow.
3. The pressure drops in all pipes have been neglected.
4. Heat transfer between the system and the surroundings occurs only in the HTC condenser and the LTC evaporator; i.e. all of the other components and all of the pipes are assumed to be insulated.
5. Streams 1 and 5 are assumed to be in a saturated vapour state; i.e. they are neither superheated nor partially condensed when they enter the compressor.
6. Streams 3 and 7 are assumed to be in a saturated liquid state; i.e. they are neither subcooled nor partially vaporised when they enter the expansion valve.
7. The expansion in the expansion valves is assumed to be isenthalpic.
8. The processes $4 - 1$ and $8 - 5$ are both phase changes at constant temperature and constant pressure.
9. The cascade heat exchanger completely transfers the heat released by the LTC refrigerant to the HTC refrigerant.

Table 4: Assumed Input Parameters

Parameter	Assumed value	Justification
$\dot{Q}_{ref}$	40 kW	Desired refrigeration effect
T_1	-65 ℃	Desired LTC evaporator stream temperature[1]
$\eta_{isen,L}$	0.6	Isentropic efficiency of the LTC compressor[6]
$\eta_{isen,H}$	0.75	Isentropic efficiency of the HTC compressor[6]
T_5	-33 ℃	Inlet temperature for the HTC compressor[7]

ΔT_{ov}	5 ℃	Minimum temperature difference in the CHX, i.e. T3 - T5[8] (the "overlap" between the cycles)
T_{amb}	25 ℃	Approximate ambient temperature in Mumbai
ΔT_{gap}	10 ℃	Minimum temperature difference in the HTC condenser[7]

These assumed values are in accordance with existing literature on cascade refrigeration systems used in similar applications with similar refrigerants. For parameters relating to heat transfer, i.e. ΔT_{ov} and ΔT_{gap}, it is assumed that their associated system components have suitable specifications and capabilities.

2.3.2 Equations used

Following is a compilation of all equations used:

(1) $T_3 = T_5 + \Delta T_{ov}$

(2) $P_2 = P_3$

(3) $h_2 = \frac{h_{2,s} - h_1}{\eta_{isen,L}} + h_1$

(4) $P_4 = P_1$

(5) $h_4 = h_3$

(6) $T_7 = T_{amb} + \Delta T_{gap}$

(7) $P_6 = P_7$

(8) $h_6 = \frac{h_{6,s} - h_5}{\eta_{isen,H}} + h_5$

(9) $P_8 = P_5$

(10) $h_8 = h_7$

(11) $\dot{m}_L = \frac{\dot{Q}_{ref}}{h_1 - h_4}$

(12) $\dot{W}_{comp,L} = (h_2 - h_1) \times \dot{m}_L$

(13) $\dot{Q}_{out} = (h_6 - h_7) \times \dot{m}_H$

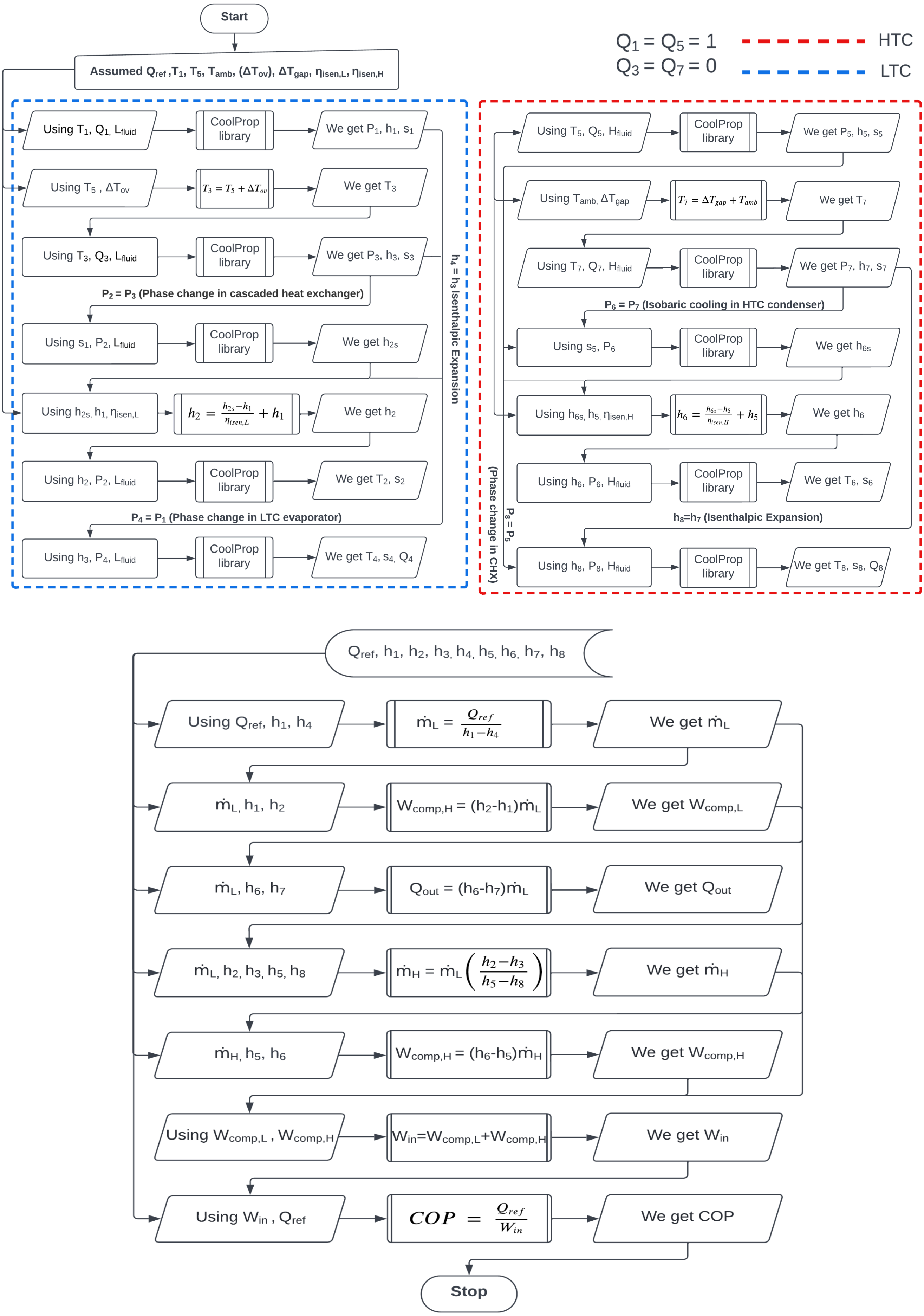
(14) $\dot{Q}_{CHX} = \dot{m}_L(h_2 - h_3) = \dot{m}_H(h_5 - h_8)$

(15) $\dot{W}_{comp,H} = (h_6 - h_5) \times \dot{m}_H$

(16) $\dot{W}_{in} = \dot{W}_{comp,L} + \dot{W}_{comp,H}$

(17) $COP_{NET} = \frac{\dot{Q}_{ref}}{\dot{W}_{in}}$

Fig 3: Complete flowchart of the calculation algorithm



2.3.3 Step-by-step Calculation

All of the above equations and the flowchart in Fig 3 are in accordance with the model description (described process-by-process in Table 1) and our initial assumptions (described in Section 2.1). The step-by-step calculation procedure for all the unknown variables was performed as follows:

1. CoolProp was used to determine P_1, h_1, s_1 from T_1, Q_1.
2. T_3 was determined from T_5, ΔT_{ov} as per equation (1).
3. CoolProp was used to determine P_3, h_3, s_3 from T_3, Q_3.
4. P_2 was determined from P_3 as per equation (2).
5. CoolProp was used to determine h_{2s} from s_1, P_2.

