HW 3

Alex Kholer

(1) - Much faster computationally. Don't have to go through iterations that can take awhile.

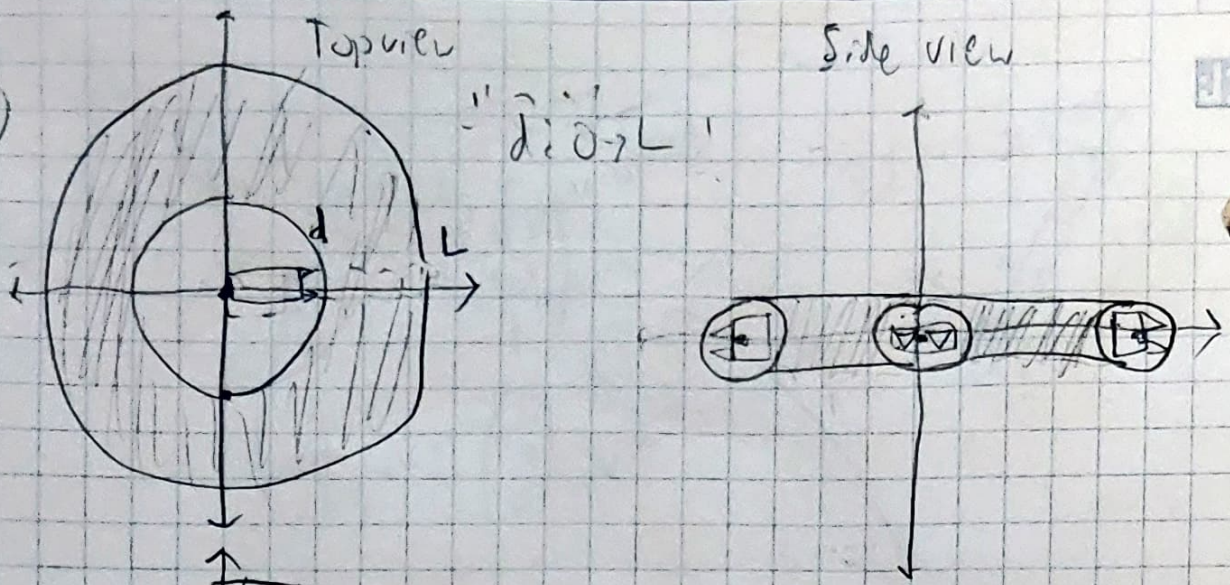
- Gives the result of the joint values needed to reach a specific endeffector position.

(2) No. 3 DOF systems are easier to derive algebraically

(3) a) Only one position. End effector is on a prismatic joint which only allows for one configuration to reach the said position.

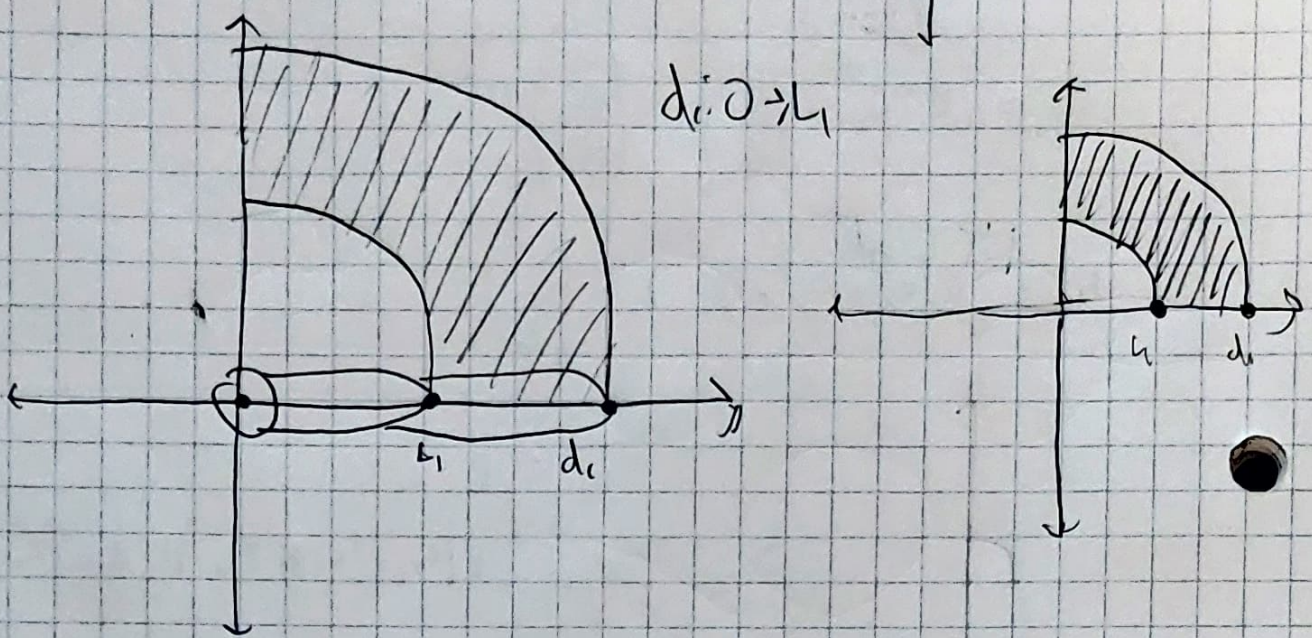
b) Potentially infinite solutions. Have two rotations on the same axis that determine the end effector position so many different configurations can put the end effector at the given position

(4)

