Mathematical Modelling for Starlings simulation

COP 290 - Design Practices

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1 Structure of the bird

Let us assume we have the following parameters for each bird:

- *index* - uid of bird
- (x, y, z) - The position coordinate of the bird
- v - The speed of the bird
- *acceleration* - The acceleration of the bird
- (u_x, u_y, u_z) - This is the **unit vector** along which the bird is headed

2 How to decide bird's parameters at any instant of time?

2.1 Position

Given the bird's parameters at time t, the position of the bird at time $t + \delta t$ is given by:

$$x(t + \delta t) = x(t) + vu_x\delta t$$
$$y(t + \delta t) = y(t) + vu_y\delta t$$
$$z(t + \delta t) = z(t) + vu_z\delta t$$

This comes directly from the definition of velocity vector

2.2 Velocity of bird

The velocity of the bird is decided by the **weighted** contribution from many factors. Please note that these weights may change with time(as discussed in sections below)

- **Position of other birds in the flock**:
  All the birds distant less than R (where R is a hyperparameter, which stands for the size of the flock) from bird i are said to be in the ith bird's flock. Each bird would want to stick with its neighbours. To account for this factor, we would calculate the average position of each neighbour of bird i.
  Let it be represented by (x_c^i, y_c^i, z_c^i).
  So the unit vector in which bird i would be headed is given by:

$$\left(\frac{x_c^i - x^i}{D}, \frac{y_c^i - y^i}{D}, \frac{z_c^i - z^i}{D}\right) \ where$$
$$D = \sqrt{(x_c^i - x^i)^2 + (y_c^i - y^i)^2 + (z_c^i - z^i)^2}$$

- **Direction in which the other birds in the flock are headed**:
  For bird i, not only do we have to care about the position of each neighbour, but also their direction. This is accounted by averaging the unit vectors of all the neighbours. Let it be represented by $(u_{xc}^i, u_{yc}^i, u_{zc}^i)$, which is equal to the new direction of the bird.

- **Repulsion due to a very close neighbour**:
  Birds have a tendency of not colliding with each other that is why a bird should repel away if it gets too close to some bird.
  For each bird i, for each bird k which is at some distance less than a hyper parameter r, we would calculate a *distance vector* and a *weight* given by:

$$d_k^i = (x^i - x^k, y^i - y^k, z^i - z^k)$$
$$w_k^i = r - |d_k^i|$$

  The new direction would be given by unit vector along:

$$\frac{\sum_k w_k^i d_k^i}{\sum_k w_k^i}$$

  The concept of weight is introduced to give more repulsion to a neighbour which is closer than the rest.

- **Boundary of the system**:
  We just assume that the bird tunnels through the boundary and reaches the other surface. This is done so that we donot face any restrictions due to the size of the screen.

As we already said, all these factors would have different importance at different time instants. Let us visualize this in the following example. Imagine all the neighbours of a bird are present together at a distance slightly less than R and coming towards it. In this case we would want the bird **NOT** to follow their direction, but rather move towards the centre of mass.
Let us formalize this by associating weights F_1, F_2 and F_3 with each factor discussed above and we must have that they sum up to 1.

- A bird colliding with another at such high speeds would be critical to their health. Therefore we would want to set F_3 to a high value of say 0.8 and F_1, F_2 equal to 0.1, when a neighbour becomes closer than r.

- If the bird is neither on a boundary nor close to some other bird, it would be guided by average direction and velocity. As described in the example above, the influence of average position would be more if it's distance from the centre of mass is high whereas if it is low, the bird would be guided by average direction.
  To put this mathematically we are defining:

$$F_1 = \frac{D^i_{CoM}}{R}$$
$$F_2 = 1 - \frac{D^i_{CoM}}{R}$$

  where D^i_{CoM} is the distance of $bird_i$ from it's flock centre of mass
  and R is the hyper parameter defining the flock size

3 How to change velocity?

$$v_x = v * u_x + (a * newDir_x) * \delta t$$
$$v_y = v * u_y + (a * newDir_y) * \delta t$$
$$v_z = v * u_z + (a * newDir_z) * \delta t$$

4 How to change acceleration?

We are a defining model where the acceleration can range from acc_{min} to acc_{max}.
Inspired by the idea that, a bird would be wanting to increase its acceleration when it gets separated by its flock or it comes too close to another bird, we have set

$$a = a + (a_{min} + (a_{max} - a_{min}) * [F_1 + F_2 + F_3]) * \delta t$$

5 How to measure physical quantities?

- **Force** is defined as mass * acceleration. We can calculate acceleration as the rate of change of velocity vector.

- **Angular momentum** with respect to the origin can be calculated as the cross product between position vector and velocity vector multiplied with mass.

- **Power** is the dot product between force experienced and velocity.

$By -$
Atishya Jain (2016CS50393)
Mayank Singh Chauhan (2016CS50394)
IIT Delhi