(enthalpy after perfectly isentropic compression)

6. h_2 was determined from h_{2s}, h1, ηisen,L as per equation (3).
7. CoolProp was used to determine T2, s2 from P2, h2.
8. P4 was determined from P1 as per equation (4).
9. h4 was determined from h3 as per equation (5).
10. CoolProp was used to determine s4, T4, Q4 from h4, P4.

This concludes the calculation of variables for the LTC.

11. CoolProp was used to determine P5, h5, s5 from T5, Q5.
12. T7 was determined from Tamb, ΔTgap as per equation (6).
13. CoolProp was used to determine P7, h7, s7 from T7, Q7.
14. P6 was determined from P7 as per equation (7).
15. CoolProp was used to determine h6s from s5, P6.

(enthalpy after perfectly isentropic compression)

16. h6 was determined from h6s, h5, ηisen,H as per equation (8).
17. CoolProp was used to determine T6, s6 from h6, P2.
18. P8 was determined from P5 as per equation (9).
19. h8 was determined from h7 as per equation (10).
20. CoolProp was used to determine T8, s8, Q8, from h8, P8.

This concludes the calculation of variables for the HTC.

All the performance and sizing parameters ($\dot{m}_L$, $\dot{W}_{comp,L}$, $\dot{Q}_{out}$, $\dot{m}_H$, $\dot{W}_{comp,H}$, $\dot{W}_{in}$, COPs) were determined from $\dot{Q}_{ref}$, and h_1, h_2, … h_8; using the component equations (11) to (17).

2.3.4 Parametric Study

As demonstrated in Fig 6, the performance of any refrigeration system depends on the ambient conditions at the location of installation (most importantly, on the ambient temperature). Thus, for a fixed value of T_{gap} = 10℃, a parametric study was conducted

by varying T_{amb} from 15℃ to 35℃ (with a step temperature of 1℃). All the calculations were repeated accordingly. Finally, the corresponding values of COP_{NET} were graphically plotted against each of these values of T_{amb}.
In our cascade refrigeration cycle, a change in T_{amb} causes an equal change in T_7, as per equation (6).

2.4 Sizing of the Cascade Heat Exchanger

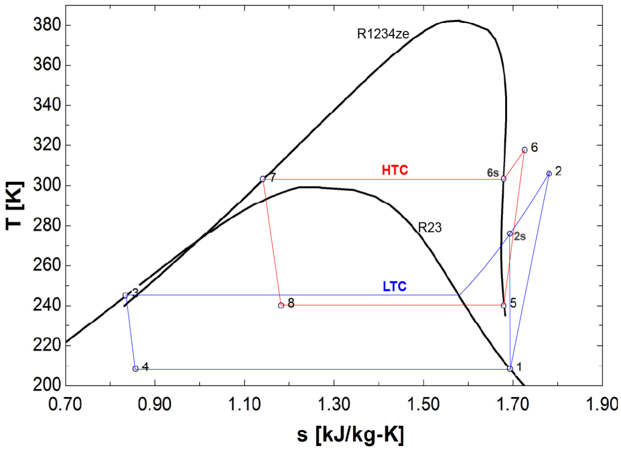


Fig 4(a) T-s curve (using EES)

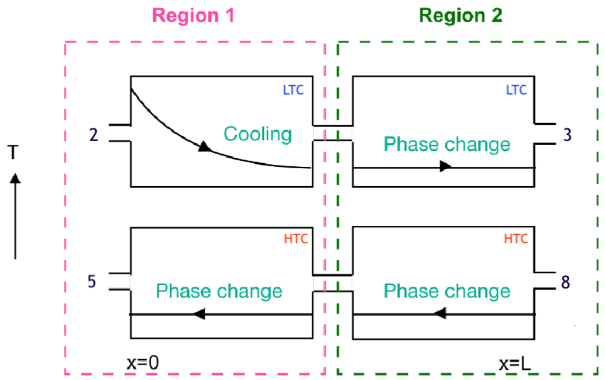


Fig 4(b) Temperature profile in CHX

From the T-s curve (seen in **Fig 4(a)**), the CHX (of counter-flow type) can be sensibly divided into two discrete regions:

(i) Isobaric cooling (of pure gas) in LTC, phase change (partial evaporation) in HTC

(ii) Phase change (full condensation) in LTC, phase change (full evaporation) in HTC

Using energy balance for Region 1, the quality at its endpoint on the HTC side ($Q_{R1H,o}$) can be determined.

2.4.1 Assumptions

1. To calculate the Nusselt number in LTC Region 1, the refrigerant temperature was assumed to be the average of the temperatures of Region 1 endpoints.
2. To calculate heat transfer coefficients on the HTC side in Region1 and both sides in Region 2, the qualities of refrigerants were assumed to be the averages of the qualities of respective Region endpoints.
3. The thermal resistance of the bulk material of all the heat exchangers (between their respective pairs of fluids), including their piping, were assumed to be negligible. Fouling resistances were also neglected.
4. The system was assumed to be perfectly thermally insulated from the environment except at the HTC condenser (which rejects heat to the ambient atmosphere) and at the LTC evaporator (which extracts heat from the refrigerated space).
5. In the brazed plate heat exchanger, the hydraulic diameter (meaningful distance between plates) and chevron angle for the plate were assumed to be 7.5 mm and 45° respectively.