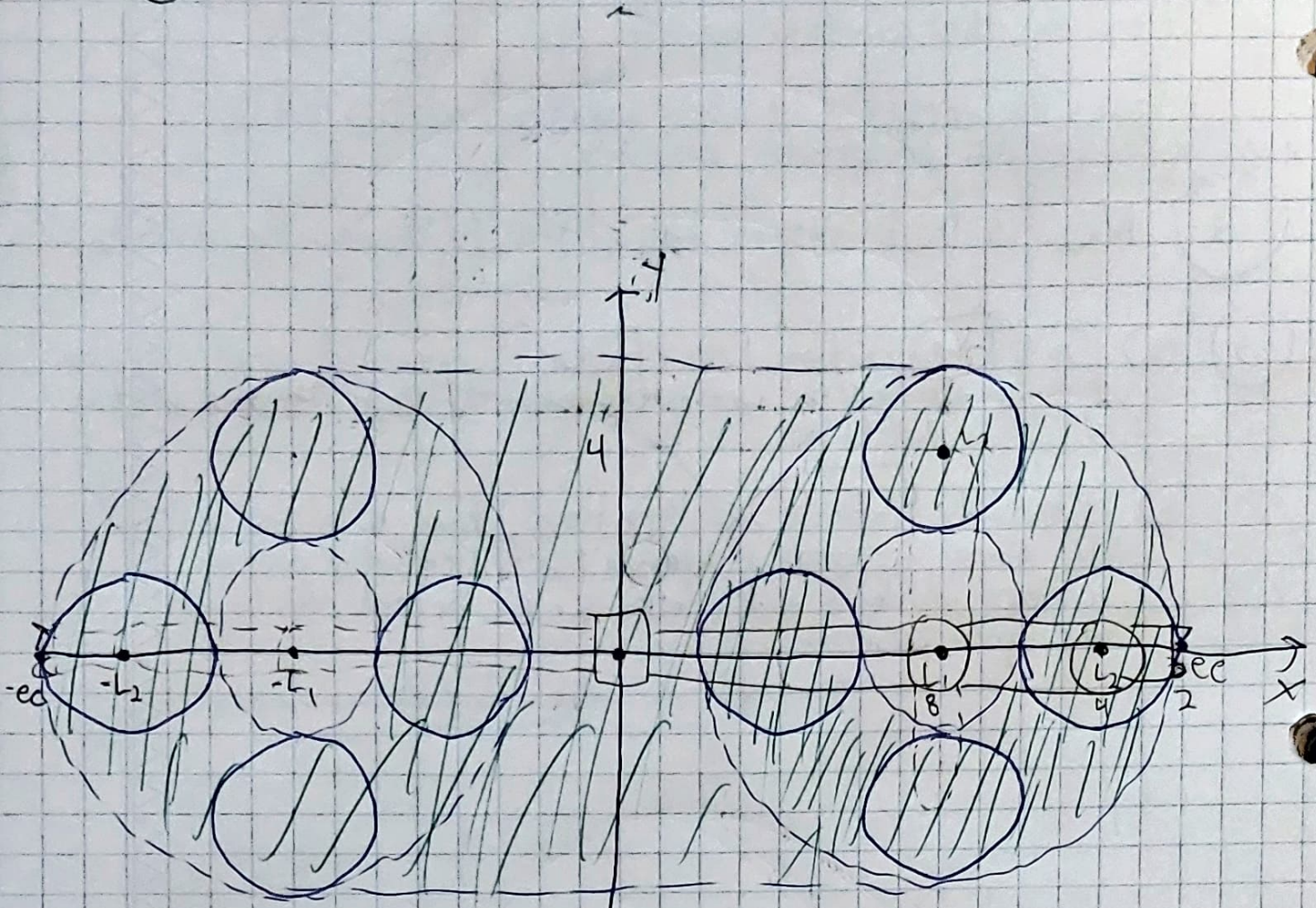


"d: 0 → L"

(5)

