Merve Nur Öztürk	Atakan Süslü	Betül Rana Kuran
2311322	*2311371*	*2311173*

1 Introduction

Finite Difference Method uses finite-difference relations which are obtained from Taylor series expansions in order to approximate the partial derivatives in a partial differential equation. The equations representing the partial differentials are called Finite Difference Equations. In this homework, given a convection-diffusion wave equation:

$$\frac{\partial \omega}{\partial t} + V\frac{\partial \omega}{\partial x} = \nu\frac{\partial^2 \omega}{\partial x^2} \tag{1}$$

where V is the constant convection velocity, ν is the diffusion (viscosity) coefficient, and ω is distributed initially as follows:

$$\omega(x, t = 0) = exp(-bln2(x/\Delta x)^2) \tag{2}$$

where $\Delta x = 0.1$, and $b = 0.025$, it is requested to solve Equation 1 by modifying the given fortran code and using forward-time central-space FDE, and prove that the equation is conditionally stable by using different σ (Courant number) and d values where σ and d values are as the following:

$$\begin{aligned} \sigma &= \frac{V\Delta t}{\Delta x} \\ d &= \frac{\nu\Delta t}{\Delta x^2} \end{aligned} \tag{3}$$

It is also asked to solve both Equation 1 and convection equation, where d equals zero, by forward-time backward-space FDE and compare the solutions. Afterwards, given a high order accuracy FDE, first, it is requested to perform consistency analysis on it and then solving Equation 1 by using this FDE:

$$\frac{-\omega_i^{n+1} + 4\omega_i^n - 3\omega_i^{n-1}}{2\Delta t} + V\frac{\omega_{i+1}^n - \omega_{i-1}^n}{2\Delta x} = \nu\frac{\omega_{i+1}^n - 2\omega_i^n + \omega_{i-1}^n}{\Delta x^2} \tag{4}$$

Finally, implicit FDE (backward-time central-space) is used to solve Equation 1. In all solutions the range of x is such that -L/2≤x≤L/2 where L=40.

2 Method

2.1 Finite Difference Method

Finite Difference method is used to solve partial differential equations by approximating partial derivatives in these equations with finite difference relations. In this homework, 1-D, linear, convection-diffusion equation and convection equation are solved with a combination of finite difference approximations.

$$\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x} = \nu\frac{\partial^2 w}{\partial x^2} \tag{5}$$

is convective diffusion equation and

$$\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x} = 0 \tag{6}$$

is convective equation.
Two different derivations are used for the explicit finite difference equation of Equation 5 in this homework.The first derivation of FDE is also used for the explicit FDE of Equation 6. In both derivations, the first term of the equation forward time approximation is used.

$$\frac{\partial w}{\partial t}|^n = \frac{w^{n+1} - w^n}{\Delta t} + O(\Delta t) \tag{7}$$

In the first derivation, for second and third terms of the Equation 5, central space approximations of $\frac{\partial w}{\partial x}|_i$ of order (Δx^2) and $\frac{\partial^2 w}{\partial x^2}|_i$ of order (Δx^2), respectively, are used.

$$\frac{\partial w}{\partial x}|_i = \frac{w_{i+1} - w_{i-1}}{2\Delta x} + O(\Delta x^2) \tag{8}$$

$$\frac{\partial^2 w}{\partial x^2}|_i = \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} + O(\Delta x^2) \tag{9}$$

By substituting Equation 7, 8, and 9 into Equation 5, it becomes:

$$\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{w_{i+1}^n - w_{i-1}^n}{2\Delta x} = \frac{w_{i+1}^n - 2w_i^n + w_{i-1}^n}{\Delta x^2} + O(\Delta x^2, \Delta t) \tag{10}$$

The explicit FDE of Equation 5 based on forward time and central spatial differences (FTCS) is obtained:

$$w_i^{n+1} = w_i^n - \frac{\sigma}{2}(w_{i+1}^n - w_{i-1}^n) + d(w_{i+1}^n - 2w_i^n + w_{i-1}^n) + O(\Delta x^2, \Delta t) \tag{11}$$

where $\sigma = \frac{V\Delta t}{\Delta x}$ and $d = \frac{\nu \Delta t}{\Delta x^2}$.
To derive the explicit FDE of Equation 6, Equation 7 and 8 are substituted into Equation 6:

$$\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{w_{i+1}^n - w_{i-1}^n}{2\Delta x} = 0 + O(\Delta x^2, \Delta t) \tag{12}$$

The explicit FDE of Equation 6 based on forward time and central spatial differences (FTCS) is obtained:

$$w_i^{n+1} = w_i^n - \frac{\sigma}{2} w_{i+1}^n - w_{i-1}^n + O(\Delta x^2, \Delta t) \tag{13}$$

where $\sigma = \frac{V\Delta t}{\Delta x}$.

In the second derivation, instead of central space approximation, backward space approximation is used. Backward space approximation of $\frac{\partial w}{\partial x}|$ of order (Δx):

$$\frac{\partial w}{\partial x}|_i = \frac{w_i - w_{i-1}}{\Delta x} + O(\Delta x) \tag{14}$$

Backward space approximation of $\frac{\partial^2 w}{\partial x^2}|$ of order (Δx):

$$\frac{\partial^2 w}{\partial x^2}|_i = \frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2} + O(\Delta x) \tag{15}$$

By substituting Equation 7, 14, and 15 into Equation 5:

$$\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{w_i^n - w_{i-1}^n}{\Delta x} = \frac{w_i^n - 2w_{i-1}^n + w_{i-2}^n}{\Delta x^2} + O(\Delta x, \Delta t) \tag{16}$$

The explicit FDE of Equation 5 based on forward time and backward spatial differences (FTBS) is obtained:

$$w_i^{n+1} = w_i^n - \sigma(w_{i+1}^n - w_{i-1}^n) + d(w_i^n - 2w_{i-1}^n + w_{i-2}^n) + O(\Delta x, \Delta t) \tag{17}$$

where $\sigma = \frac{V\Delta t}{\Delta x}$ and $d = \frac{\nu\Delta t}{\Delta x^2}$.

To obtain the implicit FDE of Equation 5, backward time central space method is used. Backward time difference is used for the first term of Equation 5:

$$\frac{\partial w}{\partial t}|^n = \frac{w^n - w^{n-1}}{\Delta x} + O(\Delta t) \tag{18}$$

Equation 8 and 9 are used to approximate the second and third terms of Equation 5, respectively. Equation 18, 8 and 9 are inserted in the Equation 5:

$$\frac{w_i^n - w_i^{n-1}}{\Delta t} + \frac{w_{i+1}^{n-1} - w_{i-1}^{n-1}}{2\Delta x} = \frac{w_{i+1}^{n-1} - 2w_i^{n-1} + w_{i-1}^{n-1}}{\Delta x^2} + O(\Delta x^2, \Delta t) \tag{19}$$

To define a PDE as implicit, it should be discretized at time level $n+1$. So, equation becomes:

$$\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{w_{i+1}^{n+1} - w_{i-1}^{n+1}}{2\Delta x} = \frac{w_{i+1}^{n+1} - 2w_i^{n+1} + w_{i-1}^{n+1}}{\Delta x^2} + O(\Delta x^2, \Delta t) \tag{20}$$

The implicit FDE of Equation 5 based on BTCS Euler Implicit method is obtained:

$$-(\frac{\sigma}{2} + d)w_{i-1}^{n+1} + (2d+1)w_i^{n+1} + (\frac{\sigma}{2} - d)w_{i+1}^{n+1} = w_i^n \tag{21}$$

where $\sigma = \frac{V\Delta t}{\Delta x}$ and $d = \frac{\nu\Delta t}{\Delta x^2}$.