2.4.2 Step-by-step Calculation

The correlations used for determining heat transfer coefficients in any thermodynamic system are heavily dependent on the system conditions, the types of processes involved, the materials and fluids being used, and the assumptions made during calculations. Heat transfer Coefficient vs vapour quality, obtained from reference paper[9], is used to determine heat transfer coefficient at the middle (quality = 0.5) of Region 2 for both HTC and LTC. Similarly using the same graph, heat transfer coefficient for middle (quality = 0.78) of Region 1 of LTC side is obtained.

$$(18)\quad Nu = \phi \cdot 0.122 \cdot Pr^{1/3} \cdot \left(\frac{\mu}{\mu_w}\right)^{1/6} \cdot \left[\xi \cdot \left(\frac{Re}{\phi}\right)^2 \cdot sin(2 \cdot \varphi)\right]^{0.374}$$

In this equation, ξ is the Moody friction factor, calculated as

$$(19)\frac{1}{\sqrt{\xi}} = \frac{cos\varphi}{\sqrt{0.18tan\varphi+0.36sin\varphi+\xi_0/cos\varphi}} + \frac{1-cos\varphi}{\sqrt{3.8\cdot\xi_{1.0}}}$$

where,

$$(20)\frac{Re}{\phi} \geq 2000 \Rightarrow \{\ \xi_0 = \left(1.8 \cdot log_{10}\frac{Re}{\phi} - 1.5\right)^{-2},\ \xi_{1.0} = \frac{39}{\left(\frac{Re}{\phi}\right)^{0.289}}\}.$$

Nusselt number correlation[10] is used to obtain the Nusselt number as a function of required flow-specific properties in Region 1 of HTC, which were determined using the “CoolProp” Python library and some analytical calculations. Next, U was determined for both regions, followed by LMTD, and the required area.

<u>Note</u>:

Here, $\dot{Q}$ represents the heat transfer rate in the heat exchanger, and Q represents the quality of refrigerant. The values for all the ρ, μ, K, σ, and C_p were determined using the CoolProp library, at a particular state. As h represents specific enthalpy, α has been chosen to represent the convective heat transfer coefficient (to avoid confusion). All other symbols and notations are the same as mentioned in the nomenclature.

Following is a compilation of all the equations used for the heat exchanger modelling:

(21) $\dot{m} = \rho A v$

(22) $R_e = \frac{\rho v D}{\mu}$

(23) $P_r = \frac{\mu C_p}{k}$

(24) $Nu = \frac{\alpha D}{k}$

(25) $A_{net} = A_{R1} + A_{R2}$

(26) $\frac{1}{U} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2}$

(27) $\dot{Q} = \dot{m} \cdot (h_o - h_i)$

(28) $\dot{Q} = U \cdot A \cdot LMTD$

(29) $LMTD = \frac{(\Delta T_1 - \Delta T_2)}{ln(\frac{\Delta T_1}{\Delta T_2})}$; where, $\Delta T_1 = T_{h,i} - T_{c,o}$ and $\Delta T_2 = T_{h,o} - T_{c,i}$

(30) $Q_{mixed\ region} = \frac{Q_{endpoint\ 1} + Q_{endpoint\ 2}}{2}$

2.5 Selection of Components from Existing Models

Table 5: Chosen Components

Components	Specific Model Name
Brazed Plate Heat Exchanger (CHX)	Danfoss, BPHE B3-052[11]
Evaporator	Danfoss, H62(L)-EZU[12]
Condenser	Danfoss, H118(L)-C[12]
Throttle Valve LTC	Danfoss, TGE series[13]
Throttle Valve HTC	Danfoss, TGE series[13]
Compressor LTC	J & Hall International HSS series[14]
Compressor HTC	J & Hall International HSS series[14]

This particular heat exchanger model was chosen on the basis of its type, number of plates, and exchanger area. The evaporator and condenser models were also chosen on the basis of their type and their exchanger area. The throttle valve model was chosen on the basis of its inlet and outlet pressures. The compressor model was chosen on the basis of its capacity and rated evaporator temperature. However, the suggested refrigerants for the throttle valve and the compressor aren't the ones being used in this cascade refrigeration system, so certain adjustments might have to be made to achieve the same performance and reliability.

2.6 Economic Analysis

Table 6: Cost Function for different Components

Components	**Cost function in ₹ (C_k), 2022**
Brazed Plate Heat Exchanger (CHX)	$56380.5 \times A_{BPHE}^{0.8}$ [15]
Evaporator	$131874 \times A_{evap}^{0.89}$ [16]
Condenser	$131874 \times A_{cond}^{0.89}$ [16]
Compressor	$959791.6 \times W^{0.46}$ [16]
Throttle Valve	$10808.6 \times \dot{m}$ [16]

*Conversion rates used (as of 21:00, October 15, 2022): $1 = ₹82.42 & €1 = ₹80.12

For simplicity, the specific heat flux for the LTC evaporator was assumed to be 10^6 W/m^2, as per the application of electronics cooling[17]. This is in accordance with the assumptions made previously. The price of connecting pipes was neglected in comparison to the prices of the other cutting-edge components. To account for inflation, cost indexes were used for the original year and the current year. $\textit{Indexed Cost of Acquisition} = \textit{Original Cost of Acquisition} \times \frac{\textit{CII of the Given Year}}{\textit{CII of the Base Year}}$, the cost functions listed in Table 3 have already been adjusted for the current year (2022).

2.7 Software Used

A. **Python (3.10.4 64-bit)**[18] was used in a multi-module program in order to conduct the modelling, simulation, parametric study, and technoeconomic analysis of the system. The entire code (alongwith the generated plots and a "README" file) has been made available here on GitHub (https://github.com/syzfs/cascade-ref-simulation), and also in the Appendix. The following libraries were used in the code:
   a CoolProp (6.4.1)[19] was used for acquiring thermodynamic data for all streams once their physical states had been determined analytically.
   b matplotlib (3.5.2)[20] was used for generating the plot at the end of the parametric study.
   c numpy (1.22.3)[21] was used for facilitating the variations of the parametric study via numpy arrays.
   d pandas (1.4.3)[22] was used for data manipulation and restructuring (into DataFrame objects).

B. **Lucid**[23]was used to generate the flowchart in Fig 3.

C. **Engineering Equation Solver (Professional V9.478-3D)**[24] was used to generate Fig 4(a).

D. **Adobe Illustrator 2020**[25] was used to generate the system schematic in Fig 4(b), and to improve the readability of the process curves in Fig 4(a) and Fig 4(b).

3 Results and Observations

3.1 Thermodynamic Simulation

3.3.1 Results based on the Refrigerant Streams

Table 7: Thermodynamic/Physical properties of all streams

Stream	P (bar)	T (℃)	h (kJ/kg)	s (J/kgK)	Q	ṁ (kg/s)
1	2.4561	-65	330.594	1697.14	1	0.2287
2	10.7216	33	394.273	1784.65	–	0.2287
3	10.7216	-28	155.683	833.96	0	0.2287
4	2.4561	-65	155.683	856.22	0.2219	0.2287
5	0.5233	-33	360.843	1682.03	1	0.4819
6	6.6450	51	423.166	1731.27	–	0.4819

7	6.6450	35	247.629	1162.69	0	0.4819
8	0.5231	-33	247.629	1210.30	0.4438	0.4819

The assumed variables have been shaded light green. The rest were calculated.

