

## ① MATLAB for Computation

$${}^0T_1 = \begin{pmatrix} c_1 & s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2T_3 = \begin{pmatrix} c_3 & s_3 & 0 & l_3 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1T_2 = \begin{pmatrix} c_2 & s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^3T_{ec} = \begin{pmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0T_{ec} = \begin{pmatrix} c_1 c_2 c_3 - s_1 s_2 c_3 - c_1 c_2 s_3 + s_1 s_2 s_3 & -c_1 s_2 c_3 - s_1 c_2 c_3 - c_1 c_2 s_3 + s_1 s_2 s_3 & 0 & 0 \\ c_1 s_2 c_3 + s_1 c_2 c_3 + c_1 c_2 s_3 - s_1 s_2 s_3 & c_1 c_2 c_3 - s_1 s_2 c_3 - c_1 c_2 s_3 + s_1 s_2 s_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where  $\begin{cases} \alpha = l_2(c_1 c_2 - s_1 s_2) + l_1 c_1 + l_3(c_1 c_2 c_3 - s_1 s_2 c_3 - c_1 c_2 s_3 + s_1 s_2 s_3) \\ \beta = l_2(c_1 s_2 + c_2 s_1) + l_1 s_1 + l_3(c_1 s_2 c_3 + s_1 c_2 c_3 + c_1 c_2 s_3 - s_1 s_2 s_3) \end{cases}$

# Problem 1

```
clc;
clear;
close all;

%Technique A

syms t [1 3] %Thetas
syms L [1 3] %Length

%T01
z01 = rotz(t1);
t01 = z01

%t12
z12 = rotz(t2);
tr12 = transl(L1, 0, 0);
t12 = tr12*z12

t02 = t01*t12;

%t23
z23 = rotz(t3);
tr23 = transl(L2, 0, 0);
t23 = tr23*z23

t03 = t02*t23;

%t3ee
tr3ee = transl(L3, 0, 0);
t3ee = tr3ee

t0eee = t03*t3ee;
%Final Answer t0ee
t0ee = t01*t12*t23*t3ee
```

t01 =

```
[cos(t1), -sin(t1), 0, 0]
[sin(t1),  cos(t1), 0, 0]
[      0,          0, 1, 0]
[      0,          0, 0, 1]
```

t12 =

```
[cos(t2), -sin(t2), 0, L1]
[sin(t2),  cos(t2), 0,  0]
[      0,          0, 1,  0]
[      0,          0, 0,  1]
```

t23 =

```
[cos(t3), -sin(t3), 0, L2]
[sin(t3),  cos(t3), 0,  0]
[      0,          0, 1,  0]
[      0,          0, 0,  1]
```

t3ee =

```
[1, 0, 0, L3]
[0, 1, 0,  0]
[0, 0, 1,  0]
[0, 0, 0,  1]
```

t0ee =

```
[cos(t3)*(cos(t1)*cos(t2) - sin(t1)*sin(t2)) - sin(t3)*(cos(t1)*sin(t2) + cos(t2)*sin(t1)), - cos(t3)*(cos(t1)*sin(t2) + cos(t2)*sin(t1)) - sin(t3)*(cos(t1)*cos(t2)
[cos(t3)*(cos(t1)*sin(t2) + cos(t2)*sin(t1)) + sin(t3)*(cos(t1)*cos(t2) - sin(t1)*sin(t2)),  cos(t3)*(cos(t1)*cos(t2) - sin(t1)*sin(t2)) - sin(t3)*(cos(t1)*sin(t2)
[
                                0,
[
                                0,
```

# Problem 1

```

clc;
clear;
close all;

syms t [1 3] %Thetas
syms L [1 3] %Length

%%Technique B

Link1 = link([0 0 0 0 0], 'modified');
Link2 = link([0 L1 0 0 0], 'modified');
Link3 = link([0 L2 0 0 0], 'modified');
Linkee = link([0 L3 0 0 0], 'modified');

r1 = robot({Link1 Link2 Link3 Linkee});
Q = [0 0 0 0];
Tarm = fkine(r1, Q)

%Thetas
t1 = pi/2;
t2 = -pi/2;
t3 = -pi/2;
%Length
L1 = 10;
L2 = 10;
L3 = 5;

Link1 = link([0 0 0 0 0], 'modified');
Link2 = link([0 L1 0 0 0], 'modified');
Link3 = link([0 L2 0 0 0], 'modified');
Linkee = link([0 L3 0 0 0], 'modified');

r1 = robot({Link1 Link2 Link3 Linkee});
Q = [t1 t2 t3 0];
Tarm = fkine(r1, Q)

view(3);
grid on;
plot(r1,Q)