2.2 Thomas Algorithm

As a consequence of the usage of the BTCS Euler Implicit method to solve the convection-diffusion equation, the tridiagonal system of equations is obtained. The general form of the tridiagonal system equation with k unknowns:

$$a_i x_{i-1} + b_i x_i + c_i x_{i+1} = f_i \tag{22}$$

where a_1 and c_k is not used.
System of equations can be written as following form

$$\begin{bmatrix} b_1 & c_1 & 0 & \dots & 0 \\ a_2 & b_2 & c_2 & \dots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \dots & a_{k-1} & b_{k-1} & c_{k-1} \\ 0 & \dots & 0 & a_k & b_k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{k-1} \\ x_k \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{k-1} \\ f_k \end{Bmatrix} \tag{23}$$

The first term of Equation 23 has 3 non-zero diagonals so it is a tridiagonal matrix. Equation 23 can be solved with Thomas Algorihtm.

2.3 Consistency Analysis of the Given FDE

Consistency is one of the conditions to be able to approximate the PDE. What is meant by consistency is that as the step sizes go to zero, FDE should converge to the PDE. For a given FDE written at a point (i,n), consistency analysis is to expand the discrete values with Taylor series expansions and leaving the original PDE terms on the left-hand-side and all the other terms on the right-hand side. If the right hand side goes to zero as the step sizes go to zero, the FDE is consistent. This procedure is carried out for the following FDE:

$$\frac{-\omega_i^{n+1} + 4\omega_i^n - 3\omega_i^{n-1}}{2\Delta t} + V\frac{\omega_{i+1}^n - \omega_{i-1}^n}{2\Delta x} = \nu\frac{\omega_{i+1}^n - 2\omega_i^n + \omega_{i-1}^n}{\Delta x^2} \tag{24}$$

First, all terms except the ω_i^n term, are expanded aorund ω_i^n with TSE:

$$\begin{aligned} \omega_i^{n+1} &= w_i^n + \Delta t \omega_t|_i^n + \frac{\Delta t^2}{2}\omega_{tt}|_i^n + \frac{\Delta t^3}{6}\omega_{ttt}|_i^n ... \\ \omega_i^{n-1} &= w_i^n - \Delta t \omega_t|_i^n + \frac{\Delta t^2}{2}\omega_{tt}|_i^n - \frac{\Delta t^3}{6}\omega_{ttt}|_i^n ... \\ \omega_{i+1}^n &= w_i^n + \Delta x \omega_x|_i^n + \frac{\Delta x^2}{2}\omega_{xx}|_i^n + \frac{\Delta x^3}{6}\omega_{xxx}|_i^n ... \\ \omega_{i-1}^n &= w_i^n - \Delta x \omega_x|_i^n + \frac{\Delta x^2}{2}\omega_{xx}|_i^n - \frac{\Delta x^3}{6}\omega_{xxx}|_i^n ... \end{aligned} \tag{25}$$

Then the above relations are substituted into Equation 24:

$$\omega_t|_i^n - \Delta t\omega_{tt}|_i^n + \frac{\Delta t^2}{6}\omega_{ttt}|_i^n + V\omega_x|_i^n + V\frac{\Delta x^2}{6}\omega_{xxx}|_i^n = \nu\omega_{xx}|_i^n + O(\Delta t^4, \Delta x^4) \tag{26}$$

Collecting the original PDE terms to the RHS:

$$[\omega_t + V\omega_x - \nu\omega_{xx}]|_i^n = \Delta t\omega_{tt}|_i^n - \frac{\Delta t^2}{6}\omega_{ttt}|_i^n - V\frac{\Delta x^2}{6}\omega_{xxx}|_i^n + O(\Delta t^4, \Delta x^4) \tag{27}$$

Since the right-hand-side terms go to zero as Δt and Δx approach to zero, the FDE given in Equation 24 is consistent.

In order to understand the effect of the leading error term on the right-hand side the leading term's time derivatives are converted to spatial derivatives using the Equation 5 as follows:

$$\frac{\partial\omega}{\partial t} = \nu\frac{\partial^2\omega}{\partial x^2} - V\frac{\partial\omega}{\partial x} \tag{28}$$

Differentiating with respect to time:

$$\begin{aligned}
\frac{\partial^2\omega}{\partial t^2} &= \nu\frac{\partial^2}{\partial x^2}(\frac{\partial\omega}{\partial t}) - V\frac{\partial}{\partial x}(\frac{\partial\omega}{\partial t}) \\
\frac{\partial^2\omega}{\partial t^2} &= \nu\frac{\partial^2}{\partial x^2}(\nu\frac{\partial^2\omega}{\partial x^2} - V\frac{\partial\omega}{\partial x}) - V\frac{\partial}{\partial x}(\nu\frac{\partial^2\omega}{\partial x^2} - V\frac{\partial\omega}{\partial x}) \\
\frac{\partial^2\omega}{\partial t^2} &= \nu^2\frac{\partial^4\omega}{\partial x^4} - 2V\nu\frac{\partial^3\omega}{\partial x^3} + V^2\frac{\partial^2\omega}{\partial x^2}
\end{aligned} \tag{29}$$

Differentiating the last equation one more time:

$$\begin{aligned}
\frac{\partial^3\omega}{\partial t^3} &= \nu^2\frac{\partial^4}{\partial x^4}(\frac{\partial\omega}{\partial t}) - 2V\nu\frac{\partial^3}{\partial x^3}(\frac{\partial\omega}{\partial t}) + V^2\frac{\partial^2}{\partial x^2}(\frac{\partial\omega}{\partial t}) \\
\frac{\partial^3\omega}{\partial t^3} &= \nu^2\frac{\partial^4}{\partial x^4}(\nu\frac{\partial^2\omega}{\partial x^2} - V\frac{\partial\omega}{\partial x}) - 2V\nu\frac{\partial^3}{\partial x^3}(\nu\frac{\partial^2\omega}{\partial x^2} - V\frac{\partial\omega}{\partial x}) + V^2\frac{\partial^2}{\partial x^2}(\nu\frac{\partial^2\omega}{\partial x^2} - V\frac{\partial\omega}{\partial x}) \\
\frac{\partial^3\omega}{\partial t^3} &= \nu^3\frac{\partial^6}{\partial x^6} - 3\nu^2 V\frac{\partial^5}{\partial x^5} + 3V^2\nu\frac{\partial^4}{\partial x^4} - V^3\frac{\partial^3}{\partial x^3}
\end{aligned} \tag{30}$$

Plugging Equations 29 and 30 into Equation 27 the right hand side becomes:

$$\begin{aligned}
V^2\Delta t\frac{\partial^2\omega}{\partial x^2} - (2V\nu\Delta t &- V^3\frac{\Delta t^2}{6} + V\frac{\Delta x^2}{6})\frac{\partial^3\omega}{\partial x^3} + (\nu^2\Delta t - V^2\nu\frac{\Delta t^2}{2})\frac{\partial^4\omega}{\partial x^4} \\
&+ \nu^2 V\frac{\Delta t^2}{2}\frac{\partial^5\omega}{\partial x^5} - \nu^3\frac{\Delta t^2}{6}\frac{\partial^6\omega}{\partial x^6}
\end{aligned} \tag{31}$$

It can be concluded from Equation 31 that the 2^{nd}, 4^{th}, and 6^{th} derivatives are dissipation errors and the 3^{rd} and 5^{th} derivatives are dispersion errors.

