Date

8. 机械振动(一)

1. B

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \frac{\pi}{4})$$

$$a = \frac{dv}{dt} = -A\omega^2 \cos(\omega t + \frac{\pi}{4})$$

$t = \frac{1}{4}T$ 代入, $\omega = \frac{2\pi}{T}$

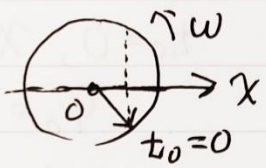
$$v = -A\omega \sin(\frac{3}{4}\pi) = -\frac{\sqrt{2}}{2}A\omega$$

$$a = -A\omega^2 \cos(\frac{3}{4}\pi) = \frac{\sqrt{2}}{2}A\omega^2$$

2. B

$t_0 = 0$, $x_0 = \frac{A}{2}$, $v_0 > 0$

$$\varphi_0 = -\frac{\pi}{3}$$



3. C

$$E_{总} = \frac{1}{2}kA^2$$

$$E_k = \frac{1}{2}kA^2 \sin^2(\omega t + \varphi_0)$$

$$x = A\cos(\omega t + \varphi_0)$$

当 $x = A\cos(\omega t' + \varphi_0) = \frac{1}{2}A$ 时

$$\cos(\omega t' + \varphi_0) = \frac{1}{2}$$

$$\sin^2(\omega t' + \varphi_0) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$E_k = \frac{3}{4} \times \frac{1}{2}kA^2 = \frac{3}{4}E_{总}$$

4. A （注意：A 选项中，根号里分母多了一个 2）

弹簧串联：$\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}$

并联：$k=k_1+k_2$

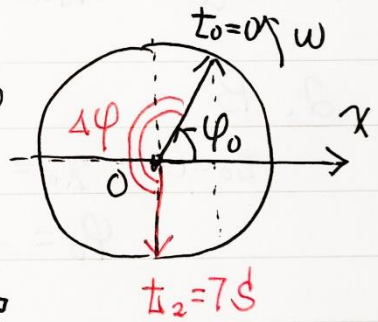
$\omega=\sqrt{\frac{k}{m}}$，$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

5. 设：$x=A\cos(\omega t+\varphi_0)$

$A=0.10\,m$

$t_0=0$，$x_0=+\frac{A}{2}$，$v_0<0$

$\varphi_0=\frac{\pi}{3}$



$t_2=7s$，

第2次回到平衡位置

$\Delta\varphi=\frac{2}{3}\pi+\frac{\pi}{2}=\frac{7}{6}\pi$

$\Delta t=t_2-t_0=7s$

$\omega=\frac{\Delta\varphi}{\Delta t}=\frac{1}{6}\pi$

$x=0.10\cos(\frac{\pi}{6}t+\frac{\pi}{3})\,m$

6. $\omega = \sqrt{\frac{k}{m}}$, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$T_1 : T_2 = \sqrt{m_1} : \sqrt{m_2} = 2 : 1$

7. $A = 0.1\ m$, $T = 2\ s \Rightarrow \omega = \frac{2\pi}{T} = \pi\ s^{-1}$

$x = A\cos(\omega t + \varphi_0) = 0.1\cos(\pi t + \varphi_0)$

1) $t_0 = 0$, $x_0 = 0$, $v_0 > 0$

$\varphi_0 = -\frac{\pi}{2}$

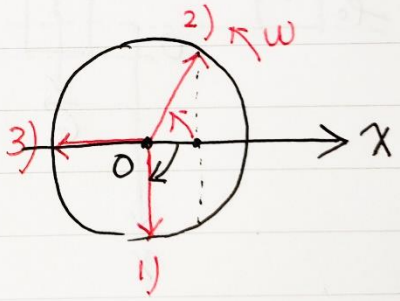
2) $t_0 = 0$, $x_0 = +\frac{A}{2}$

$v_0 < 0$

$\varphi_0 = \frac{\pi}{3}$

3) $t_0 = 0$, $x_0 = -A$

$\varphi_0 = \pi$

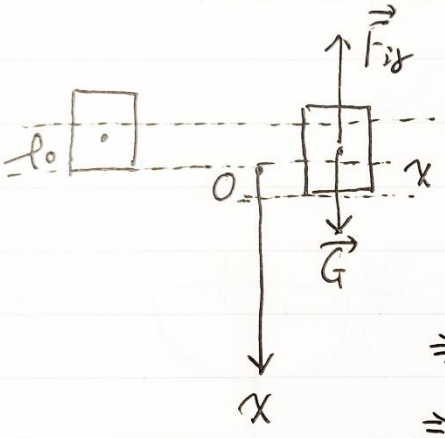


8. $S=a^2$，设木块体积 V_0，立方体？ $V_0=a^3$

1) 平衡时：$G=F_{浮} \Rightarrow mg=\rho_{木}V_0g=\rho_{水}gV_{排}$

$\Rightarrow \rho_{木}V_0g=\rho_{水}gSl_0=\rho_{水}ga^2l_0$

取平衡时位置为原点，向下为 x 轴由.



