Minimizing Interference in 1-dimensional Networks

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Background

Problem Definition:
INPUT:
Given a set of wireless nodes represented by a set of points $P \subseteq \mathbb{R}^d$
OUTPUT:
Assign a ***radius of transmission*** *to each node in P such that the resulting communication graph is* ***connected*** *and the* ***maximum interference is minimized****.*
Previous Work:

- Rickenbach et.al [3] showed that the optimal configuration results in minimum interference $O(\sqrt{n})$ and gave a $\sqrt[4]{n}$ approximation algorithm in 1D.
- Buchin [2] proved that the MMID is NP-hard in 2 or more dimensions while Von Rickenbach [3].

Receiver Centric Model

- Rickenbach's receiver-centric wireless network model:

1. Nodes are points on a plane with transmission radii $r(p)$.
2. Interference at point p is defined as:
$$I(p) = |\{q \mid q \in P \setminus \{p\}, p \in D(q, r(q))\}|$$
3. $D(p, r(p))$ represents the transmission circle centered at p with radius $r(p)$.

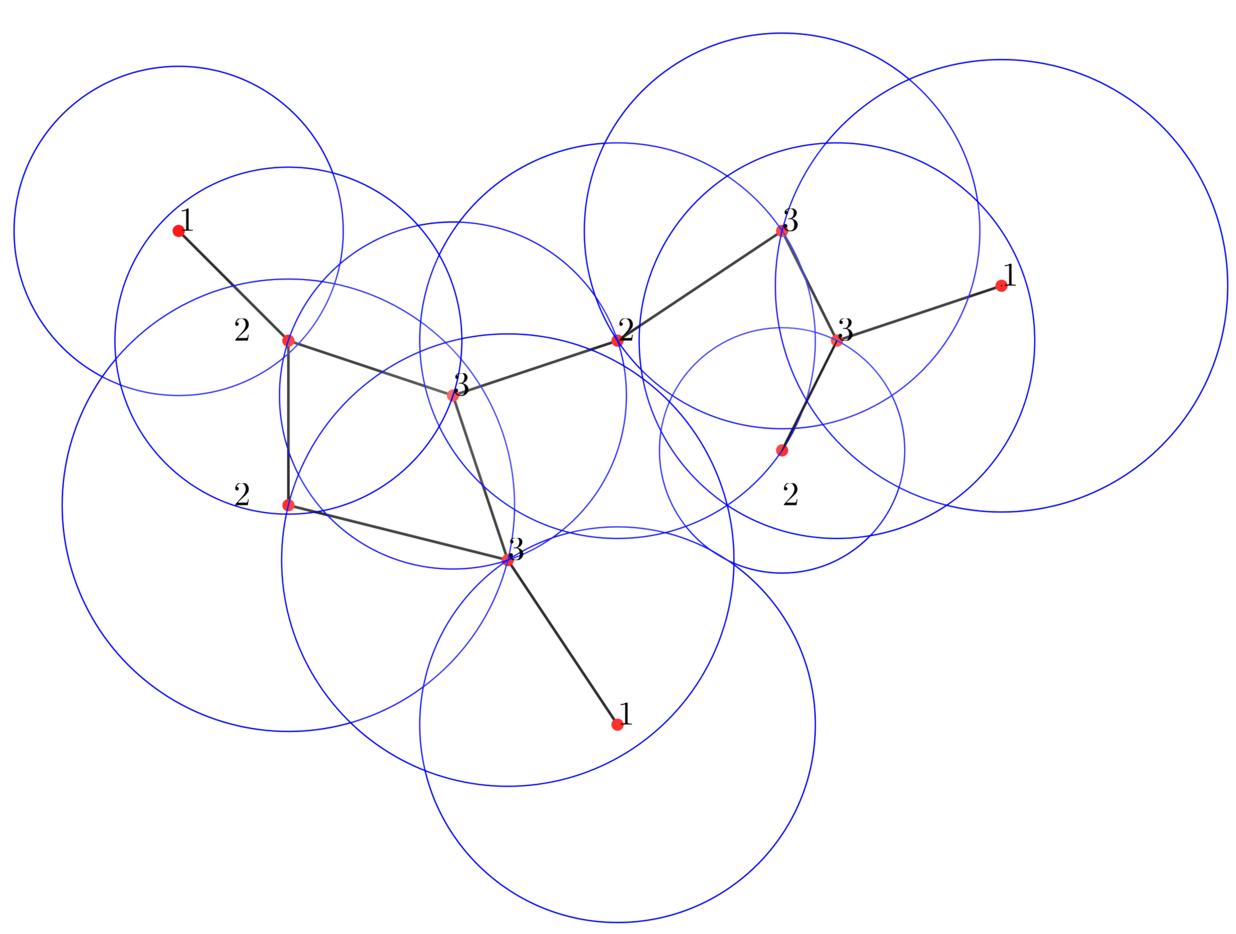

Fig. 1: Illustration of the interference model

Highway Model

- Rickenbach [3] introduced the 1-dimensional model as the highway model:
  - Points are aligned along a line, similar to vehicles on a highway.
  - Worst case: every point interferes with the point on the left.