3 Results and Discussion

3.1 Stability of the Solution of the Explicit FDE of Convection Diffusion Equation

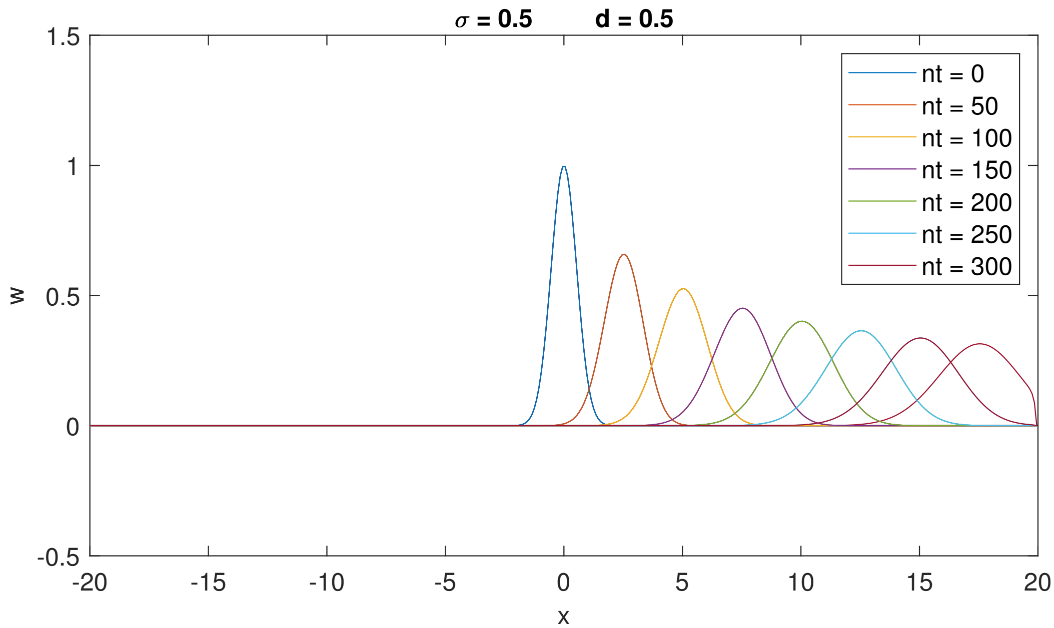

Figure 1: Convection diffusion solution by the explicit FTCS method with $\sigma = 0.5$ and $d = 0.5$ at different time steps.

Figure 1 shows the solution of the explicit FTCS method for Equation 5 with $\sigma = 0.5$ and $d = 0.5$ at various time steps. Figure 2 shows the solution of the same explicit FDE but with different d value ($d = 0.25$) at various time steps. As can be seen from the Figures 1 and 2, as time step increases, both curves continue to oscillate with a decrease in their amplitudes. It can be seen that the solution of the explicit FDE is convergent for $d = 0.5$ and $d = 0.25$ while keeping σ constant and equal to 0.5. Since the stability is a condition to be satisfied for convergence, it can be understood that both solutions are stable.

Figure 3 shows the solution of the explicit FTCS method for Equation 5 with $\sigma = 0.5$ and $d = 0.75$ at the initial condition and 25^{th} time step. The curve in the graph has some deterioration at $nt = 25$. Therefore, the explicit FDE is unstable for $\sigma = 0.5$ and $d = 0.75$.

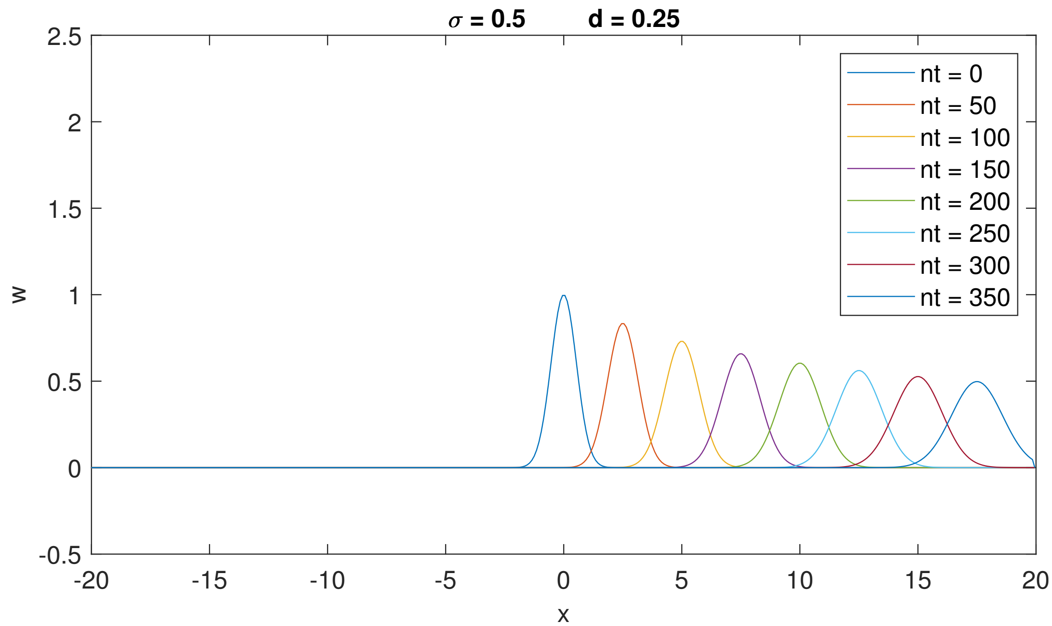

Figure 2: Convection diffusion solution by the explicit FTCS method with $\sigma = 0.5$ and $d = 0.25$ at different time steps.

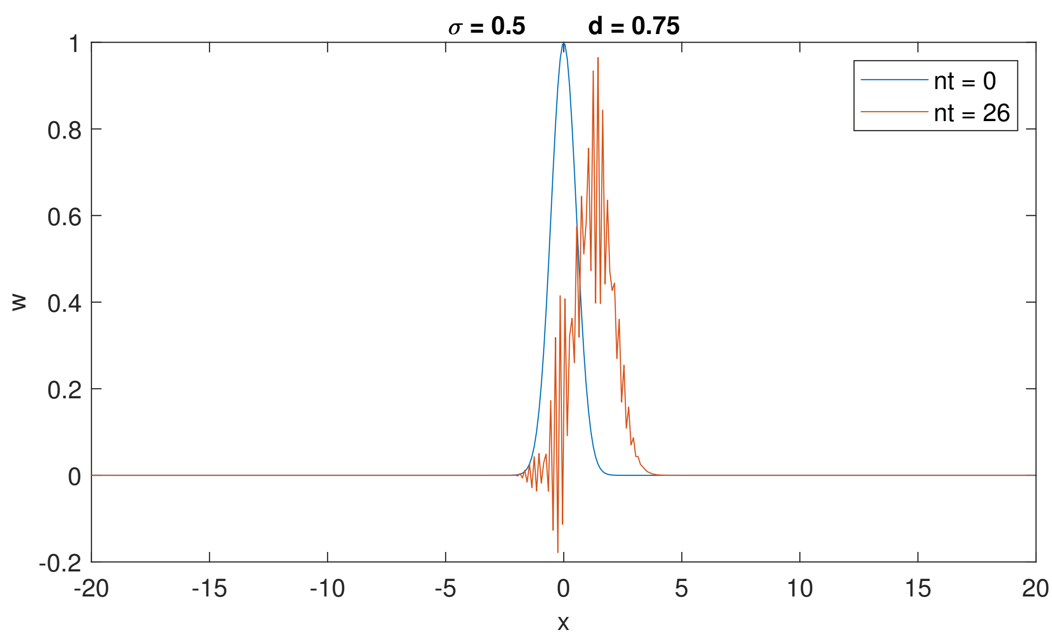

Figure 3: Convection diffusion solution by the explicit FTCS method with $\sigma = 0.5$ and $d = 0.75$ at $nt = 0$ and $nt = 25$.