```

Tarm =

```

[1, 0, 0, L1 + L2 + L3]
[0, 1, 0, 0]
[0, 0, 1, 0]
[0, 0, 0, 1]

```

Link1 =

0.000000      0.000000      0.000000      0.000000      R      (mod)

B = 0.000000

Tc = 0.000000(+) 0.000000(-)

# Problem 1

Tarm =

|         |        |        |         |
|---------|--------|--------|---------|
| 0.0000  | 1.0000 | 0      | 10.0000 |
| -1.0000 | 0.0000 | 0      | 5.0000  |
| 0       | 0      | 1.0000 | 0       |
| 0       | 0      | 0      | 1.0000  |

Warning: The DrawMode property will be removed in a future release. Use the SortMethod property instead.

Warning: The EraseMode property is no longer supported and will error in a future release.

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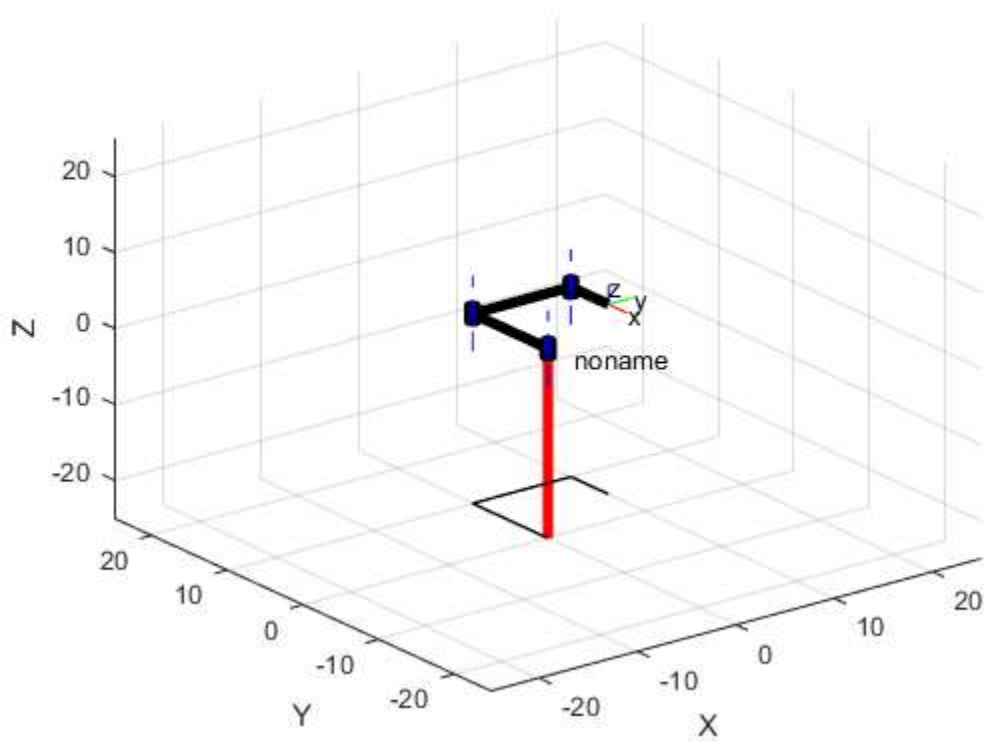
Warning: The EraseMode property is no longer supported and will error in a future release.