2) 位移为 x 时，

$\vec{F}_{合}=\vec{G}+\vec{F}_{浮}$

$F_{合}=mg-F_{浮}$

$=\rho_{木}gV_0-\rho_{水}gV_{排}$

$\Rightarrow F_{合}=\rho_{木}gV_0-\rho_{水}g(l_0+x)a^2$

$\Rightarrow F_{合}=\rho-\rho_{水}ga^2x=m\alpha=m\frac{d^2x}{dt^2}$

$\Rightarrow -\rho_{水}ga^2x=\rho_{木}V_0\frac{d^2x}{dt^2}$

$\Rightarrow \frac{d^2x}{dt^2}+\left(\frac{\rho_{水}ga^2}{\rho_{木}V_0}\right)x=0$

ω^2

$\Rightarrow \omega=\sqrt{\frac{\rho_{水}ga^2}{\rho_{木}V_0}}$

$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{\rho_{木}V_0}{\rho_{水}ga^2}}=2\pi\sqrt{\frac{\rho_{木}a}{\rho_{水}g}}$

9、

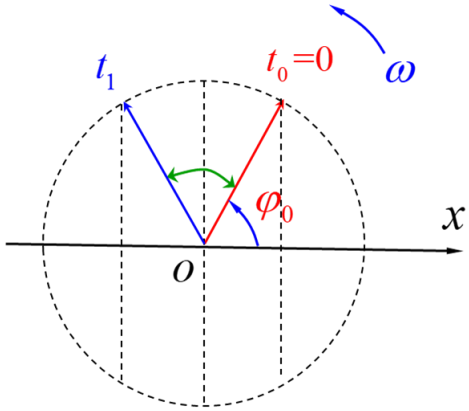
解：

$\omega = \pi\ (s^{-1})$

$t_0 = 0,\quad \varphi_0 = \dfrac{\pi}{3},$

$\Delta\varphi = \dfrac{\pi}{3},$

$\Delta t = t_1 - t_0 = \dfrac{\Delta\varphi}{\omega} = \dfrac{1}{3}\text{ s},$



10、 $A = 0.20\text{ m}$，$x = A\cos(\omega t + \varphi_0)$

$E_k = \frac{1}{2}kA^2\sin^2(\omega t + \varphi_0)$

$E_p = \frac{1}{2}kA^2\cos^2(\omega t + \varphi_0)$

$E = E_k + E_p = \frac{1}{2}kA^2$

1). $E_k = E_p \Rightarrow \cos(\omega t + \varphi_0) = \pm\frac{\sqrt{2}}{2}$

$x = A\cos(\omega t + \varphi_0) = \pm\frac{\sqrt{2}}{2}A$

$= \pm\frac{\sqrt{2}}{10}\text{ m}$

2). 当 $x' = A\cos(\omega t' + \varphi_0) = \frac{A}{2}$ 时，

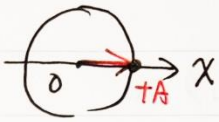
$\cos(\omega t' + \varphi_0) = \pm\frac{1}{2}$

$E_k = \frac{1}{2}kA^2 \times (1 - \frac{1}{4}) = \frac{3}{8}kA^2 = \frac{3}{4}E$

9. 机械振动（二）

1. B （反本旧）

2. B



3. C 由：$v=|v_2-v_1|$ 判断

4. $\Delta\varphi=(4\pi t+\varphi_0)-(4\pi t+\frac{\pi}{3})=\varphi_0-\frac{\pi}{3}$

① 当 $\Delta\varphi=\pm 2k\pi$ 时，$A_{max}=A_1+A_2=7\times10^{-2}m$

$\Rightarrow \varphi_0=\pm 2k\pi+\frac{\pi}{3}$，

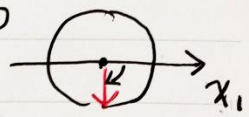
② 当 $\Delta\varphi=\pm(2k+1)\pi$ 时，$A_{min}=|A_1-A_2|$

$\Rightarrow \varphi_0=\pm(2k+1)\pi+\frac{\pi}{3}$ $=1\times10^{-2}m$

或：$\varphi_0=\pm 2k\pi+\frac{4}{3}\pi$

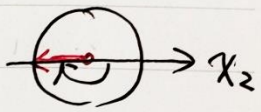
5. ① x_1：$t_0=0$，$x_{10}=0$，$v_{10}>0$

$\varphi_{01}=-\frac{\pi}{2}$



② x_2：$t_0=0$，$x_{20}=-A$.