1. The highest pressure in each cycle occurs at its compressor outlet. While the HTC compressor has a higher compression ratio (12.7:1) than the LTC compressor (4.4:1), it has a lower inlet pressure (0.523 bar) than the LTC compressor (2.4561 bar). Both compressors should be chosen accordingly.
2. Maximum temperature in either of the cycles is obtained at the output of respective compressors (T_2 and T_6). More importantly, T_6=51℃ and T_7=35℃ are sufficiently above ambient temperature which ensures the feasibility of the heat rejection from the condenser.
3. Also maximum enthalpy is obtained at the outlet of compressors because the input work superheats the refrigerant by a lot.
4. Entropy in either cycle is minimum at the condenser outlets (T_3 and T_7). This makes sense because these are in saturated liquid state, which tends to be much lower in entropy.
5. Quality of refrigerants at the LTC evaporator inlet is low (0.2219). So, the refrigerant will be able to absorb a lot of heat from the refrigerated space for complete evaporation.
6. Required mass flow rates for LTC and HTC are 0.22 kg/s and 0.48 kg/s respectively, with the mass flow ratio being $\dot{m}_{HTC}/\dot{m}_{LTC}$ = 2.11. These values are fairly realistic[15].

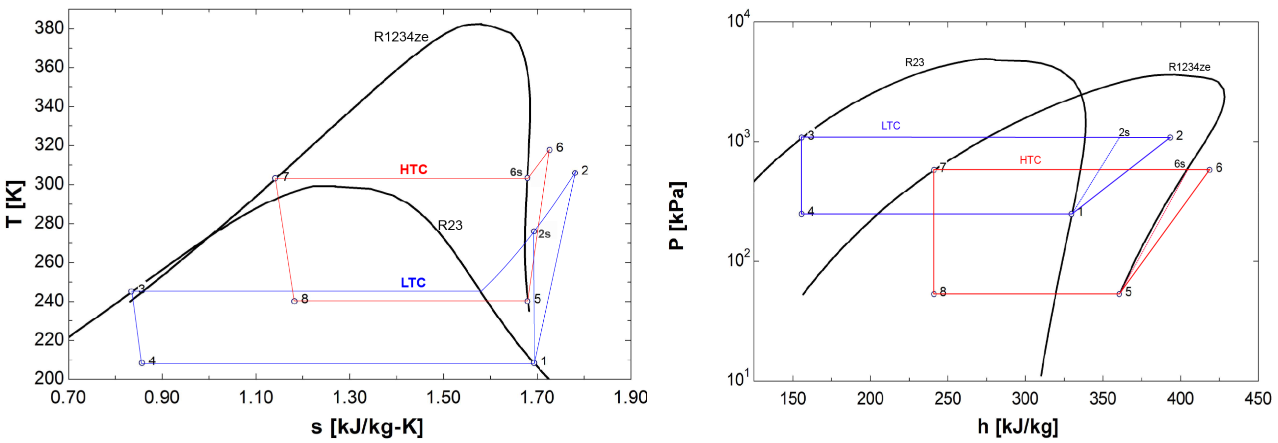


Fig 5: T-s and P-h curve (generated using EES)

3.3.2 Observations based on the Performance

Table 8: Table of results (performance and sizing parameters)

S. No.	Parameter	Values
1	LTC work input ($\dot{W}_{comp,L}$)	14.554 kW

2	HTC work input ($\dot{W}_{comp,H}$)	30.031 kW
3	Total work input ($\dot{W}_{in}$)	44.586 kW
4	Heat rejection ($\dot{Q}_{out}$)	84.586 kW
5	COP of LTC (COP_{LTC})	2.7483
6	COP of HTC (COP_{HTC})	1.8166
7	Overall COP (COP_{NET})	0.8971

1. As confirmed in Table 8, the entire system is in energy balance, i.e., $\dot{Q}_{ref} + \dot{W}_{comp,H} + \dot{W}_{comp,L} = \dot{Q}_{out}$.
2. One of the major advantages of the cascade refrigeration cycle is that over these operating temperatures (T_1 to T_6), it provides a higher refrigeration effect ($\dot{Q}_{ref}$) (and thus a higher COP_{NET} as per equation (17)) than a single vapour compression cycle within the same temperature range.
3. The obtained COP_{NET} was 0.897, similar to values observed for similar refrigerants in existing literature[6].
4. The high value of $\dot{Q}_{out}$ in the HTC condenser (for complete condensation of the HTC refrigerant) can be considered achievable because T_6=56℃ and T_7=35℃ are sufficiently above T_{amb}=25℃.

3.2 Parametric Study

Corresponding to the variation of T_{amb}, with 15℃ < T_{amb} < 35℃, the variation in T_7 was 25℃ < T_7 < 45℃. As a result, the variation in COP_{NET} was 1.050 > COP_{NET} > 0.758. This variation has been represented in Fig 6. This is indeed close to the variations observed in existing literature[7] for similar refrigerants (R507A-R23), which was as follows: 0.9274 > COP_{NET} > 0.5486 for 25℃ < T_7 < 50℃. In other words, as we increase T_{amb} over a typical range of ambient temperatures (or equivalently, as we increase T_7), we observe a decrease in COP_{NET} of our Cascade Refrigeration System. The reason for this is as follows: an increase in T_{amb} (or T_7) causes an increase in P_7 and P_6 (equation (7)), causing an increase in h_6, $W_{comp,H}$ and W_{in} (equations (15) and (16)), in turn causing a decrease in COP_{NET} (equation (17)). In other words, a higher/hotter T_{amb} requires a higher work input for the same refrigeration effect.

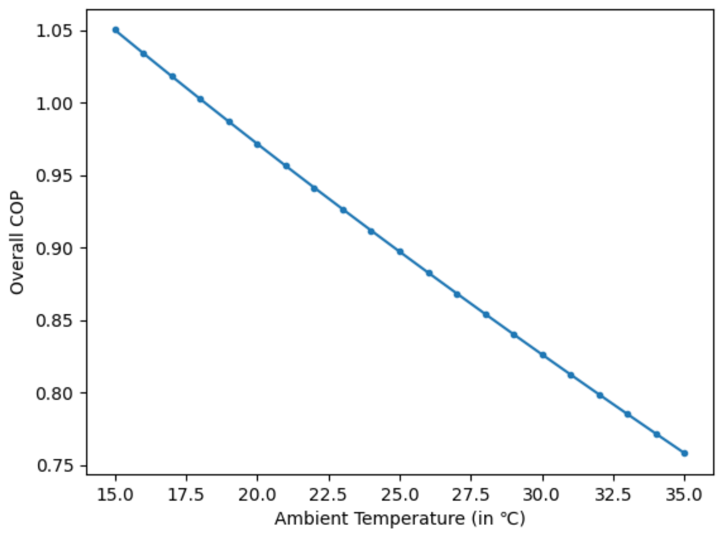


Fig 6: Parametric study with Ambient Temperature

3.3 Sizing of the Cascade Heat Exchanger Results

Continuing with Section 2.4, following are results obtained for Sizing of the Cascade Heat Exchanger.