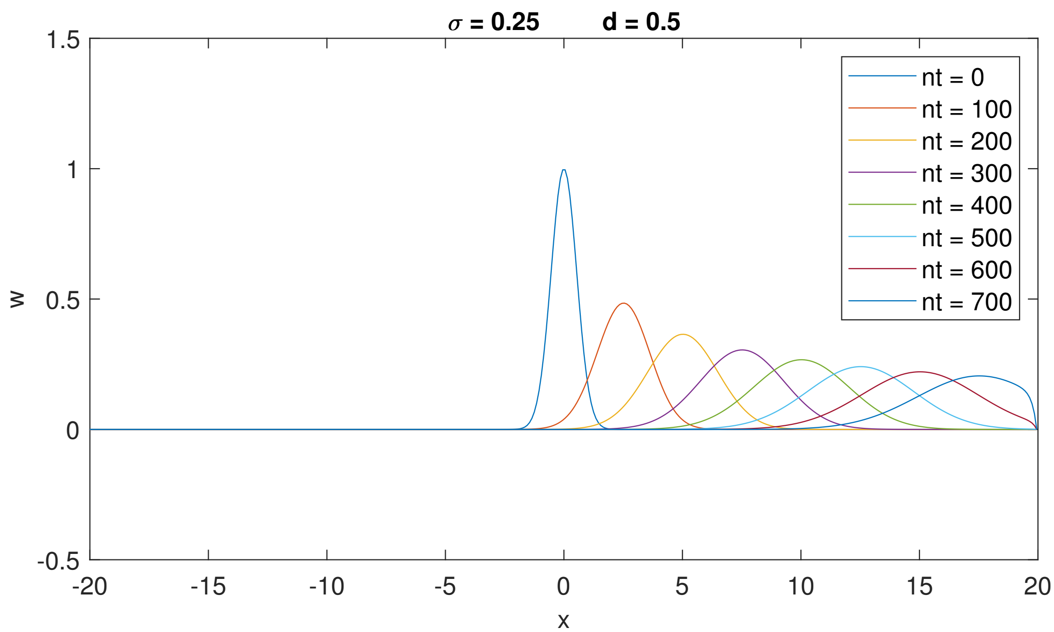

Figure 4: Convection diffusion solution by the explicit FTCS method with $\sigma = 0.25$ and $d = 0.5$ at different time steps.

Figure 4 and Figure 5 illustrates the solutions of the explicit FTCS method for Equation 5 for $d = 0.5$ with different sigma values, $\sigma = 0.25$ and $\sigma = 0.75$, respectively, at various time steps. In both Figure 4 and 5, the wave lengths of curves increases and the amplitudes of the waves are reduced with increasing time steps. Overall, it can be said that the solution of the explicit FDE is convergent, hence stable for $d = 0.5$ with $\sigma = 0.25$, and $\sigma = 0.75$ since there is no distortion observed.

Figure 6 demonstrates the solution of the explicit FTCS method for Equation 5 with $\sigma = 1.2$ and $d = 0.5$ at various time steps. For the curve in the Figure 6, the amplitude of the wave increases, and the wave becomes disrupted with increasing time step. Consequently, the solution of the explicit FDE is unstable for $\sigma = 1.2$ and $d = 0.5$

To conclude this subsection since the explicit FDE of the convection-diffusion equation is stable for some d and σ values whereas it can be unstable for some other combinations of d and σ values, it is conditionally stable.

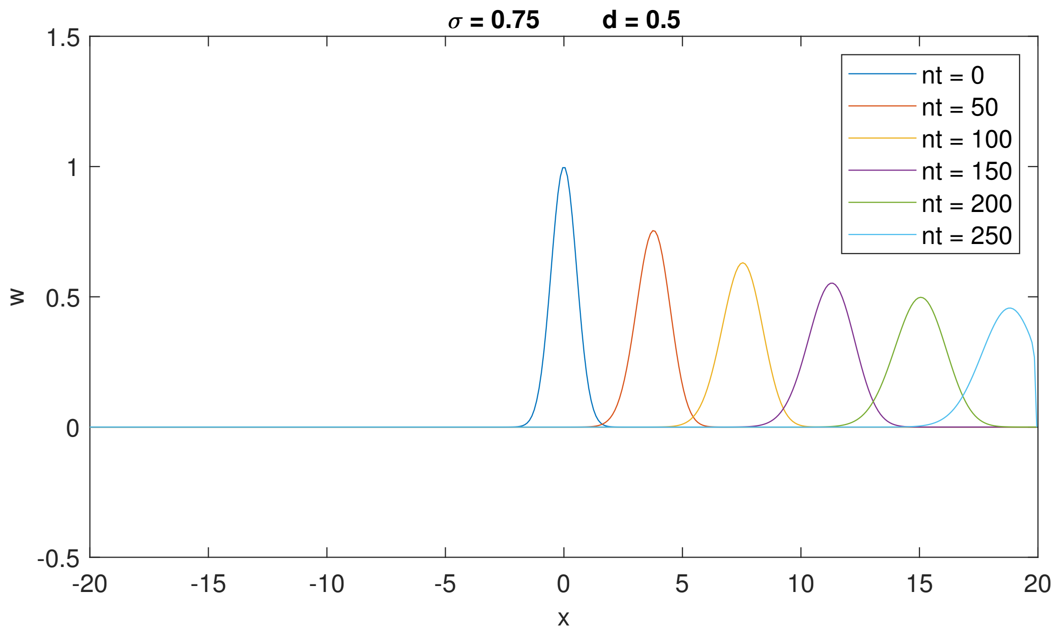

Figure 5: Convection diffusion solution by the explicit FTCS method with $\sigma = 0.75$ and $d = 0.5$ at different time steps.

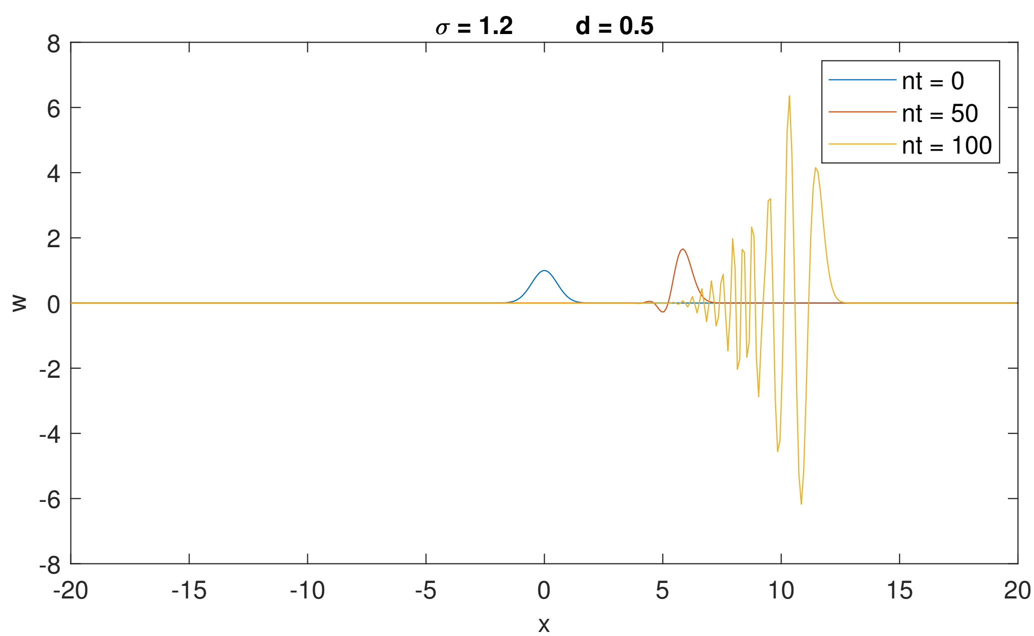

Figure 6: Convection diffusion solution by the explicit FTCS method with $\sigma = 1.2$ and $d = 0.5$ at different time steps.