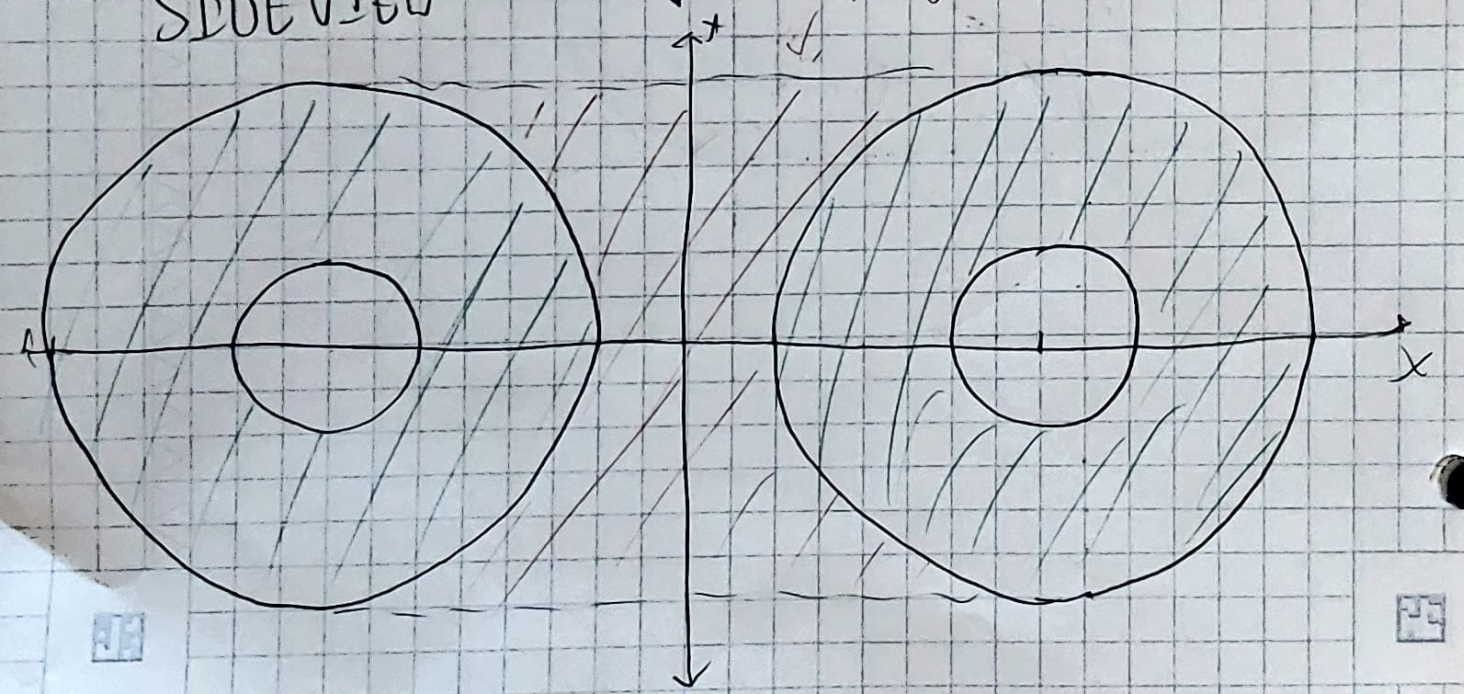


(8) .. SIDE VIEW

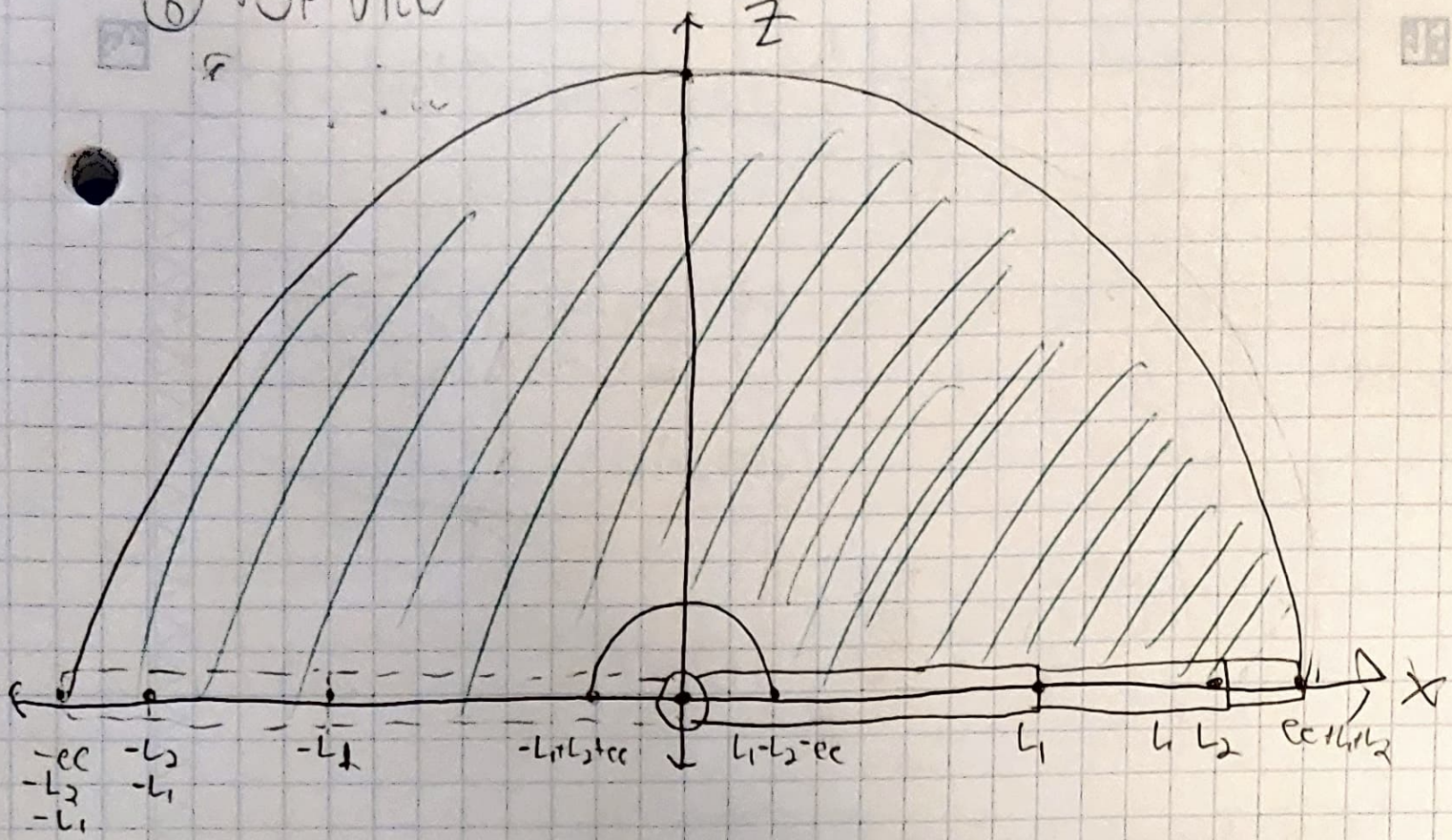


SIDE VIEW

background = -axis

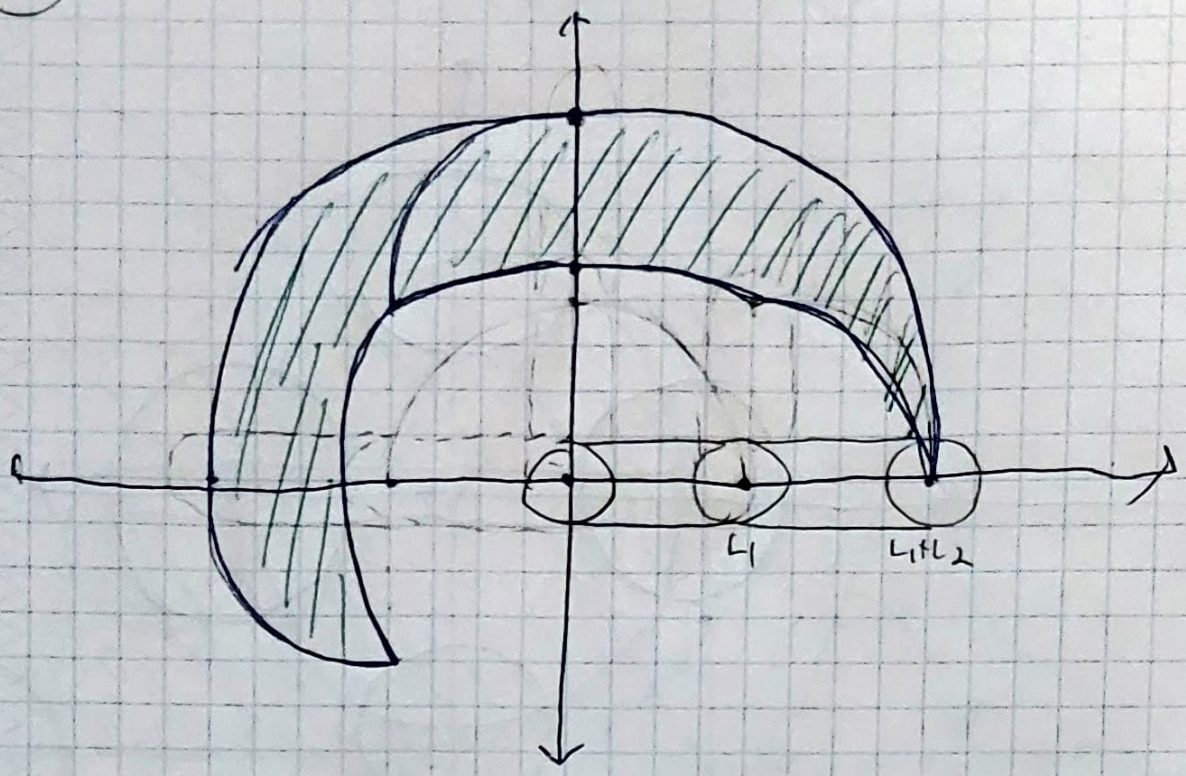


⑥ TOP VIEW

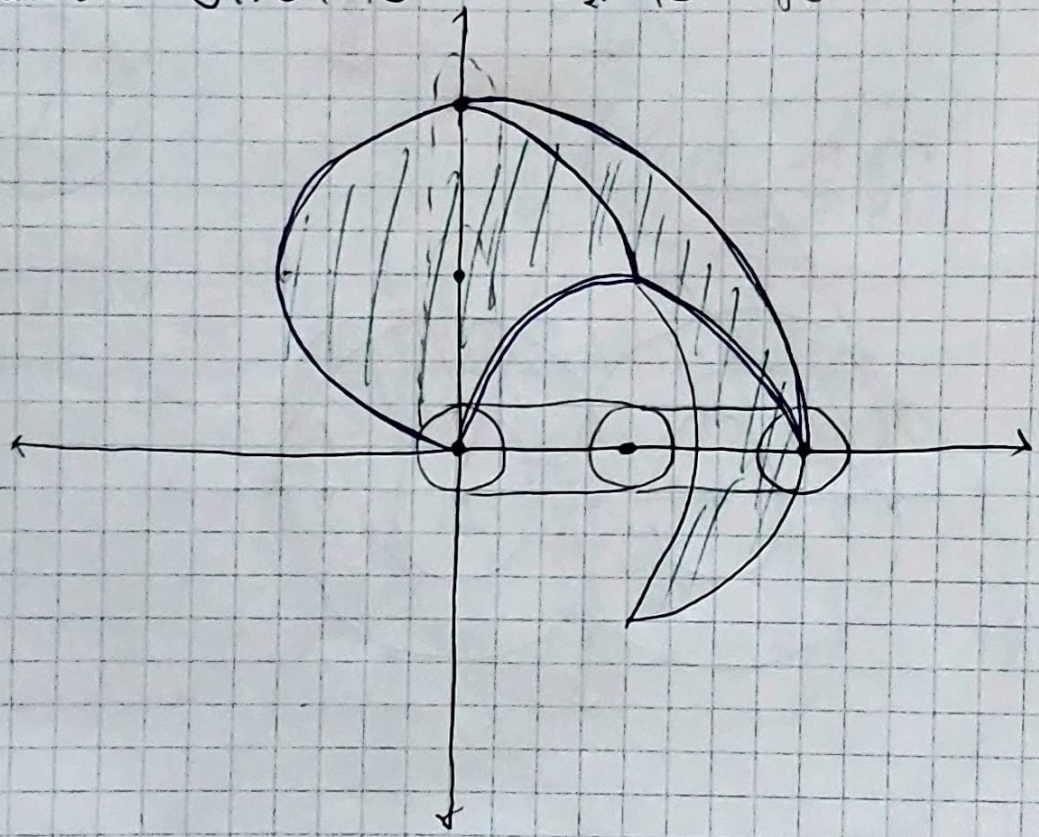


* L2 and ec can fold into L1

(7) Scenario A $\theta_1: 0 \to +180°$ $\theta_2: 0 \to +90°$



Scenario B $\theta_1: 0 \to +90°$ $\theta_2: -90 \to 180°$



8

i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	0	L_1	θ_2	0
ee	0	L_2	0	0

Ver MATLAB

$$T_{ee}^{0} = \begin{pmatrix} c_1c_2 - s_1s_2 & -c_1s_2 + c_2s_1 & 0 & L_1c_1 - L_2(c_1c_2 - s_1s_2) \\ c_1s_2 + c_2s_1 & c_1c_2 - s_1s_2 & 0 & L_1s_1 - L_2(c_1s_2 + c_2s_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -0.84 & 0.54 & 0 & 0.1 \\ -0.54 & -0.84 & 0 & 0.41 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

8b $x = L_1c_1 + L_2(c_1c_2 - s_1s_2)$
$y = L_1s_1 + L_2(c_1s_2 + c_2s_1)$

$x^2 = L_1^2c_1^2 - 2L_1L_2c_1(c_1c_2 - s_1s_2) + L_2^2(c_1c_2 - s_1s_2)^2$
$= L_1^2c_1^2 - L_1L_2c_1(\cos(\theta_1+\theta_2)) + L_2^2(\cos(\theta_1+\theta_2))^2$

$y^2 = L_1^2s_1^2 - 2L_1L_2s_1(c_1s_2 + c_2s_1) + L_2^2(c_1s_2 + c_2s_1)^2$