Table 9: Table of results for the cascaded heat exchanger

Parameters	**Region 1**	**Region 2**
Rate of heat transfer (kW)	12.392	42.161
Overall heat transfer coefficient ($kW/m^2 K$)	0.942	3.125
Logarithmic mean temperature difference (K)	23.767	5
Area (m^2)	0.554	2.698

Thus, total area of the CHX = **3.252 m^2**, for a total heat transfer rate = **54.55 kW**

3.4 Economic Analysis Of Overall System

Table 10: Results of system cost analysis

Component	**Cost in ₹ (2022)**
Brazed Plate Heat Exchanger (CHX)	188724.69
Evaporator (LTC)	338718.95
Condenser (HTC)	1317636.67
Compressor for LTC	3767773.43
Compressor for HTC	5257644.25
Throttle Valve for LTC	2831.01

Throttle Valve for HTC	5965.24

Total Capital Cost = **₹ 1,08,79,294.23 / ₹ 1.09 Crores**

3.5 Comparison with State-of-the-art Setups

This is a report on Cascade Vapour Compression Refrigeration System (CVCRS) for High Performance Computer Cooling, which could be utilised to provide a temperature of -65°C for high performance cooling in data centres. It becomes difficult to manage the high-end CPUs heat dissipation with conventional cooling, so this study deals with the Component modelling, economic analysis and simulation of CVCRS. Our T_E = -65°C is in the range of temperature generally provided by CVCRS[26]. A 40 kW cooling effect is obtained for 10 chips of 8 cm × 5 cm dimension, with heat dissipation rate of 100 W/cm^2, which is in the same order as that of current literature of 10 kW[27]. Our total power input was 44.586 kW, and after doing economic analysis, price of CVCRS came out to be **₹ 1.08 Crores**, which is pretty close to market value of 90 Lakh (obtained after phone call from manufacturer[28]), for 40kW cooling capacity, T_E = -65°C, with specified refrigerant as R22, R404A, R134A which is similar to our chosen refrigerant. Obtained COP of 0.8971, which is in range with already available CVCRS[29], makes it feasible for usage.

4. Limitations

- In a practical setup, the odd-numbered streams wouldn't exactly be saturated vapours/liquids as has been assumed. In order to protect the components from unexpected damage, some subcooling is performed on the liquid streams and some superheating is performed on the vapour streams.
  Moreover, the assumption that the system is perfectly insulated wherever needed (i.e. that it does not have any undesirable heat interaction with the surroundings) is not completely realistic.
- The chosen refrigerants are not used very commonly, especially R1234ze(E) and especially with each other. Nor is this a very commonly studied temperature range for the CHX in cascade systems. Moreover, thermophysical correlations consistently tend to show high deviations from experimental results. As a result, the values selected for the heat transfer coefficients in the mixed phase refrigerants in both regions of the CHX are only representative, and obtained from this paper-[9] at the respective qualities.
- Detailed modelling has only been performed for the CHX, and not for the LTC evaporator or the HTC condenser. The areas being used as input for their cost functions have thus been chosen as those of the market models proposed for purchase in Section 2.5.
- The cost of the piping, the charge of the refrigerants, the insulation, and the installation costs have not been computed because these could greatly vary with time, location, etc.

5. Improvements/Suggestions

The performance of a refrigeration system is evaluated in terms of COP (the ratio between refrigeration effect and the net work input). The COP of a vapour compression refrigeration system can be improved either by increasing the refrigeration effect or by reducing work input given to the system.

1. As per a parametric study performed against ambient temperature, depicted in Fig 6 of this report, the overall COP can be improved by decreasing the ambient temperature near the HTC condenser.
    a. This can be ensured by using a fan to facilitate forced convection of any heated air away from the condenser, drawing cooler air back in.
    b. Moreover, the condenser could be placed away from any direct and diffused sunlight.
    c. If possible, the project site could be shifted to a place with a naturally colder climate.
2. The throttling process in vapour compression refrigeration is an irreversible expansion process; one of the main factors responsible for exergy loss in the cycle. It occurs due to the production of flash vapours due to some practically spontaneous boiling. This reduces the cooling capacity of the refrigerant which must be counteracted with a larger evaporator size. This problem can be eliminated by adopting multi-stage expansion (with the removal of flash-vapours after each stage of expansion)[30].
3. The isentropic efficiency of the compressors and the evaporator temperature of the upper cycle have a considerable impact on the performance of the overall system. Thus, the possibility of using more isentropically efficient compressors should be considered.
4. Adjustments may be considered at the project site which allows the cooling temperature near the evaporator to be increased from -65 ℃ to something more easily achievable. This will not only reduce the desired refrigeration effect but also reduce the required work input (in fact, depending on the project constraints, this might be more preferable even at a lower system COP).

6) Conclusion

First of all, we designed a cascade vapour compression refrigeration system, chosen for the purpose of cooling high-performance computer processors. Then simulation using Python was carried out which demonstrated the feasibility of cascade vapour compression refrigeration systems, with respect to overall system performance, and also the availability of the required components.

For the chosen pair of refrigerants (R1234ze(E)-R23) and the chosen input parameters, a realistic COP_{NET} of 0.8971 was obtained. Moreover, after a parametric

study, it was determined that COP_{NET} decreases with an increase in T_{amb} over typical ambient temperature ranges, as seen in Fig 6. This analysis has significantly built upon the analysis which was described in the earlier report. Moreover, components roughly matching the requirements of this system have been proposed for purchase.
And at last, quantification for the technical and economic performance of a cascaded vapour compression refrigeration system of the proposed configuration was done, under certain feasible assumptions, and using cost functions which were developed by previous researchers. Required area of the cascaded heat exchanger was found to be 4.89 m^2 . These results are in line with existing literature about this refrigeration system.

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Conflict of Interest

None.