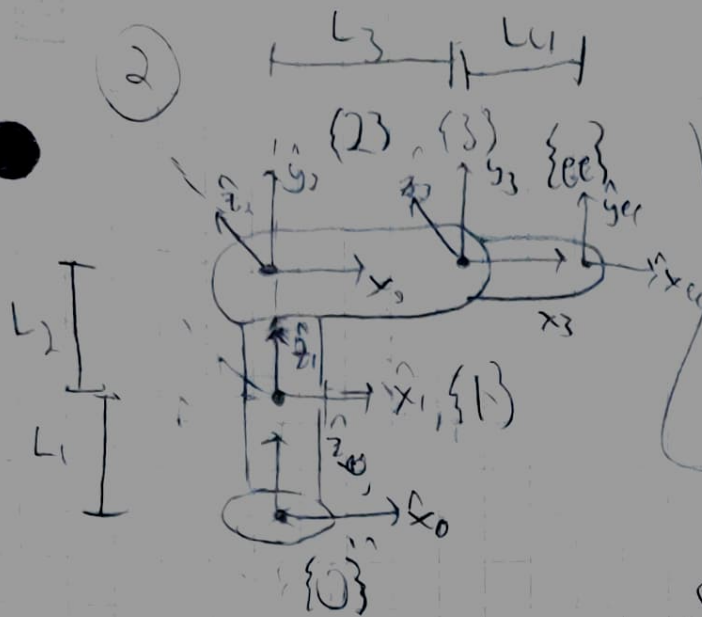
Warning: The EraseMode property is no longer supported and will error in a future release.

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## 3D view of Problem 1





| i  | $a_{i-1}$ | $\alpha_{i-1}$ | $\theta_i$ | $d_i$ |
|----|-----------|----------------|------------|-------|
| 1  | 0         | 0              | $\theta_1$ | $L_1$ |
| 2  | 0         | 0              | $\theta_2$ | $L_2$ |
| 3  | 0         | $L_3$          | $\theta_3$ | 0     |
| ee | 0         | $L_4$          | 0          | 0     |

SEE MATLAB

$${}^0_1T \Rightarrow \text{Transl}(0, 0, L_1) \text{Rot}_z(\theta_1)$$

$${}^0_1T = \begin{pmatrix} c_1 & s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1_2T \Rightarrow \text{Transl}(0, 0, L_2) \text{Rot}_x(90^\circ) \text{Rot}_z(\theta_2)$$

$${}^1_2T = \begin{pmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2_3T \Rightarrow \text{Transl}(L_3, 0, 0) \text{Rot}_z(\theta_3)$$

$${}^2_3T = \begin{pmatrix} c_3 & -s_3 & 0 & L_3 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^3_{ee}T \Rightarrow \text{Transl}(L_4, 0, 0) = \begin{pmatrix} 1 & 0 & 0 & L_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

OT =  
ee

$$\begin{pmatrix} c_1 c_2 c_3 - c_1 s_2 s_3 & -c_1 c_2 s_3 - c_1 s_2 c_3 \\ s_1 c_2 c_3 - s_1 s_2 s_3 & -s_1 c_2 s_3 - s_1 s_2 c_3 \\ c_2 s_3 + s_2 c_3 & c_2 c_3 - s_2 s_3 \end{pmatrix}$$

s<sub>1</sub>  
-c<sub>1</sub>  
0  
0

$$\begin{pmatrix} L_3 c_1 c_2 + L_4 c_1 c_2 c_3 - c_1 s_2 s_3 \\ L_4 s_1 c_2 c_3 + s_1 s_2 s_3 + L_3 s_1 c_2 \\ L_1 + L_2 + L_3 s_2 + L_4 (c_2 s_3 + s_2 c_3) \end{pmatrix}$$

## Problem 2

```
clc;
clear;
close all;

%Technique B

syms t [1 3] %Thetas
syms L [1 4] %Length

%t01
tr01 = transl(0, 0, L1);
z01 = rotz(t1);

t01 = tr01*z01

%t12
tr12 = transl(0, 0, L2);
z12 = rotz(t2);
x12 = rotx(pi/2);

t12 = vpa(tr12*x12*z12,2)

%t23
tr23 = transl(L3, 0, 0);
z23 = rotz(t3);

t23 = vpa(tr23*z23,2)

%t3ee
tr3ee = transl(L4,0,0);
t3ee = tr3ee

%Final T links
t0ee = vpa(t01*t12*t23*t3ee,2)
```

t01 =

```
[cos(t1), -sin(t1), 0, 0]
[sin(t1), cos(t1), 0, 0]
[ 0, 0, 1, L1]
[ 0, 0, 0, 1]
```

t12 =

```
[ cos(t2), -1.0*sin(t2), 0, 0]
[6.1e-17*sin(t2), 6.1e-17*cos(t2), -1.0, 0]
[ sin(t2), cos(t2), 6.1e-17, L2]
[ 0, 0, 0, 1.0]
```

t23 =

```
[cos(t3), -1.0*sin(t3), 0, L3]
[sin(t3), cos(t3), 0, 0]
[ 0, 0, 1.0, 0]
[ 0, 0, 0, 1.0]
```