$= L_1^2s_1^2 - L_1L_2s_1(\sin(\theta_1+\theta_2)) + L_2^2(\sin(\theta_1+\theta_2))^2$

$x^2+y^2 = L_1^2(\overbrace{c_1^2+s_1^2}^{1}) - 2L_1L_2[c_1\cos(\theta_1+\theta_2) + s_1\sin(\theta_1+\theta_2)] + L_2^2(\cos(\theta_1+\theta_2)^2 + \sin(\theta_1+\theta_2)^2)$

1

$2L_1L_2[\cos(\theta_1-\theta_1-\theta_2) + \cancel{\cos(\theta_1+\theta_1+\theta_2)} + \cos(\theta_1-\theta_1-\theta_2) - \cancel{\cos(\theta_1+\theta_1+\theta_2)}]$

$L_1L_2(2\cos\theta_2)$; $\cos(-\theta) = \cos\theta$

$x^2+y^2 = L_1^2 + 2L_1L_2\cos\theta_2 + L_2^2$

$\cos^{-1}\left(\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}\right) = \theta_2$

$\theta_2 = \cos^{-1}\left(\frac{x^2+y^2-2}{2}\right)$

(1) Law of Cosines

$\phi = \tan\left(\frac{y}{x}\right)$ i

$\cos(\phi) = \frac{x^2+y^2+L_1^2-L_2^2}{2L_1\sqrt{x^2+y^2}} \Rightarrow \cos^{-1}\left(\frac{x^2+y^2}{2\sqrt{x^2+y^2}}\right)$

$$\theta_1 = \tan\left(\frac{y}{x}\right) \pm \cos^{-1}\left(\frac{x^2+y^3}{2\sqrt{x^2+y^2}}\right)$$

Case	Right θ_1	Right θ_2	Left θ_1	Left θ_2
i	0.526	1.0472	1.5708	-1.0472
ii	4.7124	5.236	0.5236	-5.236
iii	[illegible]	2.0084	3.0915	-2.0084
iv	NaN	NaN	NaN	NaN

