Can FPT be Applied to the Interference Minimization Problem?

Stephane Durocher
durocher@cs.umanitoba.ca

University of Manitoba
Department of Computer Science

July 10, 2018

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Interference Minimization Problem

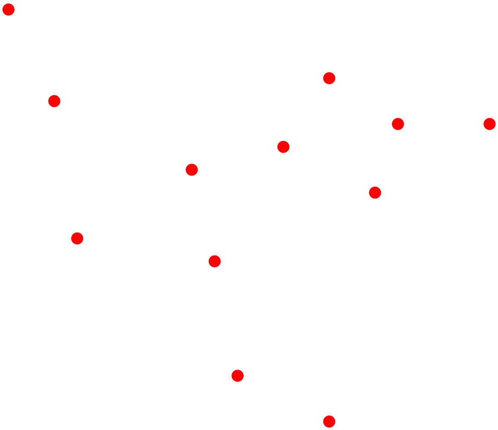
- A network must be connected for communication to be possible.
- Lower interference results in fewer collisions.

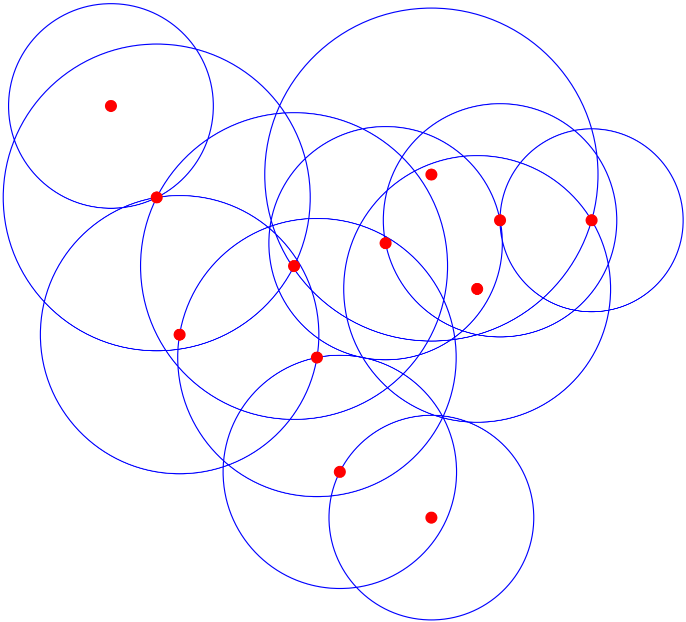
Problem Definition

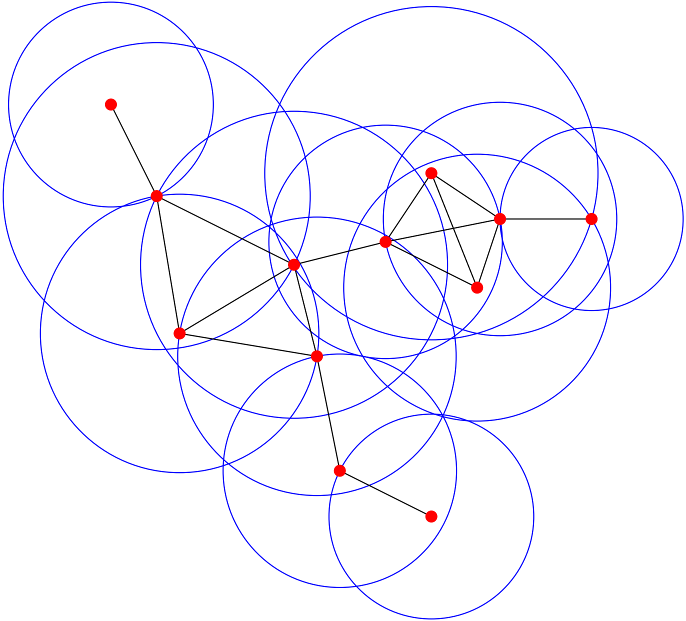
Given a set of wireless nodes represented by a set of points $P \subseteq \mathbb{R}^d$, assign a radius of transmission to each node in P such that the resulting communication graph is connected and the maximum interference is minimized.