3.2 Comparison of convective eqution with FTBS

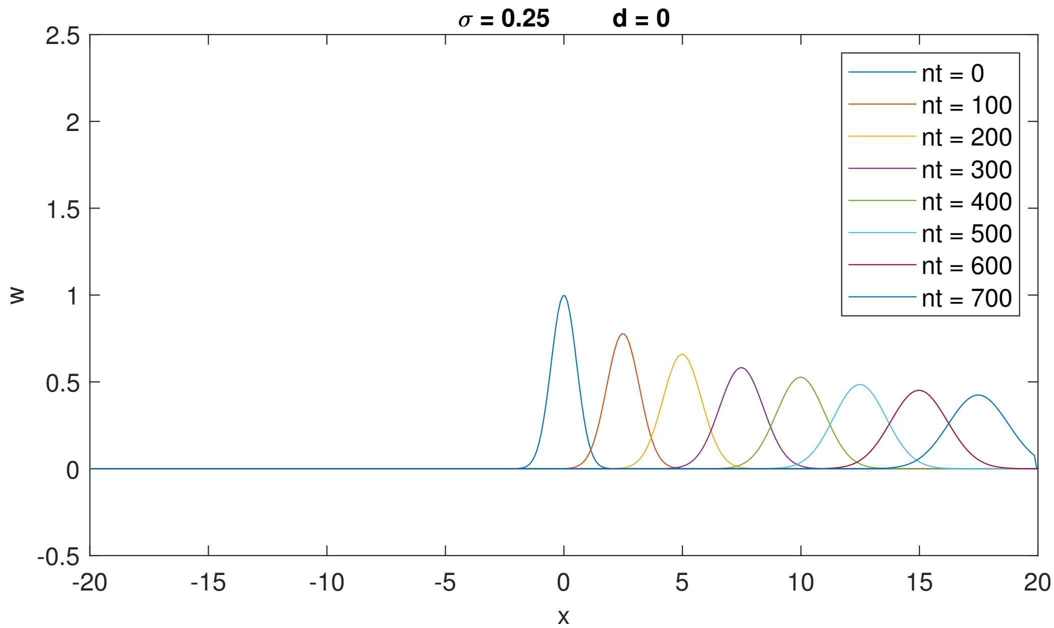

Figure 7: Convection solution by the explicit FTBS method with $\sigma = 0.25$ at different time steps.

Figures 7, 8, 9, and 10 demonstrate solutions of convection equation using FTBS with different σ values at various time steps. It can be observed that for σ values less than 1.2, the solutions obtained are convergent. However, for $\sigma = 1.2$, wave curve is distorted and the solution is divergent. Besides, comparing Figures 7, 8, and 9, it can be seen that the amplitudes of the wave curves decrease faster, whereas the wave lengths of the curves experience a more rapid increase for smaller σ values.

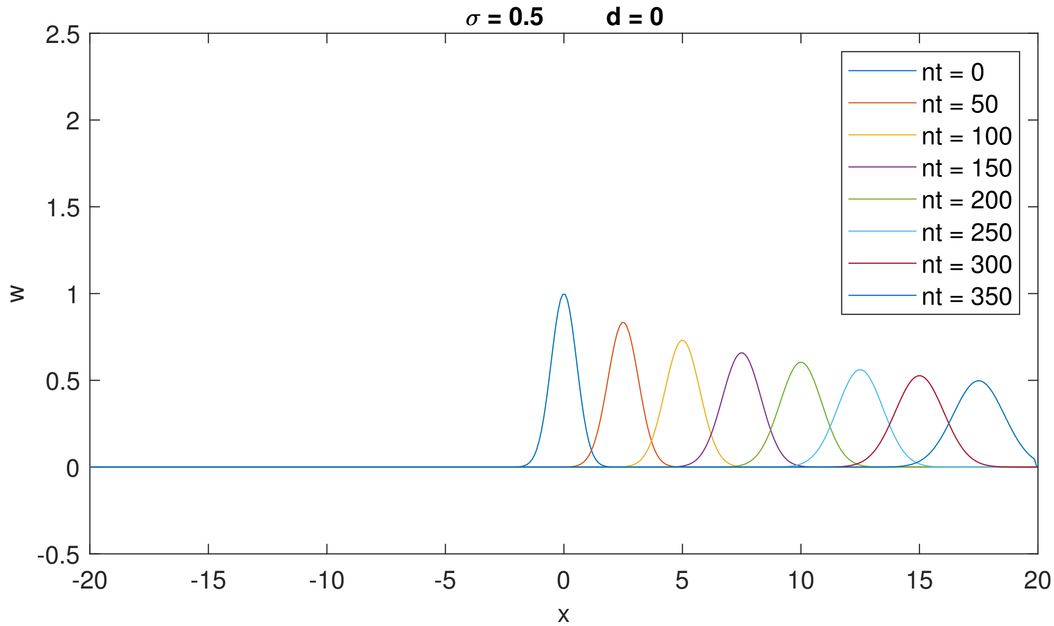

Figure 8: Convection solution by the explicit FTBS method with $\sigma = 0.5$ at different time steps.

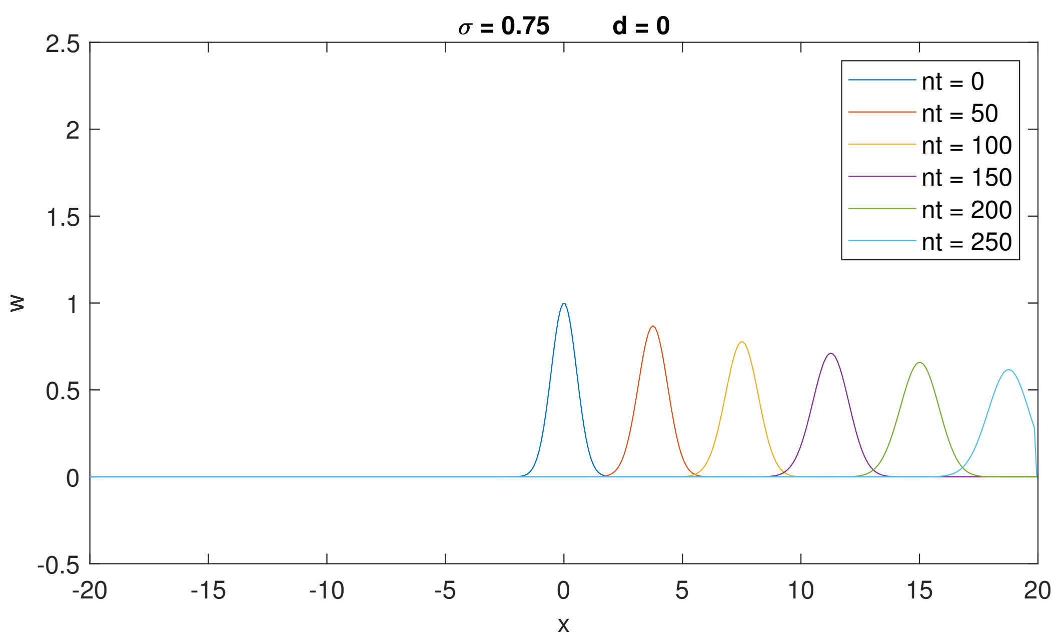

Figure 9: Convection solution by the explicit FTBS method with $\sigma = 0.75$ at different time steps.