Case iv not possible

```
clc;
clear;
close all;

syms t1 t2 L1 L2

%t1 = 360;
%t2 = 360;
%L1 = 1;
%L2 = 1;

Link1 = link([0 0 0 0 0], 'modified');
Link2 = link([0 L1 0 0 0], 'modified');
Link3 = link([0 L2 0 0 0], 'modified');

r1 = robot({Link1 Link2 Link3});
Q = [t1 t2 0];

Tarm = vpa(fkine(r1, Q),2)

%with values for transformation solution
t1 = 360;
t2 = 360;
L1 = 1;
L2 = 1;

Link1 = link([0 0 0 0 0], 'modified');
Link2 = link([0 L1 0 0 0], 'modified');
Link3 = link([0 L2 0 0 0], 'modified');

r1 = robot({Link1 Link2 Link3});
Q = [t1 t2 0];

Tarm = vpa(fkine(r1, Q),2)
```

```
Tarm =

[cos(t1)*cos(t2) - 1.0*sin(t1)*sin(t2), - 1.0*cos(t1)*sin(t2) - 1.0*cos(t2)*sin(t1),   0, L1*cos(t1) + L2*(cos(t1)*cos(t2) - 1.0*sin(t1)*sin(t2))]
[    cos(t1)*sin(t2) + cos(t2)*sin(t1),       cos(t1)*cos(t2) - 1.0*sin(t1)*sin(t2),   0,     L2*(cos(t1)*sin(t2) + cos(t2)*sin(t1)) + L1*sin(t1)]
[                                    0,                                           0, 1.0,                                                          0]
[                                    0,                                           0,   0,                                                        1.0]


Tarm =

[-0.84,  0.54,   0, -1.1]
[-0.54, -0.84,   0, 0.41]
[    0,     0, 1.0,    0]
[    0,     0,   0,  1.0]
```

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```
clc;
clear;
close all;

L1 = 1;
L2 = 1;

Link1 = link([0 0 0 0 0], 'modified');
Link2 = link([0 L1 0 0 0], 'modified');
Link3 = link([0 L2 0 0 0], 'modified');

r1 = robot({Link1 Link2 Link3});
r2 = robot({Link1 Link2 Link3});
```

```
%Case 1
x = 0.866;
y = 1.5;

rt1 = atan2(y,x)-acos((x^2+y^2)/(2*sqrt(x^2+y^2)));
rt2 = acos((x^2+y^2-2)/2);
lt1 = atan2(y,x)+acos((x^2+y^2)/(2*sqrt(x^2+y^2)));
lt2 = -acos((x^2+y^2-2)/2);


mask = [1 1 0 0 0 1];
right = [rt1 rt2 0];
left = [lt1 lt2 0];


%Tee1
Tee1 = [1 0 0 x;
        0 1 0 y;
        0 0 1 0;
        0 0 0 1];

rq1 = ikine(r1,Tee1,right,mask)
```

```
rq1 = 1×3
    0.5236    1.0472   -1.5708
```

```
lq1 = ikine(r1,Tee1,left,mask)
```

```
lq1 = 1×3
    1.5708   -1.0472   -0.5236
```

```
figure()
view(3);
grid on;
plot(r1,rq1);
```

```
Warning: The DrawMode property will be removed in a future release. Use the SortMethod property instead.
Warning: The EraseMode property is no longer supported and will error in a future release.
Warning: The EraseMode property is no longer supported and will error in a future release.
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```