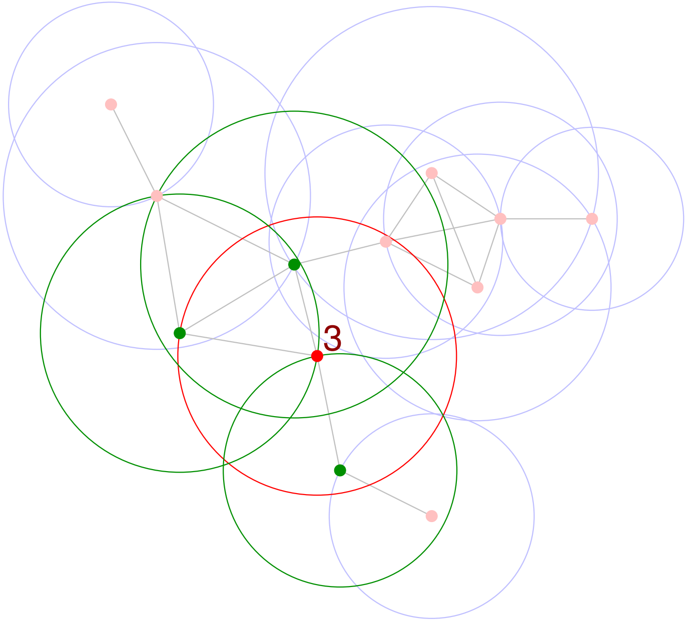
(Equivalent) Problem Definition

Given a set of wireless nodes represented by a set of points $P \subseteq \mathbb{R}^d$, define a connected graph G on P such that such that the maximum interference is minimized in the induced communication graph.





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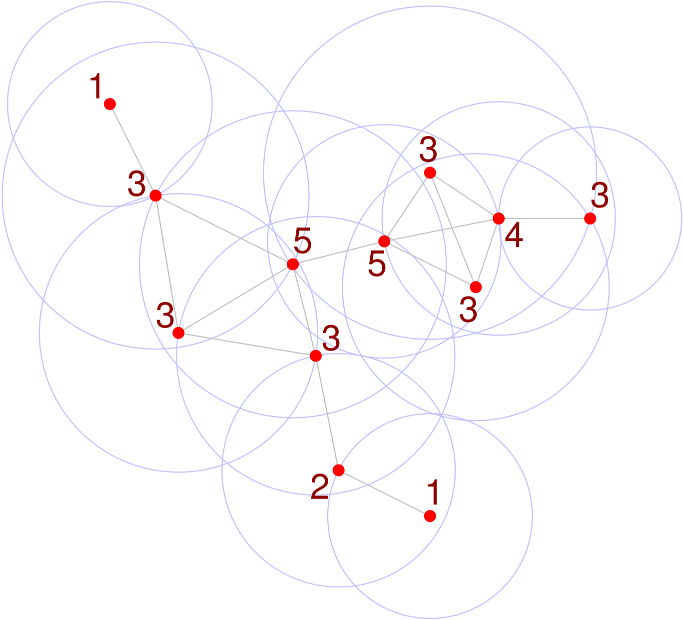
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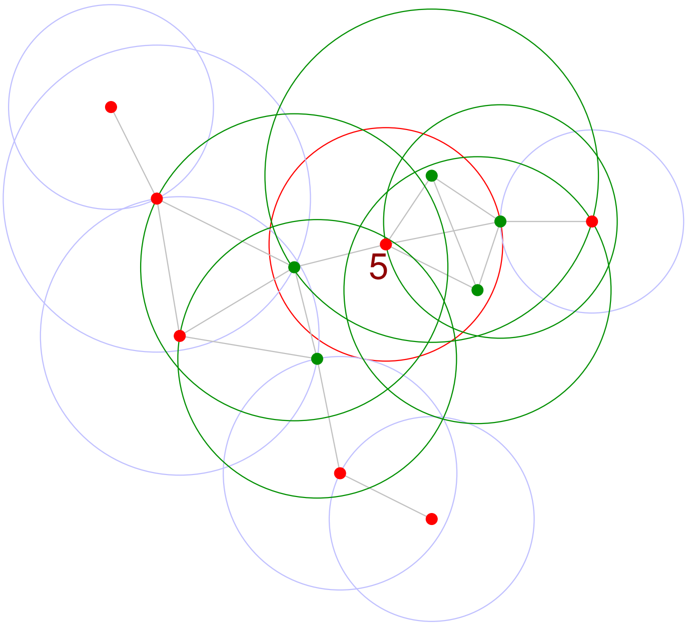
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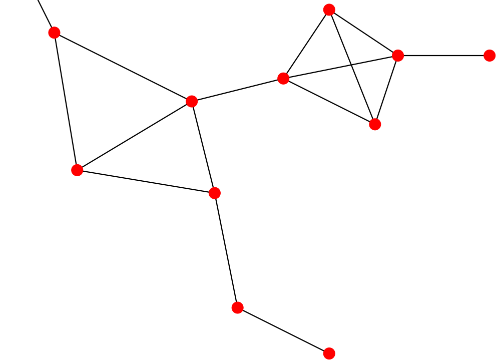