Nomenclature

Abbreviations

COP	Coefficient of performance
CHX	Cascade heat exchanger
CVCRS	Modelling and Simulation of the Cascaded Vapour Compression Refrigeration System
HTC	Higher temperature circuit
LTC	Lower temperature circuit
CP2	Course Project Part 2
CII	Cost of Inflation Index

Symbols

h	Specific enthalpy (kJ/kg)
$\dot{m}$	Mass flow rate (kg/s)
P	Pressure (bar)
Q	Quality (unitless)
$\dot{Q}$	Rate of heat transfer (kW)
s	Specific entropy (kJ/kg K)
T	Temperature (℃)
$\dot{W}$	Work input of a compressor (kW)
d	pipe diameter (m)
φ	Chevron Angle (degrees)
ϕ	Area Enlargement Factor
ξ	Moody Friction Factor

ξ_0	Intermediate Constants
$\xi_{1.0}$	Intermediate Constants
R_e	Reynolds Number (unitless)
P_r	Prandtl Number (unitless)
N_u	Nusselt Number (unitless)
α	convective heat transfer coefficient (W/(m^2 K))
k	Thermal conductivity (W/(m·K))
C_p	Specific heat capacity (J/(kg °C))
A	Area(m^2)
U	Overall Heat Transfer Coefficient (W/(m^2°C))
LMTD	Logarithmic mean temperature difference (ºC)

Greek symbols

ρ	density (kg/m^3)
σ	Surface Tension (N/m)
μ	Dynamic Viscosity (Ns/m^2)
v	velocity (m/s)
Δ	Difference between 2 quantities
η	Efficiency (as a fraction between 0 and 1)

Subscripts and superscripts

1, 2…	Stream numbers
amb	Ambient/surrounding conditions
C	Condenser
w	Water
comp	Compressor
E	Evaporator
gap	Temperature difference between T_7 and T_{amb}
H	relating to the HTC
in	Energy coming into the system
isen	isentropic (for efficiency)
L	relating to the LTC
NET	Net/Overall/Total
out	Energy going out of the system
R1	Region 1
R2	Region 2

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Appendix

- Below are the readme and code for simulation - which is also available on github

★ Readme

```
# Cascade Vapour Compression Refrigeration Cycle
## (thermodynamic simulation using `CoolProp`, and modelling using `ht`, on Python)
This repository contains the files used by Group 3, in Parts 2 & 3 of the Course Project for EN 317 (Thermo-Fluid Devices).
```
cascade-ref-simulation
├── README.md   <- This file (added for the reader's convenience).
|
├── chx.py   <- Physically models the cascade heat exchanger based on the simulation results.
                         Invoked by executing main.py.
|
├── cost.py   <- Computes the total capital cost using suitable cost functions.
                         Invoked by executing main.py.
|
├── main.py   <- The only executable file.
| Sets all the assumed input parameters.
| Generates the output.
```
```

````
|                        Broad simple overview of the program execution.
|
├── model.py          <- All thermodynamic equations, and compilation of
results.
|                        Invoked by main.py and parametrise.py.
|
├── parametrise.py    <- Varies the ambient temperature and plots the change
in COP.
|                        Invoked by executing main.py.
|
├── shorthands.py     <- Syntactical shortening of CoolProp functions.
|                        Invoked inside model.py.
|
├── paraTamb.png      <- Graphical output of the parametric study.
|                        Generated by invoking parametrise.py.
|
├── CP2 G03.pdf       <- The report for Part 2 of the project (modelling and
simulation).
|
└── CP3 G03.pdf       <- The report for Part 3 of the project
(techno-economic analysis).
```

**Python libraries needed:** `CoolProp`, `matplotlib`, `numpy`, `pandas`
````

★ Code

- Main.py the only executable file

```
import model
import parametrise
import chx
import cost
import time
import pandas as pd
sep = '%'

# Assumptions (2 refrigerants, 4 temperatures, 4 qualities, 4 component
parameters):
Hfluid = 'R1234ze(E)'
Lfluid = 'R23'
T1, T5 = 208, 240                       # in K
T_amb, T_gap, T_overlap = 298, 10, 5    # in K
n_isH, n_isL = 0.75, 0.6
Q_ref = 40000                           # in W
```
```

```
Q1, Q5 = 1, 1       # (ideal saturated vapour streams)
Q3, Q7 = 0, 0       # (ideal saturated liquid streams)
# Thus
assumptions = (Hfluid, Lfluid, T1, T5, T_amb, T_gap, T_overlap, n_isH, n_isL,
Q_ref, Q1, Q3, Q5, Q7)

# CHX Modelling-related assumptions
d_hyd = 0.0075          # hydraulic diameter for the CHX, in m

# Costs-related assumptions
area_HTCC = 100*0.186*0.613     # Heat Transfer Area of chosen condenser
model
area_LTCE = 40*0.118*0.525      # Heat Transfer Area of chosen evaporator
model

# Simulation, Tabulation, Evaluation
print('\n'+sep*80)
print('EN 317 Course Project (Group 3): Cascade Refrigeration Cycle')
print(sep*80)
print('NOTE: Printed results will be rounded for better visibility,')
print('without losing the precision of all the actual computations.')
print(sep*80)
try:
    print('Initialising simulation model with given assumptions...')
    streams, results = model.simulate(*assumptions)
    print('\nSimulation complete. Showing outputs...')
    print('\nTable of Streams, all in SI units (\'Q\' is dryness fraction):')
    print(streams.round(3))
    print('\nTable of Results:')
    print(results)
except:
    print('\nSimulation failed due to bad assumptions, try again!')
print(sep*80)
time.sleep(0.5)
try:
    print('Initialising parametric study over T_amb...')
    parametrise.T_amb(*assumptions)
    print('\nParametric study complete. Plot saved to \'paraTamb.png\'.')
except:
    print('\nParametric study failed due to bad assumptions, try again!')
print(sep*80)
time.sleep(0.5)
try:
    print('Initialising modelling of cascade heat exchanger...')
    results_R1, results_R2, area_CHX = chx.compute_area(streams, T_overlap,
Hfluid, Lfluid, d_hyd)
    print('\nModelling complete. Showing results...')
    print('\nRegion 1: LTC fluid is a pure gas, HTC fluid is a mixture')
```

```
    print(results_R1)
    print('\nRegion 2: LTC fluid is a mixture, HTC fluid is a mixture')
    print(results_R2)
    print('\nThus, total area required =',str(round(area_CHX,3)),'m^2')
except:
    print('\nModelling failed, try again!')
print(sep*80)
time.sleep(0.5)
try:
    print('Initialising capital cost calculation...')
    c_results, c_TOT = cost.capital(streams, results, area_CHX, area_HTCC,
area_LTCE)
    print('\Calculation complete. Showing results...\n')
    print(c_results)
    print('\nThus, total capital cost = ₹',str(round(c_TOT,2)))
    print('After rounding off insignificant digits = ₹',str(round(c_TOT,
-5)))
except:
    print('\nCapital cost calculation failed, try again!')
print(sep*80)
print('Execution complete.')
print(sep*80+'\n')
```