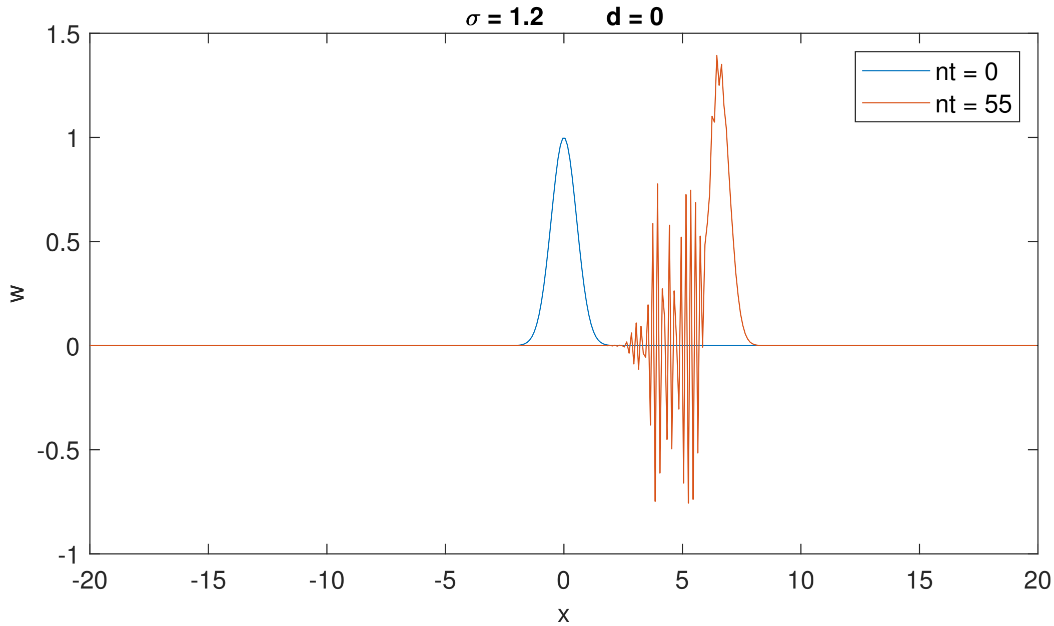

Figure 10: Convection solution by the explicit FTBS method with $\sigma = 1.2$ at $nt = 0$ and $nt = 55$.

Figures 11, 12, 13, and 14 illustrate the solutions of convection diffusion equation using FTBS with various σ values at different time steps while keeping d value constant. As can be seen from these figures, for $\sigma = 1.2$, a convergent solution is obtained for the convection diffusion equation. Nonetheless, for σ values smaller than 1.2, calculations gave divergent solutions.

Figures 14, 15, and 16 monitor the solutions of convection diffusion equation using FTBS with various d values at different time steps while keeping σ value constant. When the d value is decreased to 0.25 from 0.5, the solution remains convergent. Nevertheless, if the d value is increased, such as 0.75, distortion in wave curve is observed and the solution obtained is divergent. Comparing Figures 14 and 15, it is observed that the decrease in the amplitude of the waves is more significant for higher d values. In addition, the wave length increases faster for d = 0.5 compared to the increase in the wave length for d = 0.25.

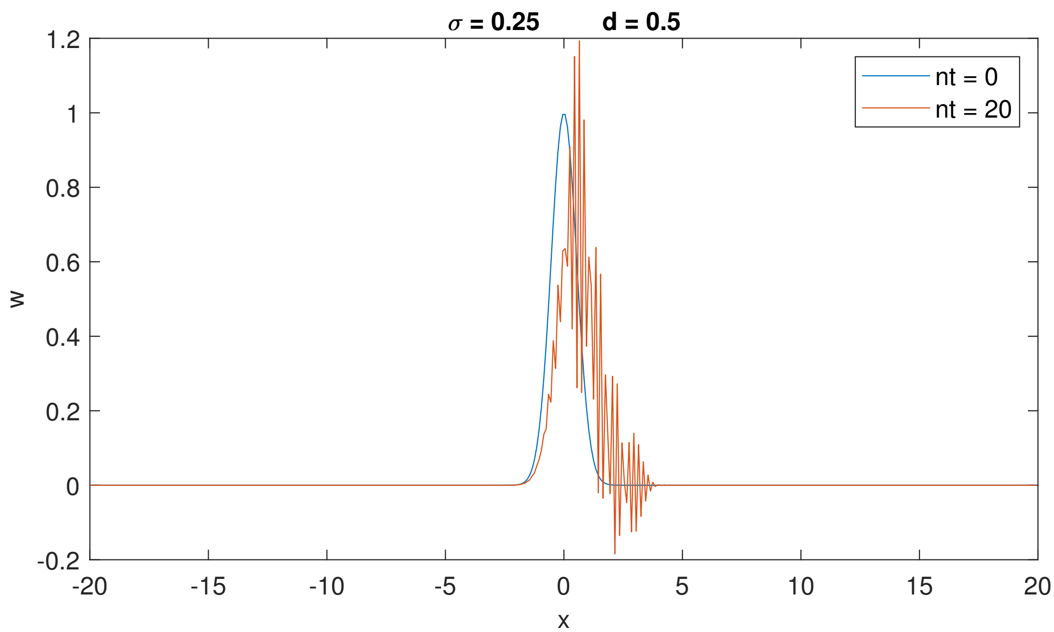

Figure 11: Convection diffusion solution by the explicit FTBS method with $\sigma = 0.25$ and $d = 0.5$ at $nt = 0$ and $nt = 20$.

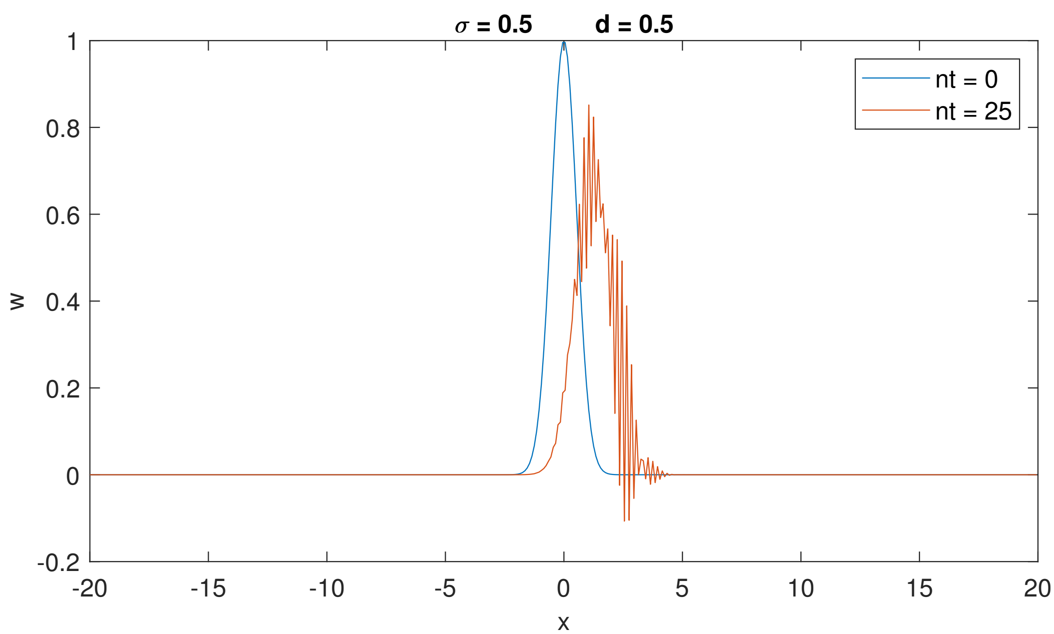

Figure 12: Convection diffusion solution by the explicit FTBS method with $\sigma = 0.5$ and $d = 0.5$ at $nt = 0$ and $nt = 25$.

