METU Department of Aerospace Eng
AE305 Numerical Methods
HW#4, Fall 2020

Consider the solution of 1-D, linear, convection diffusion equation in a discrete domain stretching in the range, $-L/2 \leq x \leq L/2$, with Δx spacing.

$$\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x} = \nu\frac{\partial^2 w}{\partial x^2}$$

where V is the constant convection velocity, and ν is the diffusion (viscosity) coefficient. The initial distribution of w is given by the Gaussian distribution

$$w(x, t = 0) = \exp(-b \ln 2(x/\Delta x)^2).$$

Take $L = 40$, $\Delta x = 0.1$, $b = 0.025$, and carry out the following tasks, using the fortran program provided for the solution of the convection equation:

1. Formulate the explicit FDE based on (forward time) and central spatial differences in terms of the two dimensionless parameters given by $\sigma = \frac{V\Delta t}{\Delta x}$ known as the Courant number, and $d = \frac{\nu\Delta t}{\Delta x^2}$ known as the diffusion number.
2. By numerical experimentationa show that the FDE is conditionally stable. Obtain stable and unstable solutions, plot the solutions in w versus t graphs, and discuss the results.
3. Compare the solutions of forward-time, backward-space (FTBS) FDEs for both the convection equation and the above convection diffusion equation.
4. Consider the following FDE

$$\frac{-w_i^{n+1} + 4w_i^n - 3w_i^{n-1}}{2\Delta t} + V\frac{w_{i+1}^n - w_{i-1}^n}{2\Delta x} = \nu\frac{w_{i+1}^n - 2w_i^n + w_{i-1}^n}{\Delta x^2}$$

   (a) Perform the consistency analysis for this FDE and identify the leading error terms.
   (b) Modify the fortran code for the solution. Note that the first step of the solution needs to be performed by the forward time, central space (FTCS) FDE. Plot the results in w versus t graphs and discuss them.

5. Obtain implicit solutions for a bonus.