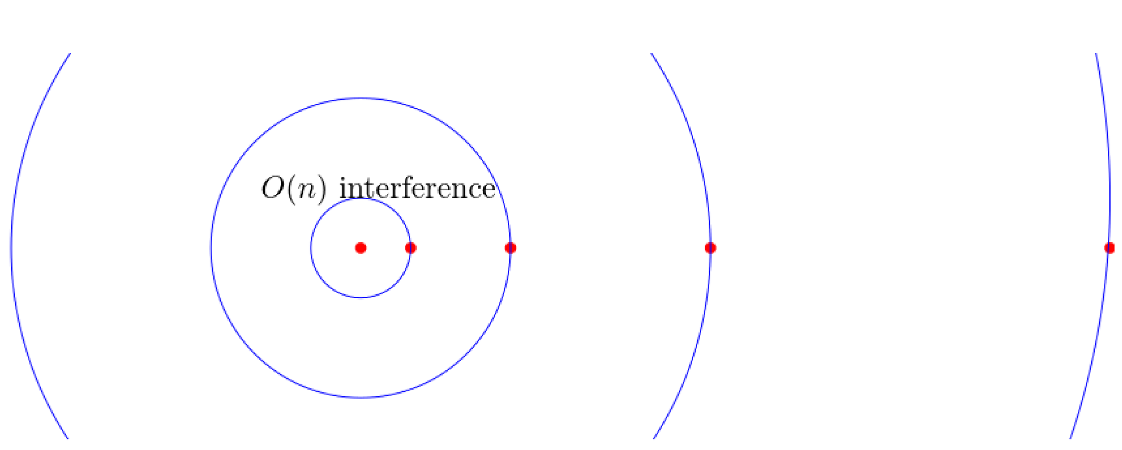

Fig. 2: The starting point has $O(n)$ interference

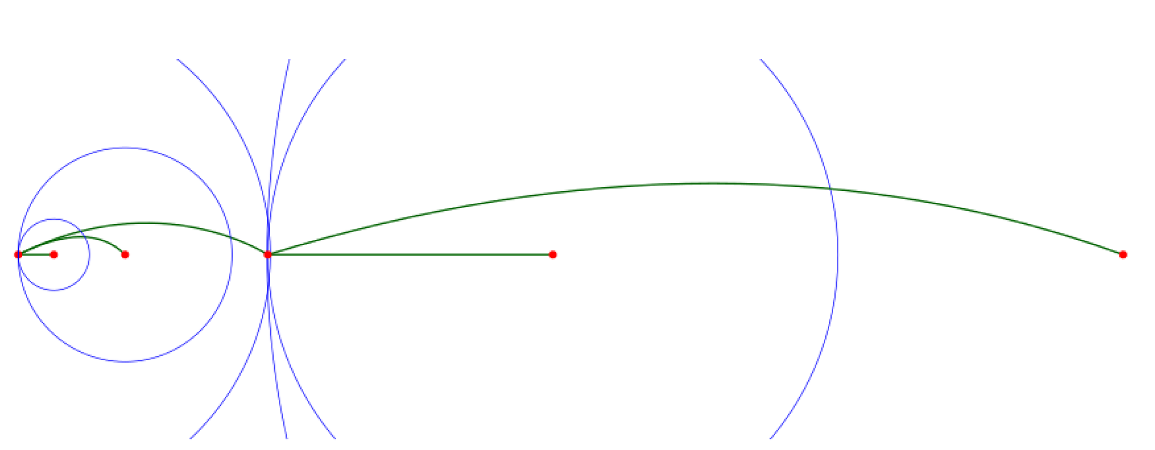

Fig. 3: The maximum interference is $O(\sqrt{n})$

Results

The research is still in progress, however here are a few of the ideas illustrated:

1. **Reduction from $VERTEXCOVER$ to $MTIP$:**

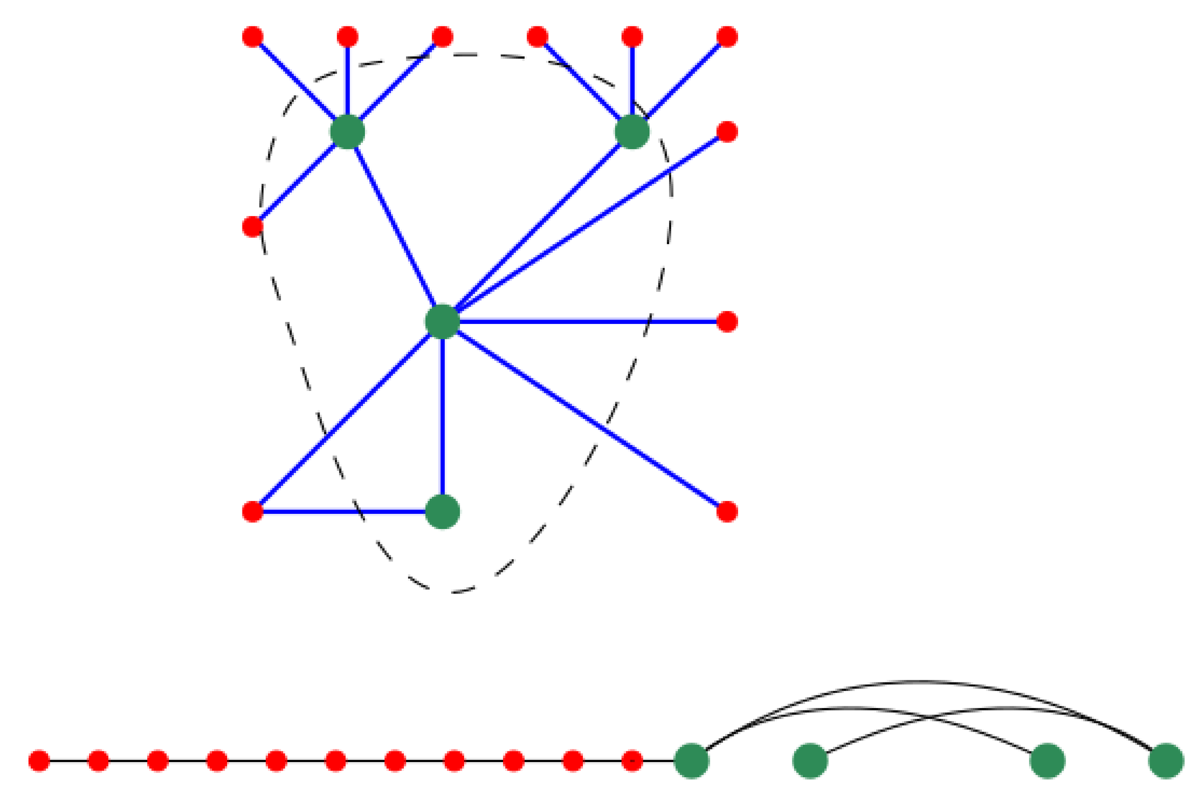

Fig. 4: Idea 1: A failed reduction from $VERTEXCOVER$ to $MTIP$. The reason why it is not valid is because the reduction presumes the existence of a k-vertex cover.