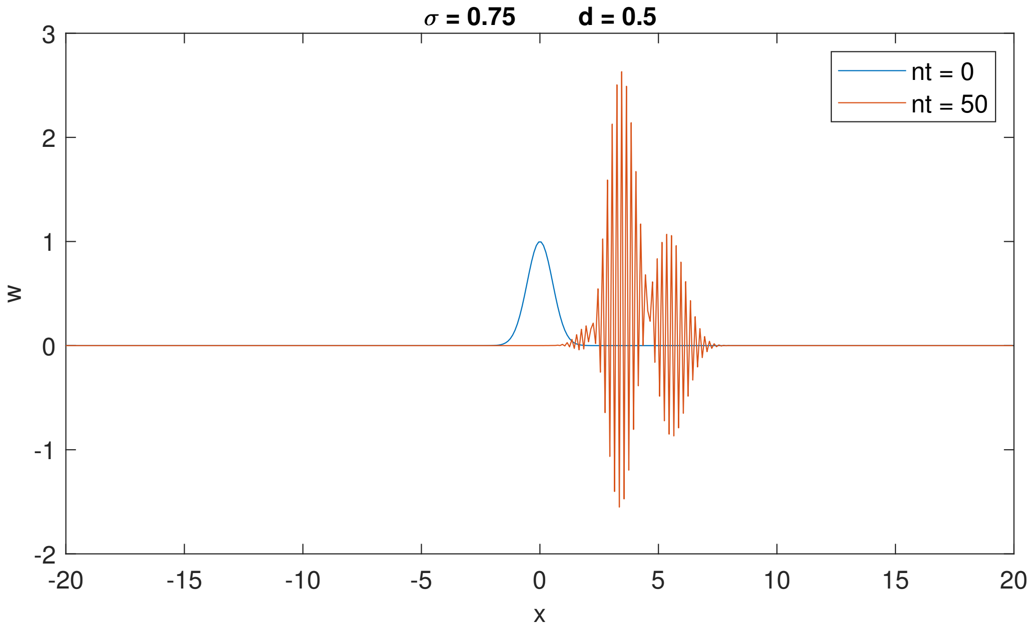

Figure 13: Convection diffusion solution by the explicit FTBS method with $\sigma = 0.75$ and $d = 0.5$ at $nt = 0$ and $nt = 55$.

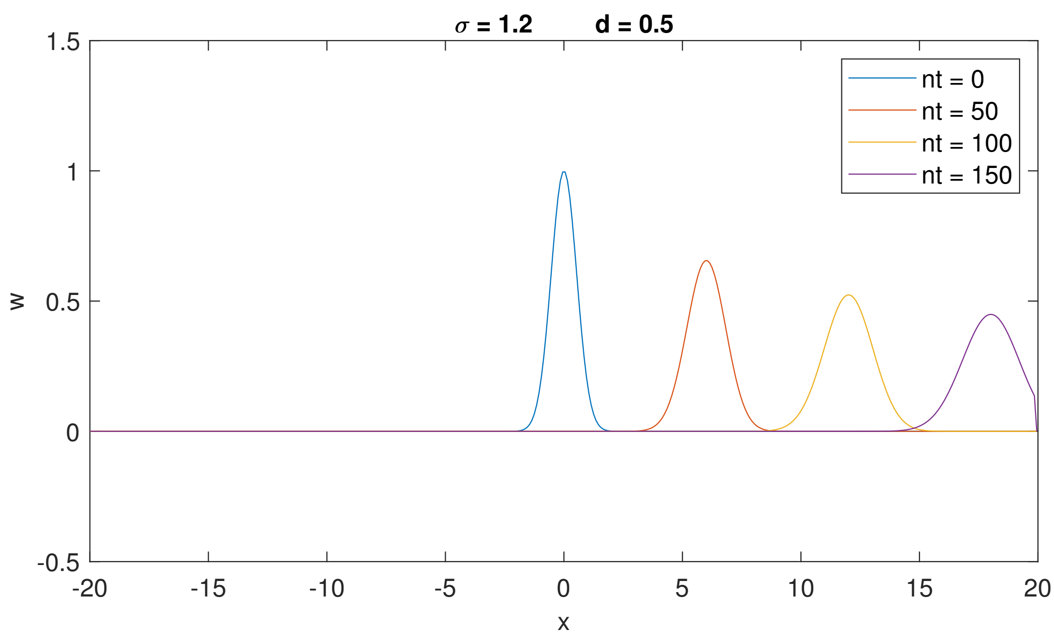

Figure 14: Convection diffusion solution by the explicit FTBS method with $\sigma = 1.2$ and $d = 0.5$ at different time steps.

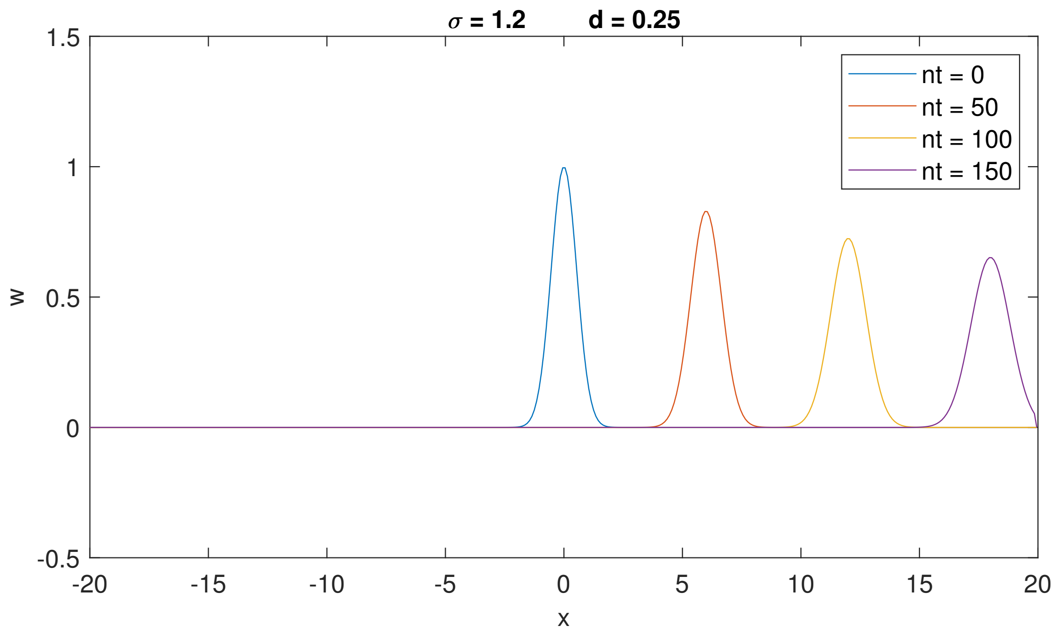

Figure 15: Convection diffusion solution by the explicit FTBS method with $\sigma = 1.2$ and $d = 0.25$ at different time steps.