t3ee =

```
[1, 0, 0, L4]
[0, 1, 0, 0]
[0, 0, 1, 0]
[0, 0, 0, 1]
```

t0ee =

```
[cos(t3)*(cos(t1)*cos(t2) - 6.1e-17*sin(t1)*sin(t2)) - 1.0*sin(t3)*(1.0*cos(t1)*sin(t2) + 6.1e-17*cos(t2)*sin(t1)), - 1.0*sin(t3)*(cos(t1)*cos(t2) - 6.1e-17*sin(t1)
[ cos(t3)*(6.1e-17*cos(t1)*sin(t2) + cos(t2)*sin(t1)) + sin(t3)*(6.1e-17*cos(t1)*cos(t2) - 1.0*sin(t1)*sin(t2)), cos(t3)*(6.1e-17*cos(t1)*cos(t2) - 1.0*sin
[ cos(t2)*sin(t3) + cos(t3)*sin(t2),
[ 0,
```



## Problem 2

```
clc;
clear;
close all;

%syms t [1 3] %Thetas
syms L [1 4] %Length

%Thetas
t1 = vpa(pi/2,2);
t2 = vpa(pi/2,2);
t3 = vpa(pi/2,2);
%Length
L1 = 1;
L2 = 1;
L3 = 1;
L4 = 1;

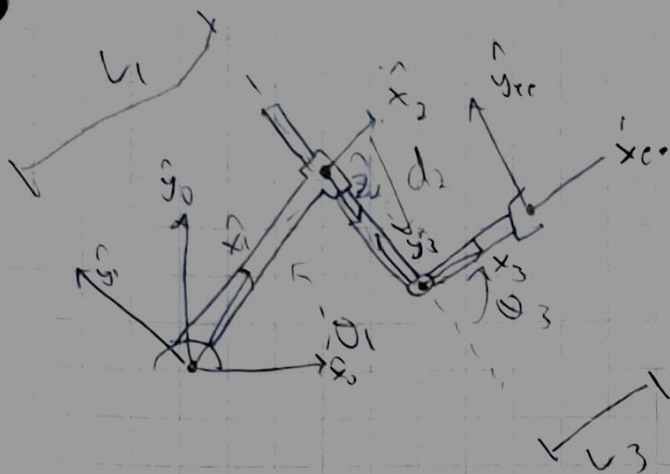
Link1 = link([0 0 0 L1 0], 'modified');
Link2 = link([pi/2 0 0 L2 0], 'modified');
Link3 = link([0 L3 0 0 0], 'modified');
Linkee = link([0 L4 0 0 0], 'modified');

r1 = robot({Link1 Link2 Link3 Linkee});
Q = [t1 t2 t3 0];

Tarm = vpa(fkine(r1, Q),2)
```

Tarm =

```
[-6.1e-11, 6.1e-17, 1.0, 1.0]
[ -1.0, -1.2e-10, -6.1e-11, -1.0]
[ 1.2e-10, -1.0, 6.1e-17, 2.0]
[ 0, 0, 0, 1.0]
```



| i  | $a_{i-1}$ | $\alpha_{i-1}$ | $\theta_i$ | $d_i$ |
|----|-----------|----------------|------------|-------|
| 1  | 0         | 0              | $\theta_1$ | 0     |
| 2  | 0         | $L_1$          | 0          | $d_2$ |
| 3  | $-90$     | 0              | $\theta_3$ | 0     |
| ee | 0         | $L_3$          | 0          | 0     |

MATLAB

$${}^0_1 T = R_{d2} \theta_1 = \begin{pmatrix} c_1 & s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$${}_1^2 T = \text{Trans}(L_1, 0, 0) = \begin{pmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}_2^3 T = \text{Trans}(0, 0, d_2) R_{d3} \theta_3 = \begin{pmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}_3^{ee} T = \text{Trans}(L_3, 0, 0) = \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_{ee} T = \begin{pmatrix} c_1 c_3 - s_1 s_3 & -c_1 s_3 - s_1 c_3 & 0 & L_3(c_1 c_3 - s_1 s_3) + L_1 c_1 \\ c_1 s_3 - c_1 s_1 & c_1 c_3 - s_1 s_3 & 0 & L_3(c_1 s_3 + c_1 s_1) + L_1 s_1 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{orm} = \begin{pmatrix} 0 & 0.8912 & -0.4538 & 0 \\ 0 & -0.4538 & -0.8912 & 1 \\ -1 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