2. **Fixed distance set**:

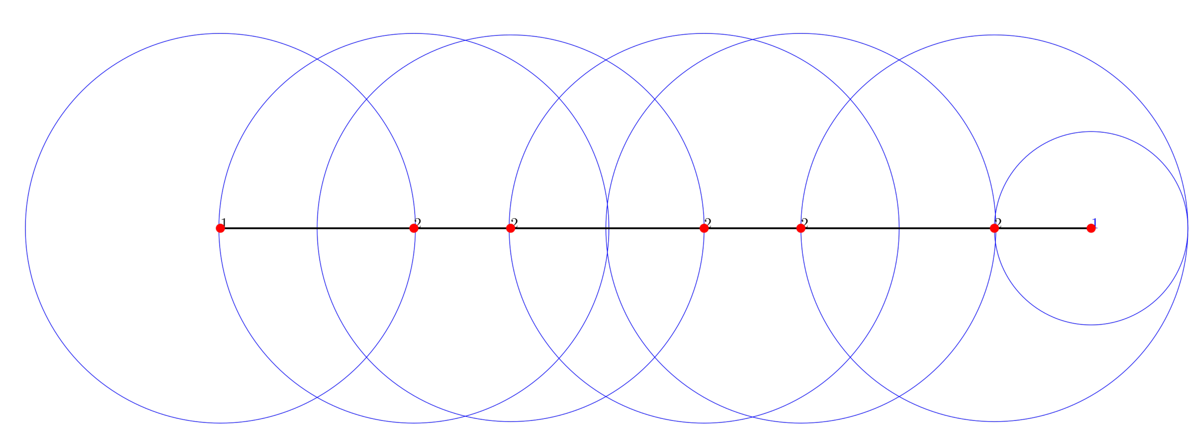

Fig. 5: Idea 2: Fixed distance set. The distance between the points is fixed and the radius of transmission is optimized. Note that the interference is the ratio of the biggest distance and the smallest distance of the distance set.

In progress ideas:

1. **Adapting the $MTIP$ algorithm for the highway model**: We are trying to adapt the Minimizing Total Interference Problem, $MTIP$, algorithm by Karim [1] for the highway model.