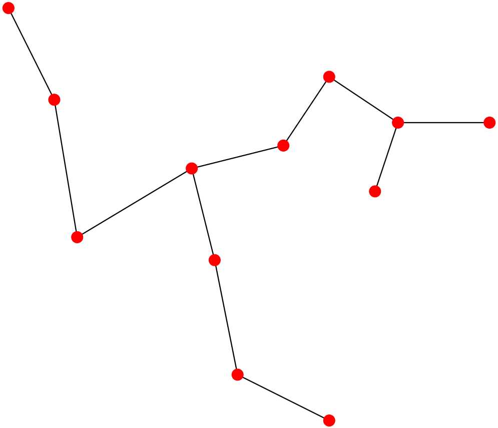

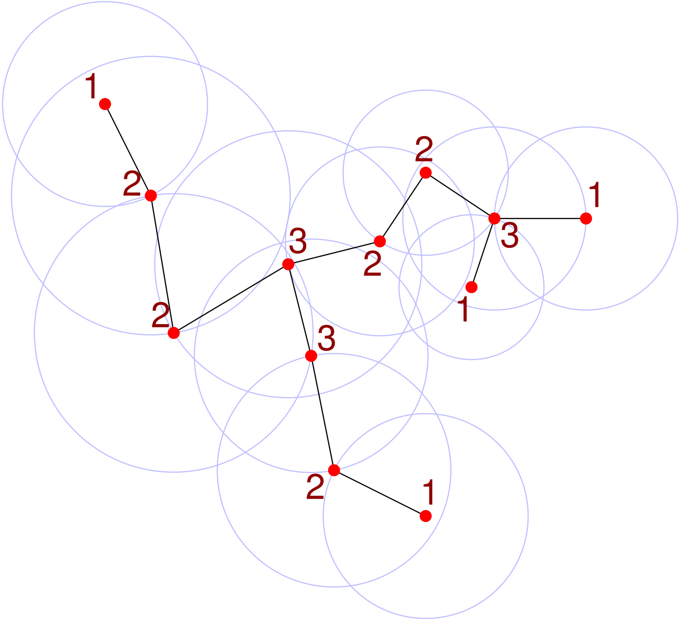
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Previous Work

Let OPT(P) denote the minimum interference attainable over all connected graphs on P.

- von Rickenbach, Schmid, Wattenhofer, & Zollinger 2005
  - $\text{OPT}(P) \in O(\sqrt{n})$
  - $\forall n,\ \exists P \subseteq \mathbb{R}$ such that $|P| = n$ and $\text{OPT}(P) \in \Omega(\sqrt{n})$.
  - Even in 1D, the MST (nearest-neighbour path) does not necessarily minimize the maximum interference.
  - $O\left(\sqrt[4]{n}\right)$-approximation algorithm in 1D
- Complexity in 2D: Buchin 2008
  - NP-complete in 2D
- 2D Algorithms:
  - Halldórsson & Tokuyama 2006: $O\left(\sqrt{n}\right)$-interference algorithm
  - Halldórsson & Tokuyama 2006: $O(\log \lambda)$-interference algorithm, where $\lambda =$ ratio of further to nearest pair distance
  - Tan et al. 2011: $n^{O(OPT(P))}$-time exact algorithm

Previous Work: Interference Minimization in 1D

von Rickenbach et al. 2005

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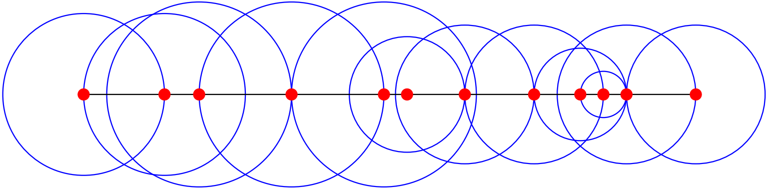

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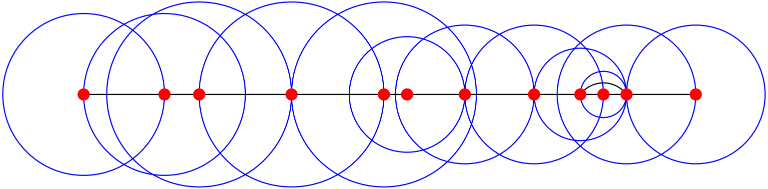