MATLAB

## Problem 3

```
clc;
clear;
close all;

syms t1 t3 L1 L3 d2;

%t01
z01 = rotz(t1);
t01 = z01;
%t12
tr12 = transl(L1, 0, 0);
t12 = tr12;
%t23
tr23 = transl(0,0,d2);
z23 = rotz(t3);
t23 = tr23*z23;
%t3ee
tr3ee = transl(L3,0,0);
t3ee = tr3ee;
%t0ee ANSWER
t0ee = t01*t12*t23*t3ee

%Endpoint location
t1 = pi/2;
d2 = 1.1;
t3 = pi/2;
L1 = 1;
L3 = 1;

Link1 = link([0 0 0 0 0], 'modified');
Link2 = link([0 L1 0 0 0], 'modified');
Link3 = link([-pi/2 0 0 0 0], 'modified');
Linkee = link([0 L3 0 0 0], 'modified');

r1 = robot({Link1 Link2 Link3 Linkee});
Q = [t1 d2 t3 0];

Tarm = fkine(r1, Q)
```

t0ee =

```
[cos(t1)*cos(t3) - sin(t1)*sin(t3), -cos(t1)*sin(t3) - cos(t3)*sin(t1), 0, L3*(cos(t1)*cos(t3) - sin(t1)*sin(t3)) + L1*cos(t1)]
[cos(t1)*sin(t3) + cos(t3)*sin(t1), cos(t1)*cos(t3) - sin(t1)*sin(t3), 0, L3*(cos(t1)*sin(t3) + cos(t3)*sin(t1)) + L1*sin(t1)]
[0, 0, 0, 1, d2]
[0, 0, 0, 0, 1]
```

Tarm =

```
-0.0000    0.8912   -0.4536   -0.0000
-0.0000   -0.4536   -0.8912    1.0000
-1.0000   -0.0000    0.0000   -1.0000
    0         0         0         1.0000
```

④ Khan  $\begin{matrix} B \\ T \\ ec \end{matrix}$   $\begin{matrix} C \\ T \\ 7 \end{matrix}$   $\begin{matrix} S \\ T \\ C \end{matrix}$   $\begin{matrix} S \\ T \\ B \end{matrix}$

{B} = Base {ec} = end effector  
{C} = camera {T} = Target {S} = station

• Find  $\begin{matrix} S \\ T \\ T \end{matrix}$

$$\begin{pmatrix} S \\ T \\ T \end{pmatrix} = \begin{pmatrix} S \\ T \\ C \end{pmatrix} \cdot \begin{pmatrix} C \\ T \\ T \end{pmatrix}$$

• And  $\begin{matrix} B \\ T \\ T \end{matrix}$

$$\begin{pmatrix} B \\ T \\ T \end{pmatrix} = \left( \begin{pmatrix} S \\ T \\ C \end{pmatrix} \right)^{-1} \cdot \begin{pmatrix} S \\ T \\ C \end{pmatrix} \cdot \begin{pmatrix} C \\ T \\ T \end{pmatrix}$$

$$\begin{pmatrix} B \\ T \\ T \end{pmatrix} = \begin{pmatrix} B \\ T \\ S \end{pmatrix} \cdot \begin{pmatrix} S \\ T \\ C \end{pmatrix} \cdot \begin{pmatrix} C \\ T \\ T \end{pmatrix}$$

• And  $\begin{matrix} S \\ T \\ ec \end{matrix}$

$$\begin{pmatrix} S \\ T \\ ec \end{pmatrix} = \begin{pmatrix} S \\ T \\ B \end{pmatrix} \cdot \begin{pmatrix} B \\ T \\ ec \end{pmatrix}$$

## Problem 5

```
clc;
clear;
close all;
```

```
syms ti ai di
alpha = vpa(-pi/2,2);
```

```
Link1 = link([alpha 0 0 0 0], 'modified');
Link2 = link([0 0 0 0 0], 'modified');
Link3 = link([0 0 0 0 0], 'modified');
Linkee = link([0 0 0 0 0], 'modified');
```

```
r1 = robot({Link1 Link2 Link3 Linkee});
Q = [0 ai ti di];
```

```
Tarm = vpa(fkine(r1, Q),2)
```