❖ chx.py - Supporting file

```
from shorthands import *
import math
import pandas as pd

def compute_area(streams, T_overlap, Hfluid, Lfluid, d_hyd):
    area_CHX = 0                    # Initialise the summation term

    # Region 1: LTC gas, HTC mixture
    # Pressure in LTC stays at P2, throughout the CHX
    mdot_L = streams['mdot'][2]
    P_L = streams['P'][2]
    h2 = streams['h'][2]                            # Specific enthalpy of
LTC fluid at the Region 1 left endpoint
    h2_cool = HST_from_PQ(P_L, 1, Lfluid)[0]        # Specific enthalpy of
LTC fluid at the Region 1 right endpoint
    Qdot_R1 = mdot_L*(h2-h2_cool)
    # Found rate of heat transfer in Region 1!

    T2 = streams['T'][2]
    T3 = streams['T'][3]
```

```
    T_L = (T2+T3)/2               # Take average of T2 and T2cool (which is
equal to T3)
    Nu_L = 615.27                 # Representative value from the correlation
    h_L_R1 = Nu_L*cp.PropsSI('L', 'T', 275.5, 'P', P_L, Lfluid)/d_hyd    #
k=0.013
    # Found convective heat transfer coefficient on LTC side!

    P_H = streams['P'][5]         # Pressure stays at P5, throughout Region 1
    T_H = streams['T'][5]         # Temperature stays at T5
    mdot_H = streams['mdot'][5]
    h5 = streams['h'][5]                              # Specific enthalpy of
HTC fluid at the Region 1 left endpoint
    h8_heat = h5 - (Qdot_R1/mdot_H)                   # Specific enthalpy of
HTC fluid at the Region 1 right endpoint
    Q5 = streams['Q'][5]                              # Quality of HTC fluid at
Region 1 left endpoint
    Q8_heat = Q_from_PH(P_H, h8_heat, Hfluid)         # Quality of HTC fluid at
Region 1 right endpoint
    Q_H_R1 = (Q5+Q8_heat)/2       # Use average quality for further
calculations. Equal to 0.78
    h_H_R1 = 5375                 # Representative value from the graph
    # Found convective heat transfer coefficient on HTC side!

    U_R1 = (h_L_R1*h_H_R1)/(h_L_R1+h_H_R1)
    # Found overall heat transfer coefficient for Region 1!
    LMTD_R1 = ((T2-T_H) - (T3-T_H))/math.log((T2-T_H)/(T3-T_H))
    # Found LMTD for Region 1!
    area_R1 = Qdot_R1/(U_R1*LMTD_R1)
    # Found area required for Region 1!

    results_R1 = pd.DataFrame([['Rate of HT in Region 1 (W)',
round(Qdot_R1,3)],
                              #  ['Convective HT coefficient on LTC side
(W/m^2 K)', h_L_R1],
                              #  ['Convective HT coefficient on HTC side
(W/m^2 K)', h_H_R1],
                               ['Overall HT coefficient (W/m^2 K)',
round(U_R1,3)],
                               ['Logarithmic mean temperature difference
(K)', round(LMTD_R1,3)],
                               ['Thus, total area of Region 1 (m^2)',
round(area_R1,3)]], columns=['Parameter', 'Value'])
    results_R1.index += 1
    # Stored results of Region 1

    area_CHX += area_R1

    # Region 2: LTC mixture, HTC mixture
```

```
    # Pressure in LTC stays at P2, Temperature stays at T3
    h3 = streams['h'][3]            # Specific enthalpy of LTC fluid at
Region 2 right endpoint
    Qdot_R2 = mdot_L*(h2_cool-h3)
    # Found rate of heat transfer in Region 2!

    Q_L_R2 = (1+0)/2                # Use average quality for further
calculations. Equal to 0.5
    h_L_R2 = 6250                   # Representative value from graph
    # Found convective heat transfer coefficient on LTC side!

    Q8 = streams['Q'][8]
    Q_H_R2 = (Q8_heat+Q8)/2         # Equal to approximately 0.5
    h_H_R2 = 6250                   # Representative value from graph
    # Found convective heat transfer coefficient on HTC side!

    U_R2 = (h_L_R2*h_H_R2)/(h_L_R2+h_H_R2)
    # Found overall heat transfer coefficient for Region 2!
    LMTD_R2 = T_overlap         # Simple phase change process, both
temperature profiles are flat and parallel
    # Found LMTD for Region 2!
    area_R2 = Qdot_R2/(U_R2*LMTD_R2)
    # Found area required for Region 2!

    results_R2 = pd.DataFrame([['Rate of HT in Region 2 (W)',
round(Qdot_R2,3)],
                            #    ['Convective HT coefficient on LTC side
(W/m^2 K)', h_L_R2],
                            #    ['Convective HT coefficient on HTC side
(W/m^2 K)', h_H_R2],
                               ['Overall HT coefficient (W/m^2 K)',
round(U_R2,3)],
                               ['Logarithmic mean temperature difference
(K)', round(LMTD_R2,3)],
                               ['Thus, total area of Region 2 (m^2)',
round(area_R2,3)]], columns=['Parameter', 'Value'])
    results_R2.index += 1
    # Stored results for Region 2!

    area_CHX += area_R2
    # Found total area for the entire CHX!
    return results_R1, results_R2, area_CHX
```

- cost.py - Supporting file

```
import pandas as pd

# Get simulation outputs, find costs
def capital(streams, results, area_CHX, area_HTCC, area_LTCE):      # For
CHX, we may have to add more input parameters to compute c_CHX
    ci_ratio_22by19 = 331/289
    ci_ratio_22by15 = 331/254
    # LTC Compressor: Use cost indices on 2019 cost function
    c_compL = 959791.6*((results.iloc[0,1]/1000)**0.46) * ci_ratio_22by19
    # LTC EV: Use cost indices on 2019 cost function
    c_evL = 10808.6*(streams['mdot'][3]) * ci_ratio_22by19
    # LTC Evaporator: Use cost indices on 2019 cost function
    c_evapL = 131874.0*(area_LTCE**0.89) * ci_ratio_22by19

    # CHX: Use cost indices on 2015 cost function
    c_CHX = 56380.5*(area_CHX**0.8) * ci_ratio_22by15

    # HTC Compressor: Use cost indices on 2019 cost function
    c_compH = 959791.6*((results.iloc[1,1]/1000)**0.46) * ci_ratio_22by19
    # HTC EV: Use cost indices on 2019 cost function
    c_evH = 10808.6*(streams['mdot'][7]) * ci_ratio_22by19
    # HTC Condenser: Use cost indices on 2019 cost function
    c_condH = 131874.0*(area_HTCC**0.89) * ci_ratio_22by19

    # Total
    c_results = pd.DataFrame([['LTC Compressor', round(c_compL,2)],
                          ['LTC Throttle Valve', round(c_evL,2)],
                          ['LTC Evaporator', round(c_evapL,2)],
                          ['Cascade Heat Exchanger', round(c_CHX,2)],
                          ['HTC Compressor', round(c_compH,2)],
                          ['HTC Throttle Valve', round(c_evH,2)],
                          ['HTC Condenser', round(c_condH,2)]],
columns=['Component', 'Price in ₹ (2022)'])
    c_results.index += 1
    c_TOT = c_compL+c_evL+c_evapL+c_CHX+c_compH+c_evH+c_condH
    return c_results, c_TOT
```