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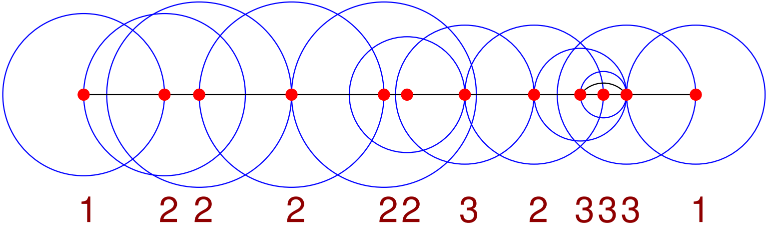

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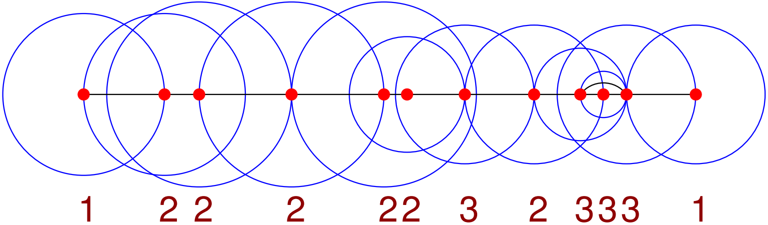

O(1) maximum interference?

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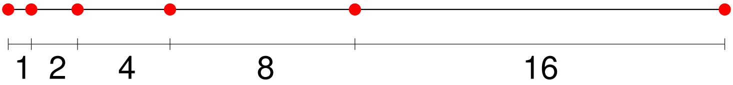

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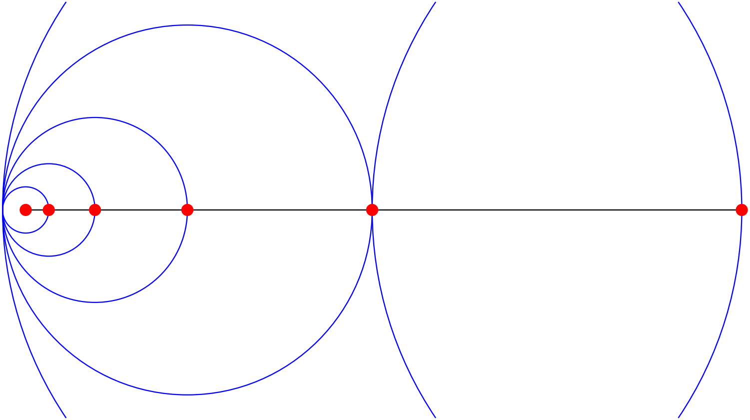

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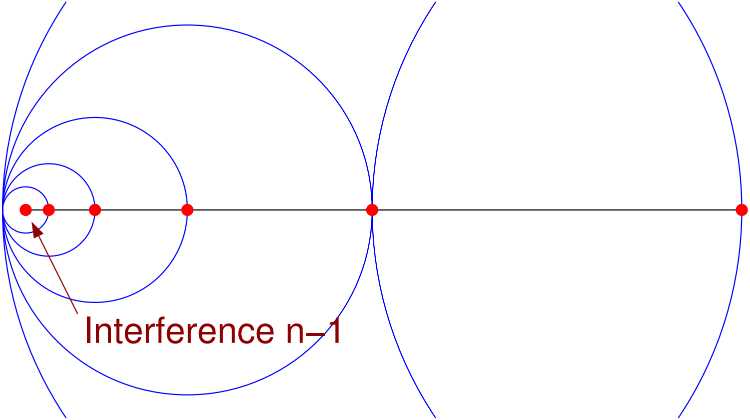

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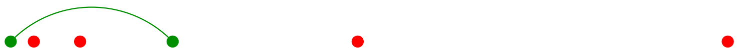

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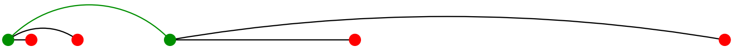

