HW#1 Robotics

Aly Khater

① ✓

② ${}^{A}P_2 = s_1 \, {}^{A}P_1 + c_2 \, {}^{A}P_2$

$\theta_1 = \frac{\pi}{3} \; ; \; \theta_2 = \frac{\pi}{6} \; ; \; {}^{A}P_1^T = (3,1,5), \; {}^{A}P_2^T = (2,6,9)$

$${}^{A}P_2 = \sin\left(\frac{\pi}{3}\right)\begin{pmatrix}3\\1\\5\end{pmatrix} + \cos\left(\frac{\pi}{6}\right)\begin{pmatrix}2\\6\\9\end{pmatrix}$$

$$= 0.866\begin{pmatrix}3\\1\\5\end{pmatrix} + 0.866\begin{pmatrix}2\\6\\9\end{pmatrix}$$

$$= \begin{pmatrix}2.599\\0.866\\4.330\end{pmatrix} + \begin{pmatrix}1.732\\5.196\\7.794\end{pmatrix} = \boxed{\begin{pmatrix}4.331\\6.062\\12.124\end{pmatrix}}$$

③ See MATLAB

```matlab
%Robotics HW1 Problem 3, Aly Khater
clc;
clear;
close all;
%Vectors and Matrices definition
a = [1 2 3];
b = [1;2;3];
C = [1 1 1;1 2 2;1 2 3];
D = [2 1 3;2 1 1;2 2 2];

a3 = a*b;
b3 = b*a;
c3 = a*C;
%d3 = C*a; Incorrect dimensions
%e3 = b*D; Incorrect dimensions
f3 = D*b;
g3 = C*D;
h3 = D*C;

disp("a) "), disp(a3);
disp("b) "), disp(b3);
disp("c) "), disp(c3);
disp("d) Incorrect Dimensions");
disp("e) Incorrect Dimensions");
disp("f) "), disp(f3);
disp("g) "), disp(g3);
disp("h) "), disp(h3);
```

```
a)
    14

b)
     1     2     3
     2     4     6
     3     6     9

c)
     6    11    14

d) Incorrect Dimensions
e) Incorrect Dimensions
f)
    13
     7
    12

g)
     6     4     6
    10     7     9
    12     9    11

h)
     6    10    13
     4     6     7
```

```
     6    10    12
```

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(4) $Rotz(\theta)=\begin{pmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $Rotx(\Phi)=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c\Phi & -s\Phi & 0 \\ 0 & s\Phi & c\Phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

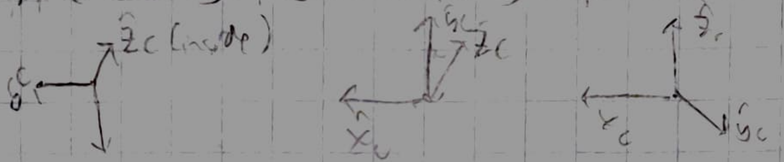
$Rotx(\Phi)\,Rotz(\theta)$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{MATLAB}} = \boxed{\begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

(5) If the rotatons are about the same axis.

(6) $^{C}_{A}T = {}^{C}_{\cancel{B}}T \cdot {}^{\cancel{B}}_{A}T$

$= Rot\,x(60^\circ)\, Rot\,z(90^\circ)\, Rot\,x(-90^\circ)$



$Trans(2,0,2)\, Rot\,x(-90)\, Rot\,z(90)\, Rot\,x(60)^{T}$

MATLAB

$$^{C}_{B}T = \begin{pmatrix} 0 & -.5 & 0.866 & 2 \\ 0 & 0.866 & 0.5 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$Trans(3,0,0)\, Rot\,z(180^\circ)$

MATLAB

$$^{B}_{A}T = \begin{pmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{C}_{A}T = \begin{pmatrix} 0 & 0.5 & 0.866 & 2 \\ 0 & -0.866 & 0.5 & 0 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(7) MATLAB

$${}^{A}_{B}T = Trans(0,4,2) \quad Rot_x(90)\, Rot_z(180)$$

$${}^{A}_{B}T = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(8) MATLAB

$${}^{C}_{A}T = {}^{C}_{B}T \cdot {}^{B}_{A}T$$

$${}^{C}_{B}T = Trans(3\cos 30^\circ, 3\sin 60^\circ, 0) \quad Rot_x(90^\circ)\, Rot_z(-150^\circ)$$

$${}^{C}_{B}T = \begin{pmatrix} -0.866 & -0.5 & 0 & 2.6 \\ 0 & 0 & -1 & 2.6 \\ 0.5 & -0.866 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{B}_{A}T = \left({}^{A}_{B}T\right)^{-1}$$

$${}^{C}_{A}T = \begin{pmatrix} 0.866 & 0 & 0.5 & 1.6 \\ 0 & 1 & 0 & -4.4 \\ -0.5 & 0 & 0.866 & -1.7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
%Robotics HW1 Problems 6,7,8, Aly Khater
clc;
clear;
close all;
%Problem 6
%cbT
x1a = rotx(-pi/2);
z1a = rotz(pi/2);
x1b = rotx(pi/3);
t1a = transl(2, 0, 0);

cbTa = t1a*x1a*z1a*x1b;
%baT
z2a = rotz(pi);
t2a = transl(3,0,0);

baTa = t2a*z2a;
%Answer for Problem 6; caT
caTa = cbTa*baTa

%Problem 7
x1b = rotx(pi/2);
z1b = rotz(pi);
t1b = transl(0,4,2);
%Answer for Problem 7; abT
abTb = t1b*x1b*z1b