Tarm =

```
[
    - 1.0*sin(di)*(cos(ai)*sin(ti) + sin(ai)*cos(ti)) - 1.0*cos(di)*(sin(ai)*sin(ti) - 1.0*cos(ai)*cos(ti)),
cos(di)*(6.1e-11*cos(ai)*sin(ti) + 6.1e-11*sin(ai)*cos(ti)) - 1.0*sin(di)*(6.1e-11*sin(ai)*sin(ti) - 6.1e-11*cos(ai)*cos(ti)), - 1.0*cos(di)*(6.1e-11*sin(ai)*sin(t
[
    sin(di)*(1.0*sin(ai)*sin(ti) - 1.0*cos(ai)*cos(ti)) - 1.0*cos(di)*(1.0*cos(ai)*sin(ti) + 1.0*sin(ai)*cos(ti)),
cos(di)*(1
[
    0,
```



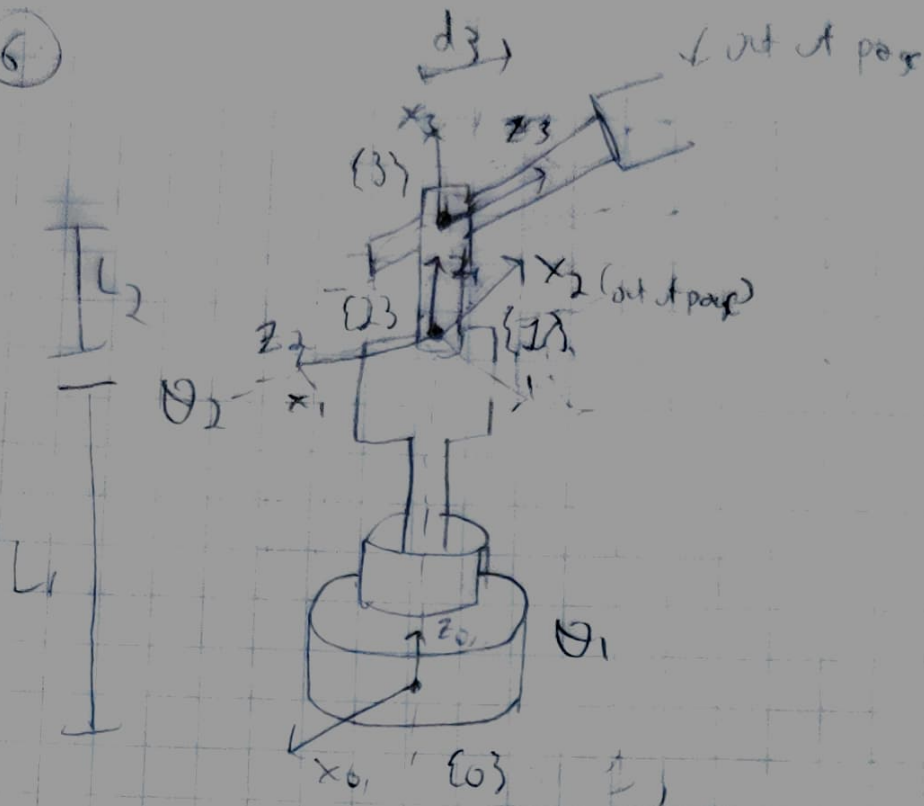
5

cc  
p  
o  
r

| $i$ | $a_{i-1}$ | $a_i$     | $O_i$ | $d_i$ |
|-----|-----------|-----------|-------|-------|
| 1   | $a_0$     | $O_1$     | $O_1$ | $d_1$ |
| 2   | $O_1$     | $O_{i-1}$ | $O_i$ | $d_i$ |
| 3   | $O_2$     | $O_i$     | $O_i$ | $d_i$ |
| 4   | $O_3$     | $O_i$     | $O_i$ | $d_i$ |