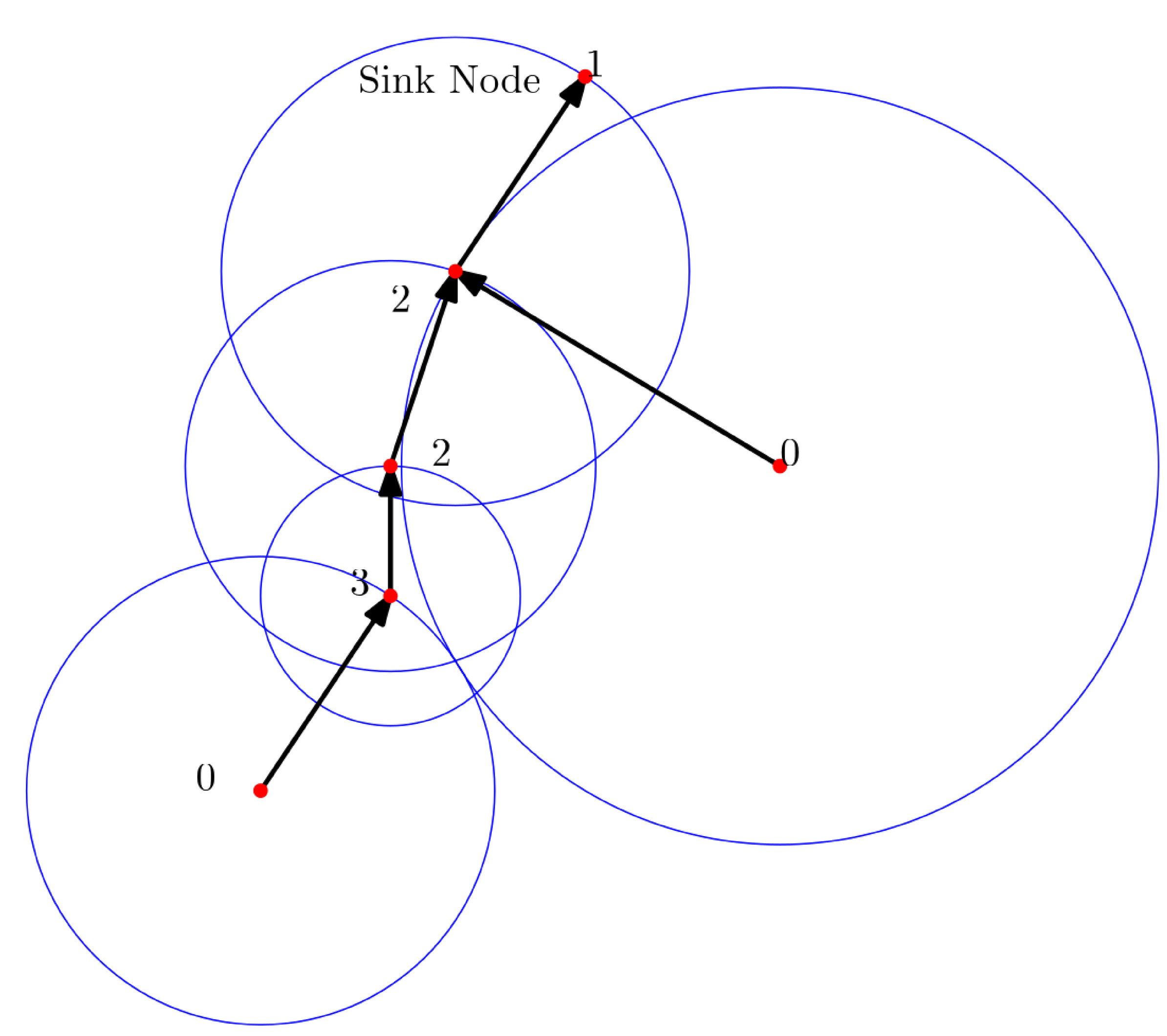

Fig. 6: Sink tree with 6 nodes, where the uppermost node is the sink node. The maximum interference of this network is 3. The idea of sink trees is vital for

2. **Fixed Parameter Tractable methods**: Here is the problem definition that we are currently working towards:

Problem Definition 2.
Given $P = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}$ such that $\forall i, p_i < p_{i+1}$, find an algorithm that finds the optimal solution for P in $O(f(k) \cdot n^c$ time, for some $f(k)$ and $c \in O(1)$.

3. **Finding a better approximation algorithm than $\sqrt[4]{n}$**: Finding a better α approximation algorithm such that $\alpha < \sqrt[4]{n}$.

Comparison

Recent developments in the minimizing interference in networks include looking at minimizing the total interference instead of the maximum interference in a given point set. Karim et. al[1] showed that you can actually obtain a $O(n^3)$ algorithm that minimizes the total interference. Their definition of interference was as follows:

$$I(G_\rho) = \sum_{p \in P}^{RI(p)}$$

where $RI(p)$ denotes the asymmetric transmission radius of a point p.
Our problem defines the interference as:

$$I(G) = \max_{e \in E} Cov(e)$$

where $Cov(e)$ denotes the coverage of an edge e which is given by:

$$Cov(e) = |\{w \in V | w \text{ is covered by } D(u, |uv|)\} \cup \{w \in$$

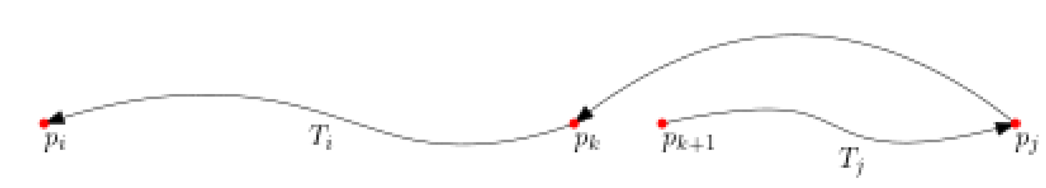

Fig. 7: Karim's model of the minimum weight sink tree for asymmetric sensor networks.

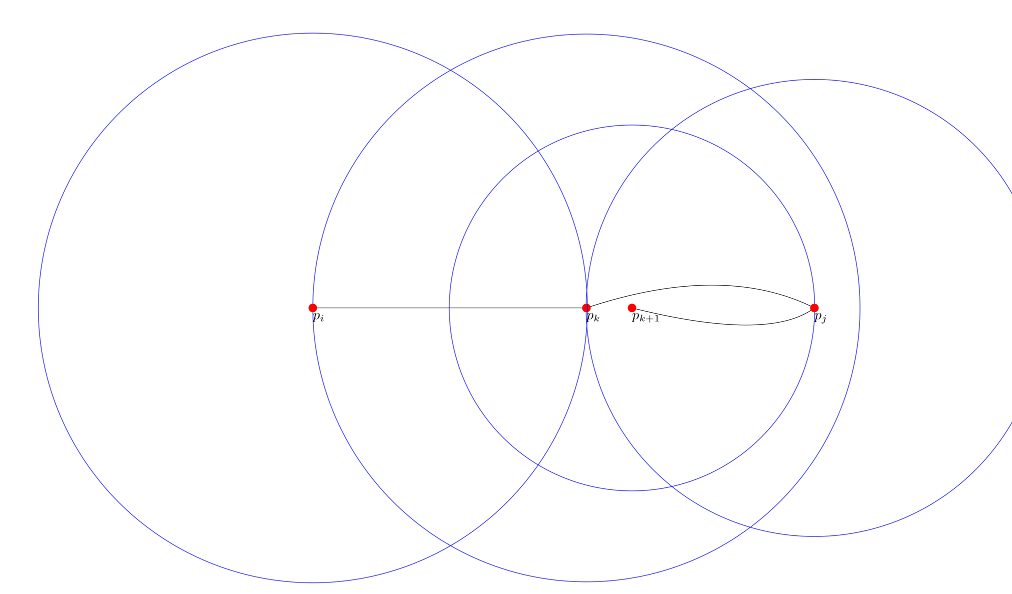

Fig. 8: Karim's model adapted with symmetric edges which does not reduce the interference level but instead increases it.

Future Works

- Initially believed the problem was NP-hard.
- Recent work suggests a high probability of finding a polynomial-time algorithm in possibly other models than the highway model.
- Current focus:
  - Fixed Parameter Tractable methods.
  - Adapting the $MTIP$ algorithm for the highway model.
- Successful adaptation may yield a polynomial-time algorithm.

References

[1] A. Karim Abu-Affash, Paz Carmi, and Matthew J. Katz. "Minimizing total interference in asymmetric sensor networks". In: *Theoretical Computer Science* 889 (2021), pp. 171–181. ISSN: 0304-3975. DOI: https://doi.org/10.1016/j.tcs.2021.08.003. URL: https://www.sciencedirect.com/science/article/pii/S0304397521004461.

[2] Kevin Buchin. *Minimizing the Maximum Interference is Hard.* 2011. arXiv: 0802.2134.

[3] Pascal von Rickenbach, Roger Wattenhofer, and Aaron Zollinger. "Algorithmic Models of Interference in Wireless Ad Hoc and Sensor Networks". In: *IEEE/ACM Transactions on Networking* 17.1 (2009), pp. 172–185. DOI: 10.1109/TNET.2008.926506.