%Problem 8
x1c = rotx(pi/2);
z1c = rotz(5*pi/6);
t1c = transl(3*cos(30*pi/180), 3*sin(60*pi/180), 0);

cbTc = t1c*x1c*z1c;
baTc = inv(abTb);%inverse of problem 7

caTc = cbTc*baTc
```

```
caTa =

   -0.0000    0.5000    0.8660    2.0000
    0.0000   -0.8660    0.5000    0.0000
    1.0000    0.0000    0.0000   -3.0000
         0         0         0    1.0000


abTb =

   -1.0000   -0.0000         0         0
    0.0000   -0.0000   -1.0000    4.0000
    0.0000   -1.0000    0.0000    2.0000
         0         0         0    1.0000


caTc =
```

```
    0.8660    0.0000    0.5000    1.5981
   -0.0000    1.0000   -0.0000   -1.4019
   -0.5000   -0.0000    0.8660   -1.7321
         0         0         0    1.0000
```

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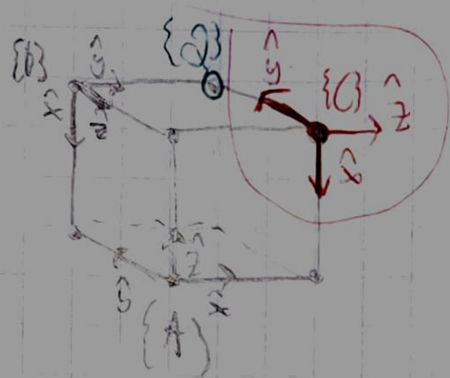
9 a)



$${}^A_B T = \begin{pmatrix} {}^A_B R & {}^A_B P \\ (0\,0\,0) & 1 \end{pmatrix}$$

Translates 1 unit in $\hat{z}$
Translates 1 unit in $\hat{y}$
Rotate around z $-90°$
Rotate around y $-90°$

$${}^A_B P = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$${}^A_B R = Rot_y(-90)\, Rot_z(-90)$$

$$= \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = {}^A_B R$$

MATLAB

$${}^A_B T = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) $${}^A_C T = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

See MATLAB

tr2eul = (0, 90°, 0)
trans(1,0,1) Roty(90°)

See cube above in Red

```
%RBT_HW1_Prob9b
acT = [0 0 1 1; 0 1 0 0; -1 0 0 1; 0 0 0 1];
rot = tr2eul(acT)
```

```
rot =

         0    1.5708         0
```

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(10) a) ${}^{A}_{C}P = (1,0,1)^T$

b) ${}^{C}_{A}T = ({}^{A}_{C}T)^{-1} = \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

${}^{C}_{A}P = (1,0,-1)^T$

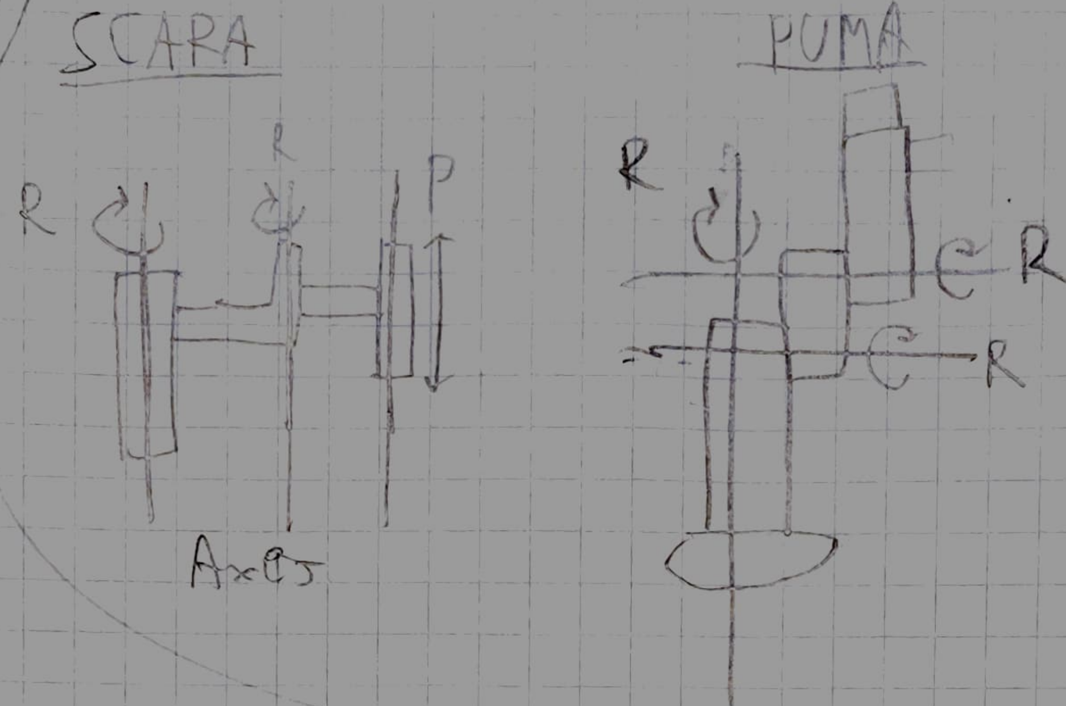
c) ${}^{A}\vec{P}_Q = (1\ 1\ 1)^T$ Find ${}^{B}P_Q$

${}^{B}P_Q = (0\ 1\ 0)^T$

d) ${}^{A}\vec{V}_Q = (-1,0,0)^T$ Find ${}^{B}\vec{V}_Q$

${}^{B}V_Q = (0,-1,0)^T$

11



12 See attached Picture for axes

- Slightly Less than 1 meter
- As long as it does not reach singularity, yes.
  (x, y, z, r, p, y)