$\varphi_{02}=-\pi$



$\Delta\varphi=\varphi_2-\varphi_1=(\omega t+\varphi_{02})-(\omega t+\varphi_{01})$

$\Rightarrow \Delta\varphi=-\frac{\pi}{2}$，$\varphi_2$ 比 φ_1 落后 $\frac{\pi}{2}$

6. 1) 0.5 s, 1.5 s

2) 0, 1, 2 s

7. $x=0.1\cos(3\pi t+\frac{2}{3}\pi)$ m

$A=0.1$ m, $\omega=3\pi$, $T=\frac{2\pi}{\omega}=\frac{2}{3}$ s

$\varphi_0=\frac{2}{3}\pi$

$v=\frac{dx}{dt}=-0.3\pi\sin(3\pi t+\frac{2}{3}\pi)$

$v_m=0.3\pi$ m/s

$a=\frac{dv}{dt}=-0.9\pi^2\cos(3\pi t+\frac{2}{3}\pi)$

$a_m=0.9\pi^2$ m/s^2

8、

解：

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{k}{m_1+m_2}}=10(\mathrm{s}^{-1})$$

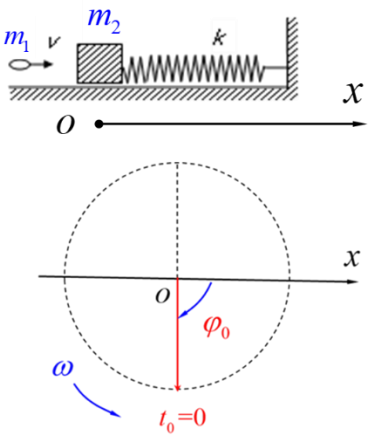
$$m_1v=(m_1+m_2)v_0$$

$$v_0=(\frac{m_1}{m_1+m_2})v=0.8\ (\mathrm{m/s})$$

$t_0=0, x_0=0, v_0=0.8,$

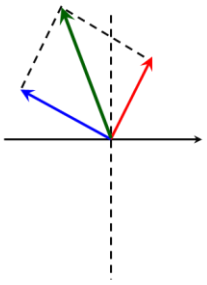
$$A=\sqrt{x_0^2+\frac{v_0^2}{\omega^2}}=0.08\ (\mathrm{m})$$

$$\varphi_0=-\frac{\pi}{2},\qquad x=0.08\cos(10t-\frac{\pi}{2})\ (\mathrm{m}),$$



9、

解：$A_1 = 6\text{cm}$，$\varphi_{01} = \dfrac{\pi}{3}$，$A_2 = 8\text{cm}$，$\varphi_{02} = \dfrac{5\pi}{6}$



$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_{02} - \varphi_{01})}$$

$$A = \sqrt{6^2 + 8^2 + 2\times 6\times 8\cos(\frac{5\pi}{6} - \frac{\pi}{3})} = 10(\text{cm})$$

$$\tan\varphi_0 = \frac{A_1\sin\varphi_{01} + A_2\sin\varphi_{02}}{A_1\cos\varphi_{01} + A_2\cos\varphi_{02}} = -2.344$$

$$\varphi_0 = 113^0 = 0.63\pi(\text{rad})$$

10. a：$A_1 = 2\text{cm}$

$t_0 = 0$，$x_{01} = \dfrac{A_1}{2}$，$v_{01} > 0$

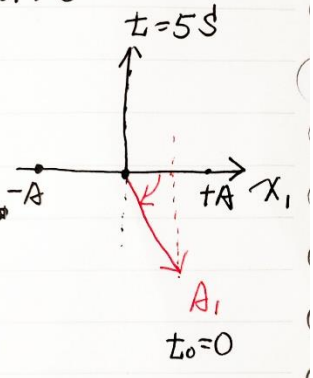
$\Rightarrow \varphi_{01} = -\dfrac{\pi}{3}$

$t = 5\text{s}$，$x_1 = 0$

$\Delta\varphi = \dfrac{\pi}{2} + \dfrac{\pi}{3} = \dfrac{5}{6}\pi$

$\Delta t = t - t_0 = 5\text{s}$

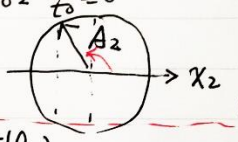
$\omega = \dfrac{\Delta\varphi}{\Delta t} = \dfrac{\pi}{6}\text{s}^{-1}$



b：$A_2 = 1\text{cm}$

$t_0 = 0$，$x_{02} = -\dfrac{A_2}{2}$，$v_{02} < 0$

$\varphi_{02} = \dfrac{2}{3}\pi$



合振动：$x = x_1 + x_2 = A\cos(\omega t + \varphi_0)$

x_1，x_2 相位差：$\Delta\varphi = \varphi_2 - \varphi_1 = \dfrac{2}{3}\pi - (-\dfrac{\pi}{3})$

$\Rightarrow \Delta\varphi = \pi$ → 反相.

$\vec{A} = \vec{A_1} + \vec{A_2}$

$A = A_1 - A_2 = 1\text{cm}$

$\varphi_0 = -\dfrac{\pi}{3}$，$T = \dfrac{2\pi}{\omega} = 12\text{s}$