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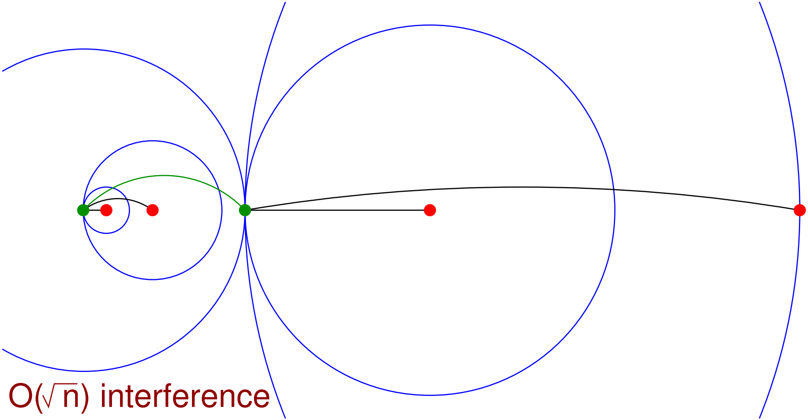

$O(\sqrt{n})$ interference

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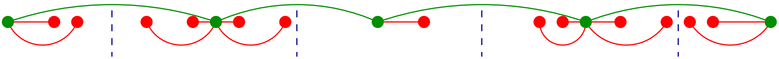

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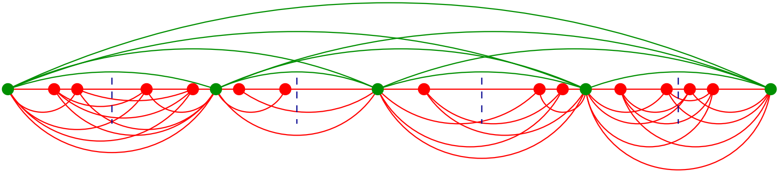

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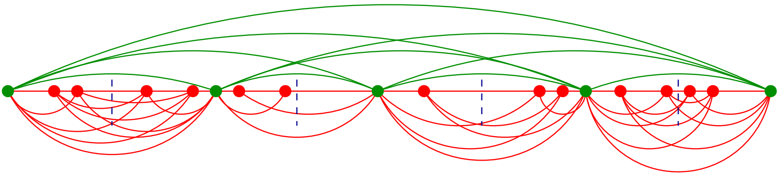

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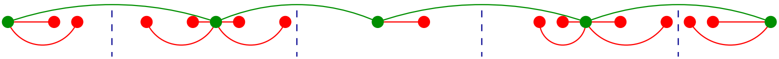


Denote this graph by $HUBS(P)$.

Previous Work: Interference Minimization in 1D

Lemma [von Rickenbach et al. 2005]

$$\mathrm{OPT}(P) \in \Omega\left(\sqrt{\mathrm{inter}(\mathrm{MST}(P))}\right).$$

Algorithm $\mathcal{A}_{1D}$ [von Rickenbach et al. 2005]

Input: a set of n points $P \subseteq \mathbb{R}$

- Measure inter(MST(P)).
- If $\mathrm{inter}(\mathrm{MST}(P)) \leq \sqrt{n}$, then return MST($P$).
- If $\mathrm{inter}(\mathrm{MST}(P)) > \sqrt{n}$, then return $HUBS(P)$.

Algorithm $\mathcal{A}_{1D}$ provides a $O\left(\sqrt[4]{n}\right)$-approximation.

Open Problems

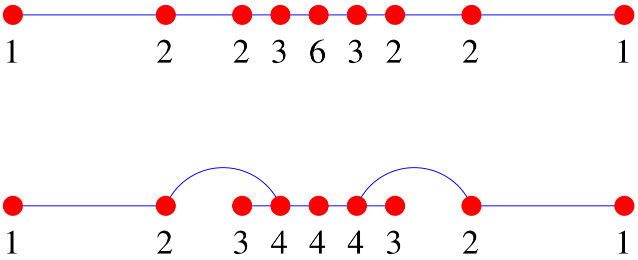
Question 1

In 1D, can an optimal solution be found in polynomial time?

- Is the problem NP-hard?
- Buchin's 2D hardness reduction does not apply in 1D.
- Consider solving specific instances (e.g., when the distance between adjacent points is 1, 2, or 4).

1-2-4 Example

MST(P) is not necessarily optimal even when distances between adjacent points in P are restricted to $\{1, 2, 4\}$.



Open Problems

Question 2

Does there exist a good FPT algorithm to solve interference minimization in 1D?

possible parameterizations:

- Let $k = \#$ of distinct lengths between neighbouring points.
  - $k = 3$ when the lengths are in $\{1, 2, 4\}$.
  - straightforward when $k \leq 2$
- Let $k = \mathrm{OPT}(P)$.

Open Problems

Question 3

In 2D, what approximation factor can be guaranteed?

- Is an $O\left(\sqrt[4]{n}\right)$ approximation algorithm possible in 2D?
- Is a PTAS or $O(1)$-approximation algorithm possible?
- What are good FPT parameters in 2D?

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