- model.py - supporting file

```
from shorthands import *
import pandas as pd

def simulate(Hfluid, Lfluid, T1, T5, T_amb, T_gap, T_overlap, n_isH, n_isL,
Q_ref, Q1, Q3, Q5, Q7):
    # Lower Temperature Cycle
```

```
    P5, h5, s5 = PHS_from_TQ(T5, Q5, Hfluid)
    P1, h1, s1 = PHS_from_TQ(T1, Q1, Lfluid)
    T3 = T5 + T_overlap
    P3, h3, s3 = PHS_from_TQ(T3, Q3, Lfluid)
    P2 = P3
    h2s = cp.PropsSI('H', 'S', s1, 'P', P2, Lfluid)
    h2 = (h2s-h1)/n_isL + h1
    s2, T2 = ST_from_PH(P2, h2, Lfluid)
    h4 = h3
    P4 = P1
    s4, T4 = ST_from_PH(P4, h4, Lfluid)
    Q4 = Q_from_PH(P4, h4, Lfluid)

    # Higher Temperature Cycle
    T7 = T_amb + T_gap
    P7, h7, s7 = PHS_from_TQ(T7, Q7, Hfluid)
    P6 = P7
    h6s = cp.PropsSI('H', 'S', s5, 'P', P6, Hfluid)
    h6 = (h6s-h5)/n_isH + h5
    s6, T6 = ST_from_PH(P6, h6, Hfluid)
    h8 = h7
    P8 = P5
    s8, T8 = ST_from_PH(P8, h8, Hfluid)
    Q8 = Q_from_PH(P8, h8, Hfluid)

    # Mass Flow Rates, Performance, COPs
    mdotL = Q_ref/(h1-h4)
    mdotH = (h2-h3)*mdotL/(h5-h8)
    W_compL = mdotL*(h2-h1)
    W_compH = mdotH*(h6-h5)
    W_inNET = W_compH+W_compL
    Q_out = mdotH*(h6-h7)
    copL, copH = (h1-h4)/(h2-h1), (h5-h8)/(h6-h5)
    # copNET = (copL*copH)/(1+copL+copH)
    copNET = Q_ref/W_inNET

    # Compile
    streams = pd.DataFrame(columns=['h', 's', 'T', 'P', 'Q', 'mdot'])
    streams['h'] = [h1, h2, h3, h4, h5, h6, h7, h8]
    streams['s'] = [s1, s2, s3, s4, s5, s6, s7, s8]
    streams['T'] = [T1, T2, T3, T4, T5, T6, T7, T8]
    streams['P'] = [P1, P2, P3, P4, P5, P6, P7, P8]
    streams['Q'] = [Q1, '(superheated)', Q3, Q4, Q5, '(superheated)', Q7, Q8]
    streams['mdot'] = [mdotL, mdotL, mdotL, mdotL, mdotH, mdotH, mdotH,
mdotH]
    streams.index += 1
    streams.index.rename('Stream', inplace=True)
    results = pd.DataFrame([['LTC work input (in W)', round(W_compL,3)],
```

```
                                 ['HTC work input (in W)', round(W_compH,3)],
                                 ['Total work input (in W)', round(W_inNET,2)],
                                 ['Refrigeration effect (in W)', round(Q_ref,3)],
                                 ['Heat rejection (in W)', round(Q_out,3)],
                                 ['COP of LTC', round(copL,3)],
                                 ['COP of HTC', round(copH,3)],
                                 ['Overall COP', round(copNET,3)]],
columns=['Parameter', 'Value'])
    results.index += 1
    # return streams, W_compL, W_compH, Q_out, copH, copL, copNET
    return streams, results
```

❖ parametrise.py - supporting file

```
import model
import numpy as np
import matplotlib.pyplot as plt

# Parametrize over T_amb
def T_amb(Hfluid, Lfluid, T1, T5, T_amb, T_gap, T_overlap, n_isH, n_isL,
Q_ref, Q1, Q3, Q5, Q7):
    T_list = np.linspace(-15,15,31) + T_amb
    COP_list = []
    for T in T_list:
        COP = model.simulate(Hfluid, Lfluid, T1, T5, T, T_gap, T_overlap,
n_isH, n_isL, Q_ref, Q1, Q3, Q5, Q7)[1].iloc[7,1]
        COP_list.append(COP)
    plt.plot(T_list-273, COP_list, '.-')
    plt.title('Parametric Study against Ambient Temperature')
    plt.xlabel('Ambient Temperature (in ℃)')
    plt.ylabel('Overall COP')
    plt.savefig('paraTamb')
    return
```

❖ shorthands.py - supporting file

```
import CoolProp.CoolProp as cp

# Super / Sub:
def HS_from_TP(t, p, fluid):
    h, s = cp.PropsSI(['H', 'S'], 'T', t, 'P', p, fluid)
    return h, s
```

```python
def ST_from_PH(p, h, fluid):
    s, t = cp.PropsSI(['S', 'T'], 'P', p, 'H', h, fluid)
    return s, t

def TP_from_HS(h, s, fluid):
    t, p = cp.PropsSI(['T', 'P'], 'H', h, 'S', s, fluid)
    return t, p

def PH_from_ST(s, t, fluid):
    p, h = cp.PropsSI(['P', 'H'], 'S', s, 'T', t, fluid)
    return p, h

def HT_from_PS(p, s, fluid):
    h, t = cp.PropsSI(['H', 'T'], 'P', p, 'S', s, fluid)
    return h, t

def PS_from_TH(t, h, fluid):
    p, s = cp.PropsSI(['P', 'S'], 'T', t, 'H', h, fluid)
    return p, s
# #####

# Saturation Dome:
# Quality Known:
def HST_from_PQ(p, q, fluid):
    h, s, t = cp.PropsSI(['H', 'S', 'T'], 'P', p, 'Q', q, fluid)
    return h, s, t

def STP_from_HQ(h, q, fluid):
    s, t, p = cp.PropsSI(['S', 'T', 'P'], 'H', h, 'Q', q, fluid)
    return s, t, p

def TPH_from_SQ(s, q, fluid):
    t, p, h = cp.PropsSI(['T', 'P', 'H'], 'S', s, 'Q', q, fluid)
    return t, p, h

def PHS_from_TQ(t, q, fluid):
    p, h, s = cp.PropsSI(['P', 'H', 'S'], 'T', t, 'Q', q, fluid)
    return p, h, s

# Quality Unknown:
def Q_from_PH(p, h, fluid):
    q = cp.PropsSI('Q', 'P', p, 'H', h, fluid)
    return q

# Write 5 more (not 6, because ..._from_PT() is useless)
# ...or maybe not needed
# #####
```