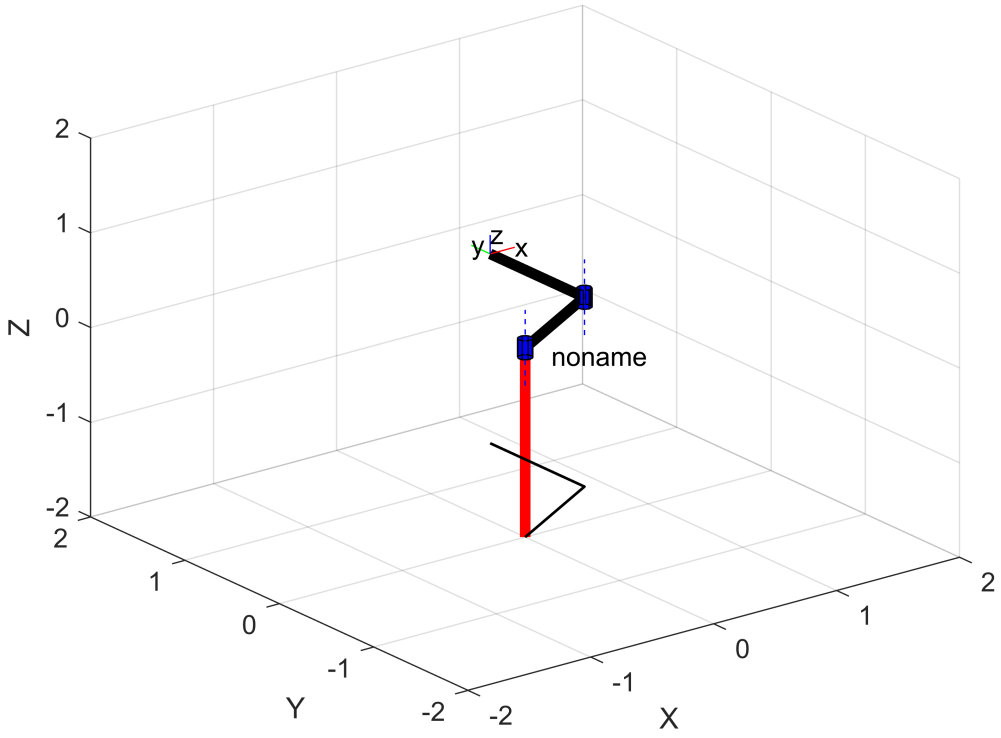


```
figure();
view(3);
grid on;
plot(r2,lq1);
```

```
Warning: The DrawMode property will be removed in a future release. Use the SortMethod property instead.
Warning: The EraseMode property is no longer supported and will error in a future release.
Warning: The EraseMode property is no longer supported and will error in a future release.
Warning: The EraseMode property is no longer supported and will error in a future release.
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```
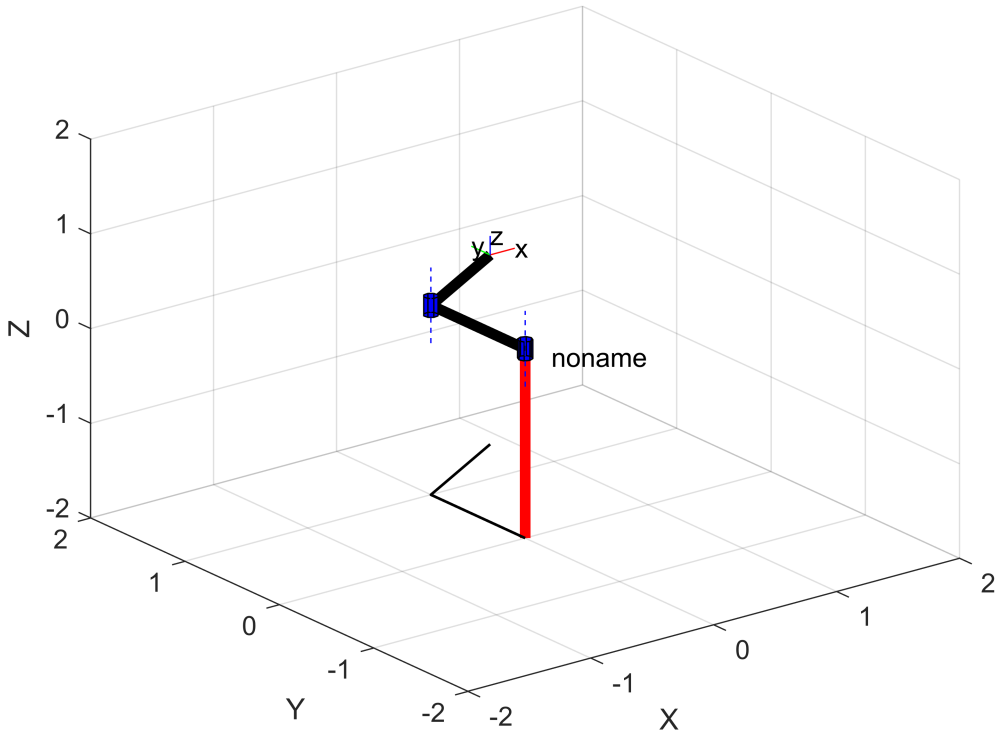
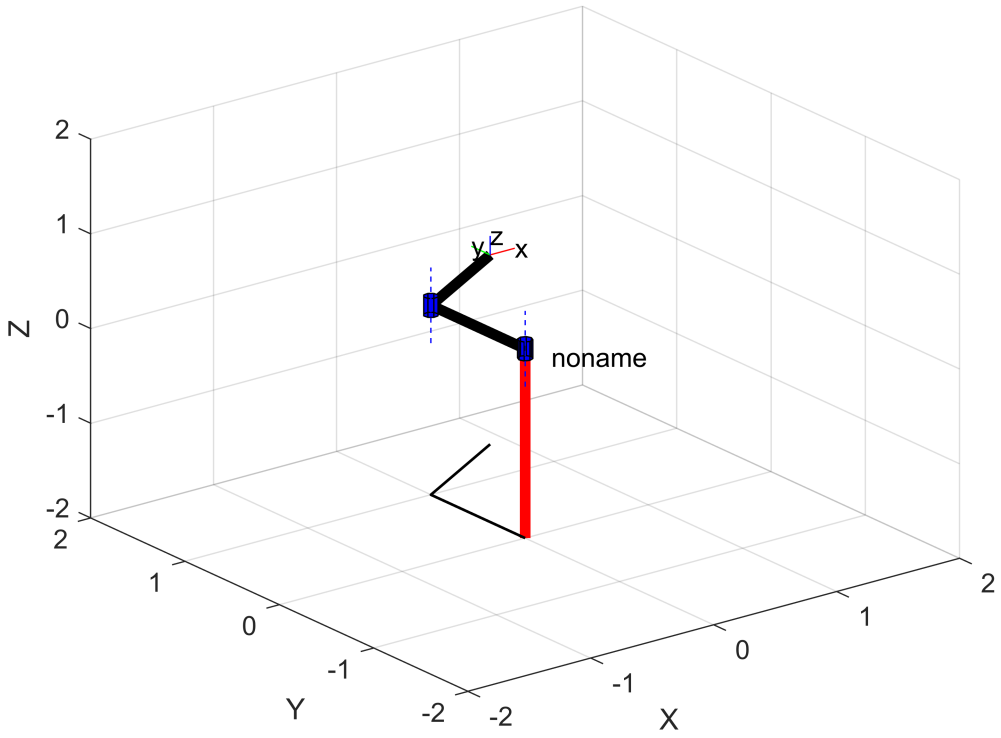


2
1
0
-1
-2
Z
y
z
x
noname
2
1
0
-1
-2
Y
2
1
0
-1
-2
X


2
1
0
-1
-2
Z
y
z
x
noname
2
1
0
-1
-2
Y
2
1
0
-1
-2
X

```
%Case 2
x = -0.134;
y = -.5;

rt1 = atan2(y,x)-acos((x^2+y^2)/(2*sqrt(x^2+y^2)));
rt2 = acos((x^2+y^2-2)/2);
lt1 = atan2(y,x)+acos((x^2+y^2)/(2*sqrt(x^2+y^2)));
lt2 = -acos((x^2+y^2-2)/2);


mask = [1 1 0 0 0 1];
right = [rt1 rt2 0];
left = [lt1 lt2 0];


%Tee2 for position [0.866 1.5]
Tee2 = [1 0 0 x;
        0 1 0 y;
        0 0 1 0;
        0 0 0 1];

rq2 = ikine(r1,Tee1,right,mask)
```

```
rq2 = 1×3
   -4.7124    5.2360   -0.5236
```

```
lq2 = ikine(r1,Tee1,left,mask)
```

```
lq2 = 1×3
    0.5236   -5.2360   -1.5708
```

```
%Case 3
x = -0.5;
y = .95;

rt1 = atan2(y,x)-acos((x^2+y^2)/(2*sqrt(x^2+y^2)));
rt2 = acos((x^2+y^2-2)/2);
lt1 = atan2(y,x)+acos((x^2+y^2)/(2*sqrt(x^2+y^2)));
lt2 = -acos((x^2+y^2-2)/2);


mask = [1 1 0 0 0 1];
right = [rt1 rt2 0];
left = [lt1 lt2 0];


%Tee3
Tee3 = [1 0 0 x;
        0 1 0 y;
```

```
        0 0 1 0;
        0 0 0 1];

rq3 = ikine(r1,Tee3,right,mask)
```

```
rq3 = 1×3
    1.0511    2.0084   -3.0595
```

```
lq3 = ikine(r1,Tee3,left,mask)
```

```
lq3 = 1×3
    3.0595   -2.0084   -1.0511
```

```
%Case 4
x = -.5;
y = 1.95;

rt1 = atan2(y,x)-acos((x^2+y^2)/(2*sqrt(x^2+y^2)));
rt2 = acos((x^2+y^2-2)/2);
lt1 = atan2(y,x)+acos((x^2+y^2)/(2*sqrt(x^2+y^2)));
lt2 = -acos((x^2+y^2-2)/2);


mask = [1 1 0 0 0 1];
right = [rt1 rt2 0];
left = [lt1 lt2 0];