See MATLAB

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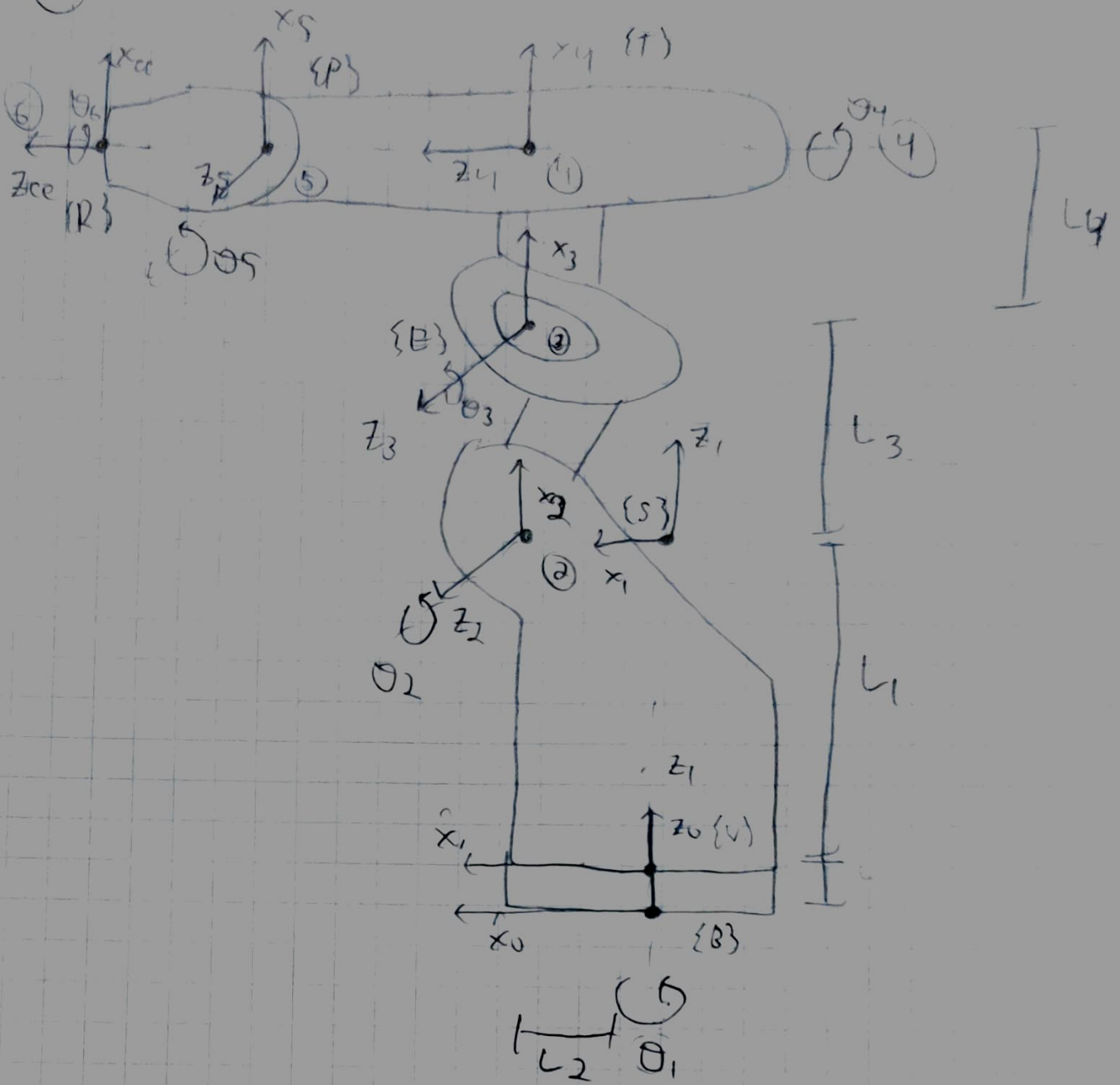


| $i$ | $d_{i-1}$  | $\alpha_{i-1}$ | $\theta_i$ | $d_i$ |
|-----|------------|----------------|------------|-------|
| 1   | 0          | 0              | $\theta_1$ | $L_1$ |
| 2   | $90^\circ$ | $L_2$          | $\theta_2$ | 0     |
| 3   | 0          | 0              | 0          | $d_3$ |

$$\theta_1 = 90^\circ$$

$$\theta_2 = 90^\circ$$

(7)



~~Between  $\{1\}$  and  $\{2\}$ , no MP~~

|   | $i$ | $\alpha_{i-1}$ | $a_{i-1}$ | $\theta_i$ | $d_i$ |
|---|-----|----------------|-----------|------------|-------|
| W | 1   | 0              | 0         | $\theta_1$ | $L_1$ |
| S | 2   | $-90$          | $L_2$     | $\theta_2$ | 0     |
| E | 3   | 0              | $L_3$     | $\theta_3$ | 0     |
| T | 4   | $90$           | $L_4$     | $\theta_4$ | 0     |
| P | 5   | $90$           | 0         | $\theta_5$ | $L_5$ |
| R | 6   | $90$           | 0         | $\theta_6$ | $L_6$ |



