MPC HW #4

1) $V(x) = 9x_1^2 + 25x_2^2 + 16x_3^2$

a) Find Q such that

$$V(x) = x^T Q x$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^T$$; Q diagonal will just be coefficients

$$\boxed{Q = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 16 \end{bmatrix}}$$

b) since the diagonal of $Q > 0$

$\boxed{Q \text{ is positive definite}}$

c) $\frac{\delta V}{\delta x} = \Delta V = \frac{\delta V}{\delta x_1} + \frac{\delta V}{\delta x_2} + \frac{\delta V}{\delta x_3}$

$V(x) = 9x_1^2 + 25x_2^2 + 16x_3^2$

$$\boxed{\Delta V = 18x_1 + 50x_2 + 32x_3}$$

$$\boxed{= \begin{bmatrix} 18 & x_1 \\ 50 & x_2 \\ 32 & x_3 \end{bmatrix}}$$

(2) $V(x) = 5x_1^2 + 2x_2^2 + x_3^2 + 100u_1^2 + 4u_2^2$

a) Find Q and R such that

$V(x) = x^T Q x + u^T R u$ $\quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad R = \begin{bmatrix} 100 & 0 \\ 0 & 4 \end{bmatrix}$$

c)

$$\frac{\partial V}{\partial x} = \nabla V_x = 10x_1 + 2x_2 + x_3 = \begin{bmatrix} 10x_1 \\ 2x_2 \\ 1x_3 \end{bmatrix} = \nabla V_x$$

$$\frac{\partial V}{\partial u} = \nabla V_u = 200u_1 + 4u_2 = \begin{bmatrix} 200u_1 \\ 4u_2 \end{bmatrix} = \nabla V_u$$

b) Q and R are positive definite.

Diagonal > 0

3) $x(k+1) = 9x(k) + 0.5u(k)$
$y(k) = x(k)$

Step disturbance introduced.

$$u(k) = (u(k) + d(k))$$

$$x(k+1) = 9x(k) + 0.5[u(k) + d(k)] \qquad d(k) = d(k+1)$$
$$y(k) = x(k)$$

$$\hat{x}(k+1) = 9\hat{x}(k) + 0.5d(k) + 0.5u(k)$$
$$y(k) = \hat{x}(k)$$

$$\hat{x}(k+1) = \begin{bmatrix} 9 & 0.5 \\ 0 & 1 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = [1\ 0]\hat{x}(k)$$

b) Design state observer $\lambda_1 = 0.1$; $\lambda_2 = 0.2$

$L = ?$ $\quad Q = \begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0.5 \end{pmatrix}$ ✓ $\quad (1\ 0)\begin{pmatrix} 9 & 0.5 \\ 0 & 1 \end{pmatrix} = (1\ 0.5)$

$\alpha_c(A) Q^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\qquad$ Observable. Rank$(Q) = 2$

Ackermann's

$$\alpha_c(A) = (\lambda - 0.1)(\lambda - 0.2)$$
$$= \lambda^2 - 0.3\lambda + 0.02$$

$$L = (A^2 - 0.3A + 0.02) \begin{pmatrix} 1 & 0 \\ 1 & 0.5 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$(A^2 - 0.3A + 0.02) \begin{pmatrix} 1 & 0 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = ($$

$$(A^2 - 0.3A + 0.02I)\begin{pmatrix}1 & 0\\ -2 & 2\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} \quad ; \quad A = \begin{pmatrix}9 & 0.5\\ 0 & 1\end{pmatrix}$$

$$A^2 = \begin{pmatrix}9 & 0.5\\ 0 & 1\end{pmatrix}\begin{pmatrix}9 & 0.5\\ 0 & 1\end{pmatrix} = \begin{pmatrix}81 & 5\\ 0 & 1\end{pmatrix}$$

$$0.3A = \begin{pmatrix}2.7 & 0.15\\ 0 & 0.3\end{pmatrix}$$

$$\begin{pmatrix}81 & 5\\ 0 & 1\end{pmatrix} - \begin{pmatrix}2.7 & 0.15\\ 0 & 0.3\end{pmatrix} + \begin{pmatrix}0.02 & 0\\ 0 & 0.02\end{pmatrix} = \begin{pmatrix}78.32 & 4.85\\ 0 & 0.72\end{pmatrix}$$

$$\begin{pmatrix}78.32 & 4.85\\ 0 & 0.72\end{pmatrix}\begin{pmatrix}1 & 0\\ -2 & 2\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \boxed{\begin{pmatrix}9.7\\ 1.44\end{pmatrix} = L}$$