%Tee3
Tee4 = [1 0 0 x;
        0 1 0 y;
        0 0 1 0;
        0 0 0 1];

rq4 = ikine(r1,Tee4,right,mask)
```

```
rq4 = 1×3 complex
      NaN +    NaNi      NaN +
```

```
lq4 = ikine(r1,Tee4,left,mask)
```

```
lq4 = 1×3 complex
      NaN +    NaNi      NaN +
```

```

```

(9) SEE MATLAB

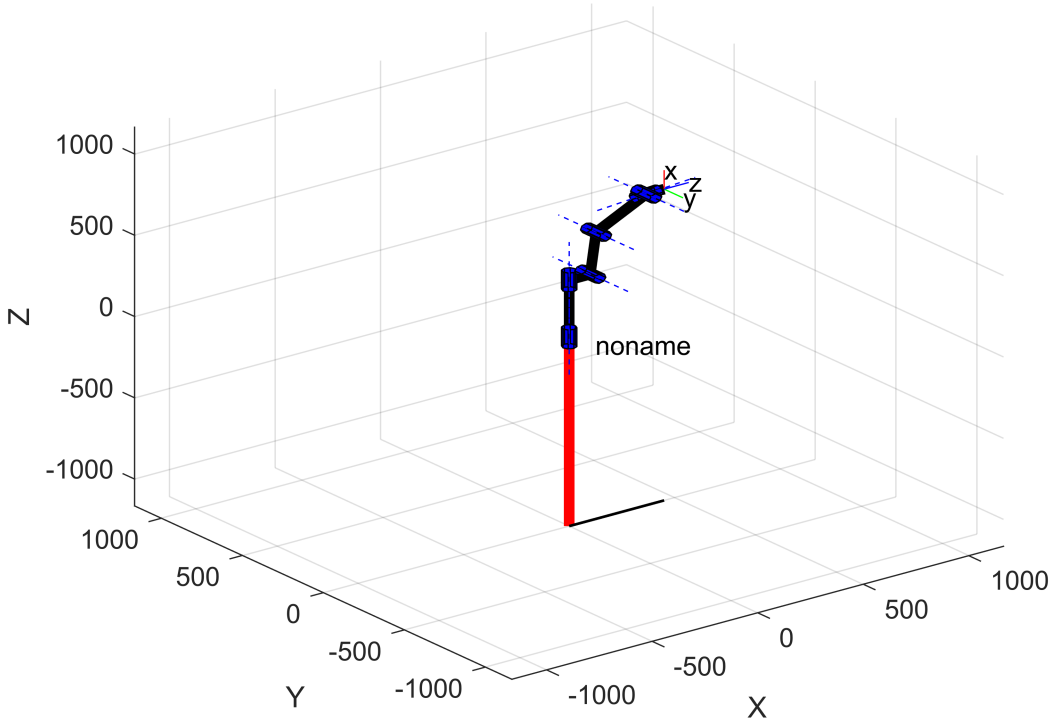
Case	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
A1	0	.1094	-.1981	0	.0882	0
A2	0	.1099	-.1981	3.14	-.0882	3.14
B1	-.7854	-.0599	.2313	0	1.3994	-.7854
B2	-.7854	-.0599	.2313	3.1416	1.3994	2.3562

```
clc;
clear;
close all;
warning('off');

syms t [1 6];
%syms L [1 6];
L1 = 350;
L2 = 100;
L3 = 250;
L4 = 130;
L5 = 250;
L6 = 85;

LinkW1 = link([0 0 0 L1 0], 'modified');
LinkS2 = link([-pi/2 L2 0 0 0 -pi/2], 'modified');
LinkE3 = link([0 L3 0 0 0], 'modified');
LinkT4 = link([-pi/2 L4 0 L5 0], 'modified');
LinkP5 = link([pi/2 0 0 0 0], 'modified');
LinkR6 = link([-pi/2 0 0 L6 0], 'modified');

r1 = robot({LinkW1 LinkS2 LinkE3 LinkT4 LinkP5 LinkR6});

Ta = [0 0 1 450;
      0 -1 0 0;
      1 0 0 750;
      0 0 0 1];

Tb = [1 0 0 250;
      0 -1 0 -250;
      0 0 -1 600;
      0 0 0 1];
```

```
close all;
%Case A 1st solution
q1 = ikine(r1,Ta)
```

```
q1 = 1×6
    0.0000    0.1099   -0.1981   -0.0000    0.0882    0.0000
```

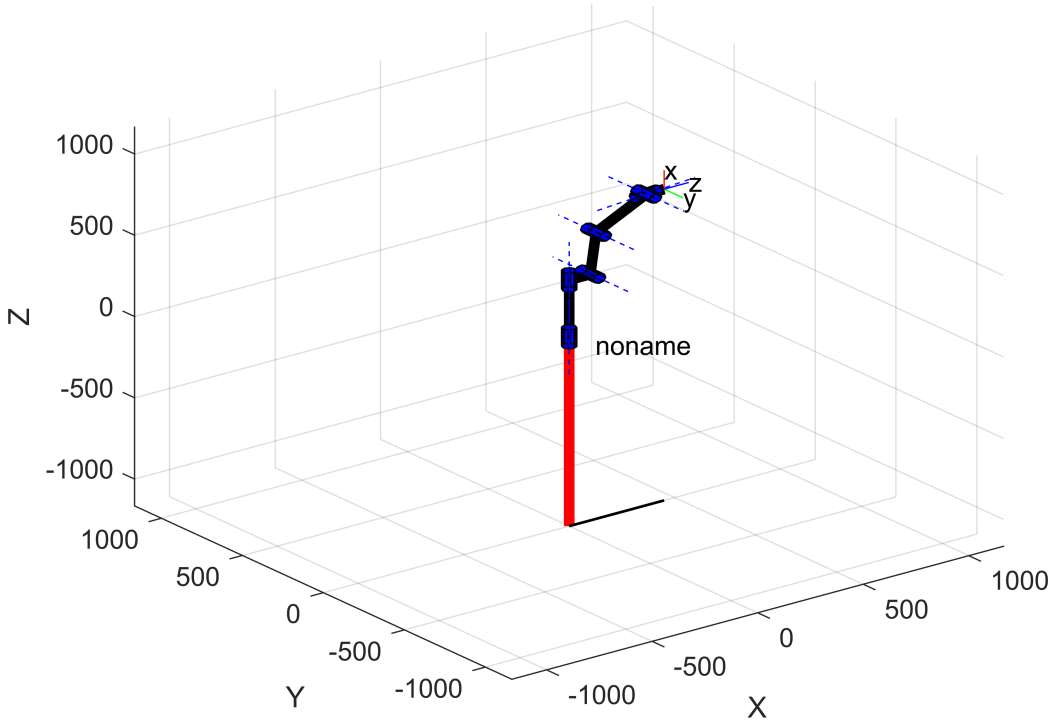
```
figure(1)
view(3);
title("Case A 1st solution");
plot(r1,q1);
```