## Problem 7

```
clc;
clear;
close all;

syms t [1 6];
%syms L [1 6];
L1 = 350;
L2 = 100;
L3 = 250;
L4 = 130;
L5 = 250;
L6 = 85;

LinkW1 = link([0 0 0 L1 0], 'modified');
LinkS2 = link([-pi/2 L2 0 0 0 -pi/2], 'modified');
LinkE3 = link([0 L3 0 0 0], 'modified');
LinkT4 = link([-pi/2 L4 0 0 0], 'modified');
LinkP5 = link([pi/2 0 0 L5 0], 'modified');
LinkR6 = link([-pi/2 0 0 L6 0], 'modified');

r1 = robot({LinkW1 LinkS2 LinkE3 LinkT4 LinkP5 LinkR6});
Q = [pi/2 pi/2 -pi/2 0 -pi/2 0];

Tarm = fkine(r1, Q);
figure(1);
view(3);
grid on;
plot(r1,Q)

figure(2);
view(3);
grid on;

Qb = [pi/2 pi/2 -pi/2 0 -pi/2 0];
plot(r1,Qb);
```

Warning: The DrawMode property will be removed in a future release. Use the SortMethod property instead.

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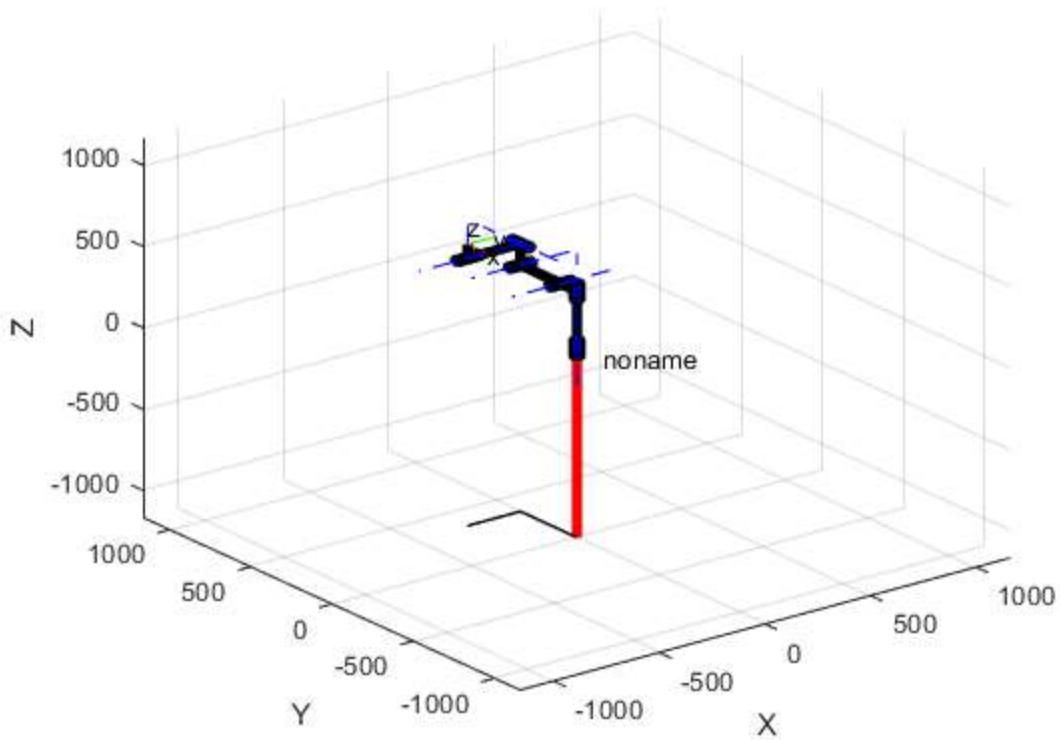
Warning: The EraseMode property is no longer supported and will error in a future release.

Warning: The EraseMode property is no longer supported and will error in a future release.

Warning: The EraseMode property is no longer supported and will error in a future release.

Warning: The EraseMode property is no longer supported and will error in a future release.

$$Q = [0 \ 0 \ 0 \ 0 \ 0 \ 0]$$



$$Q = [\pi/2 \ \pi/2 \ -\pi/2 \ 0 \ -\pi/2 \ 0]$$