```
close all;
%Case A 1st Pieper's solution
q2 = ikine(r1,Ta,[0 0 0 pi 0 pi])
```

```
q2 = 1×6
    0.0000    0.1099   -0.1981    3.1416   -0.0882    3.1416
```

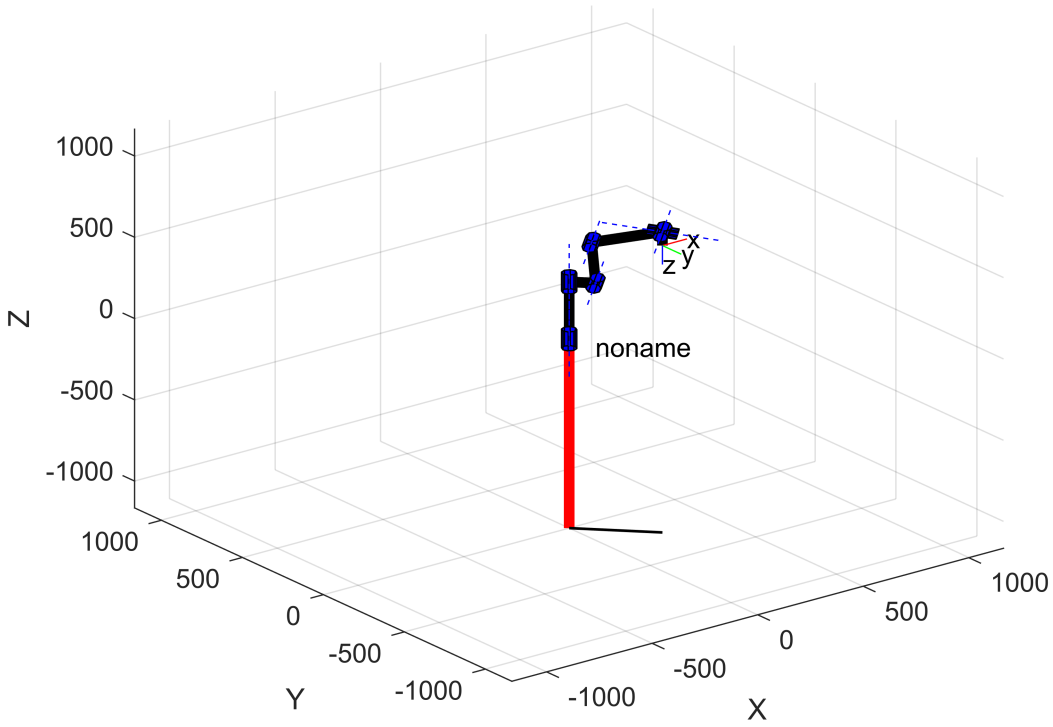
```
figure(2)
view(3);
title("Case A 1st Pieper's solution");
plot(r1,q2);
```




```
close all;
%Case B 2nd solution
q3 = ikine(r1,Tb)
```

```
q3 = 1×6
   -0.7854   -0.0599    0.2313   -0.0000    1.3994   -0.7854
```

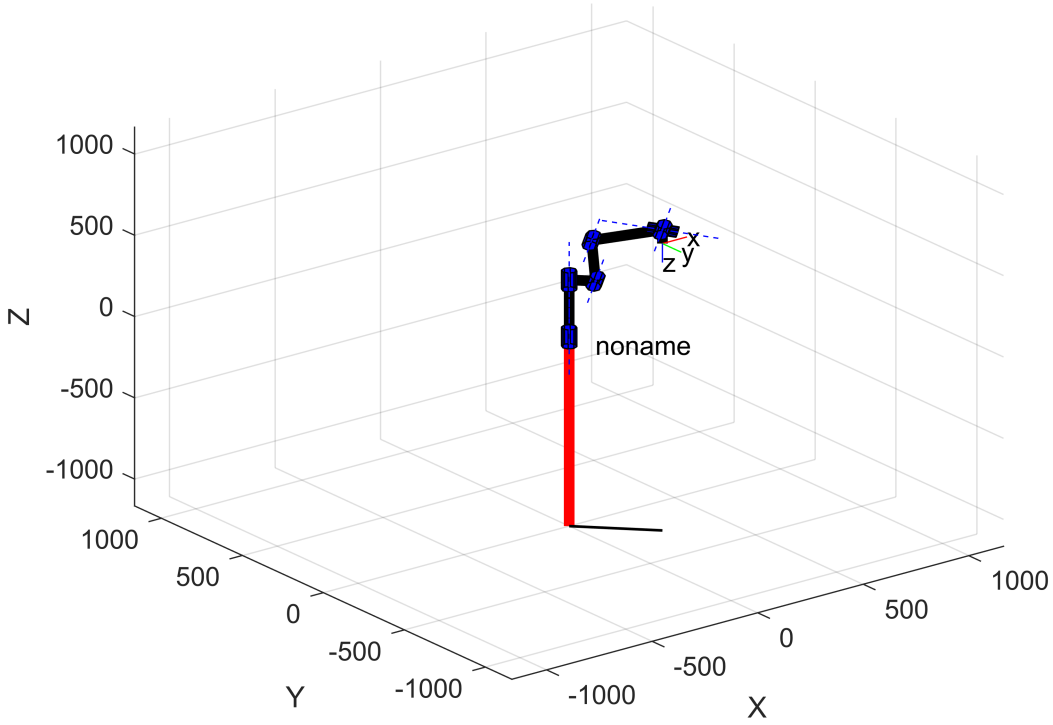
```
figure(3)
view(3);
title("Case B 2nd solution");
plot(r1,q3);
```




```
close all;
%Case B 2nd Pieper's solution
q3 = ikine(r1,Tb,[0 0 0 pi 0 pi])
```

```
q3 = 1×6
   -0.7854   -0.0599    0.2313    3.1416   -1.3994    2.3562
```

```
figure(4)
view(3);
title("Case B 2nd Pieper's solution");
plot(r1,q3);
```


Case B 2nd Pieper's solution
1000
500
0
-500
-1000
Z
1000
500
0
-500
-1000
Y
1000
500
0
-500
-1000
X
x
y
z
noname