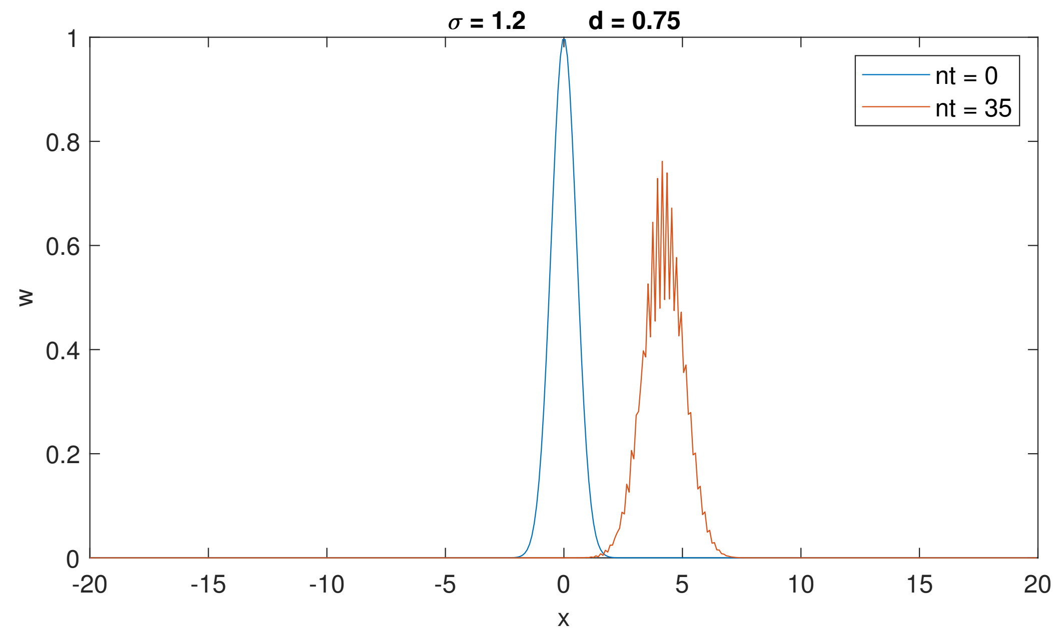

Figure 16: Convection diffusion solution by the explicit FTBS method with $\sigma = 1.2$ and $d = 0.75$ at different time steps.

Overall, if the solutions of the convection-diffusion equation and the convection equation are compared, it can be denoted that for σ values lower than 1.2 the FTBS equation gives convergent solutions for the convection equation whereas for the convection-diffusion equation the solutions obtained by FTBS equation is divergent.

3.3 Solution of the given FDE

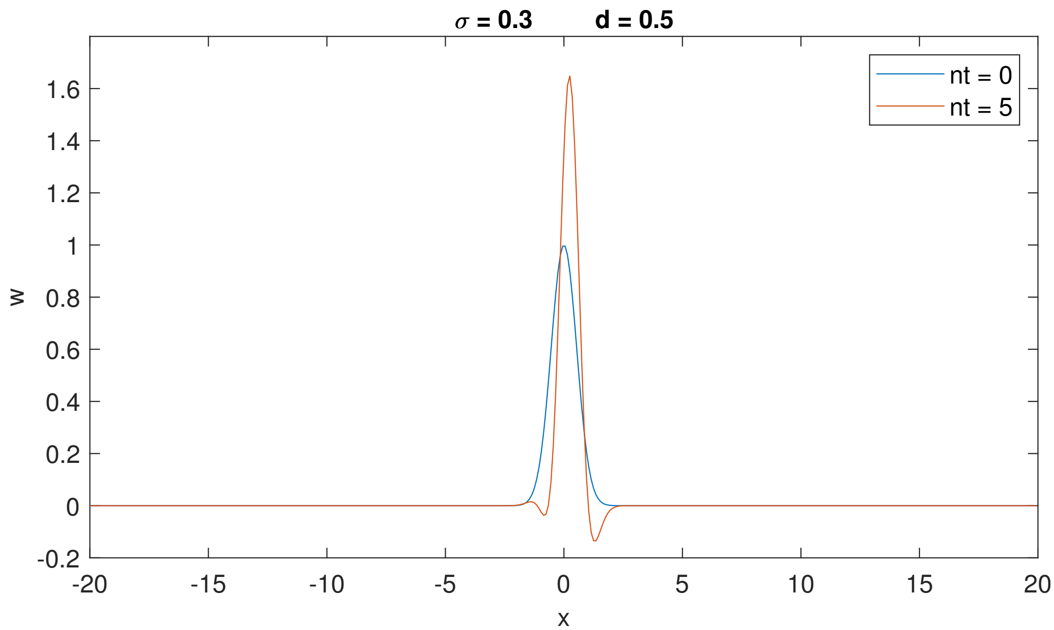

Figure 17: Solution of given FDE with $\sigma = 0.3$ and $d = 0.5$ at different time steps.

Stability of given FDE is numerically experimented for σ and d values between 0 and 2 with 0.05 spacing. Some of the results this experimentation is given in Figures 17, 18 and 19. As a result, it is concluded that given FDE is unconditionally unstable. Although given FDE is found consistent in Section 2.3, as FDE is found to be unstable, numerical solution does not converge.

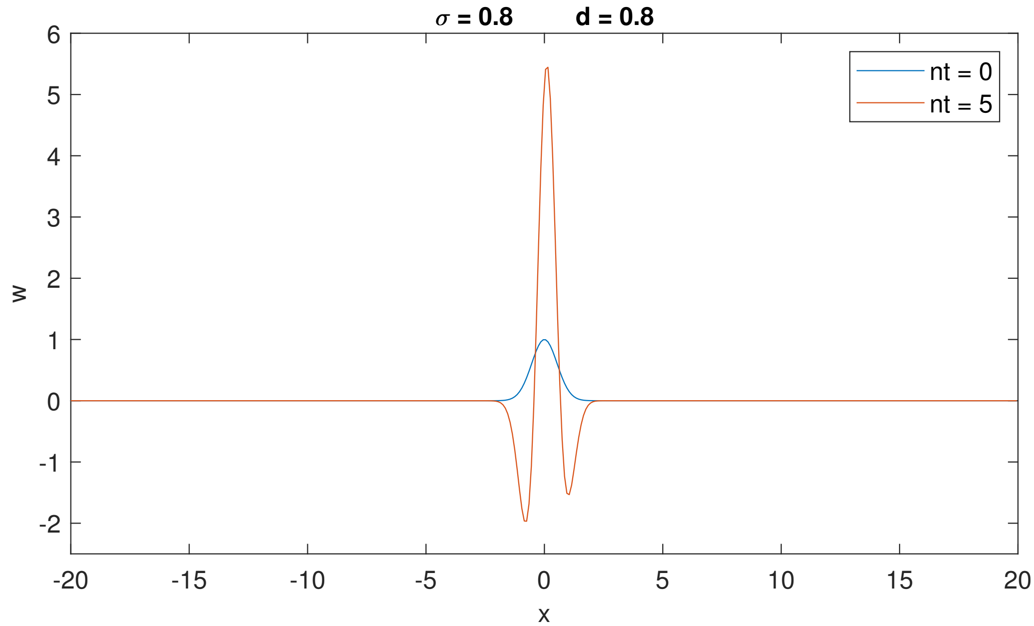

Figure 18: Solution of given FDE with $\sigma = 0.8$ and $d = 0.8$ at different time steps.

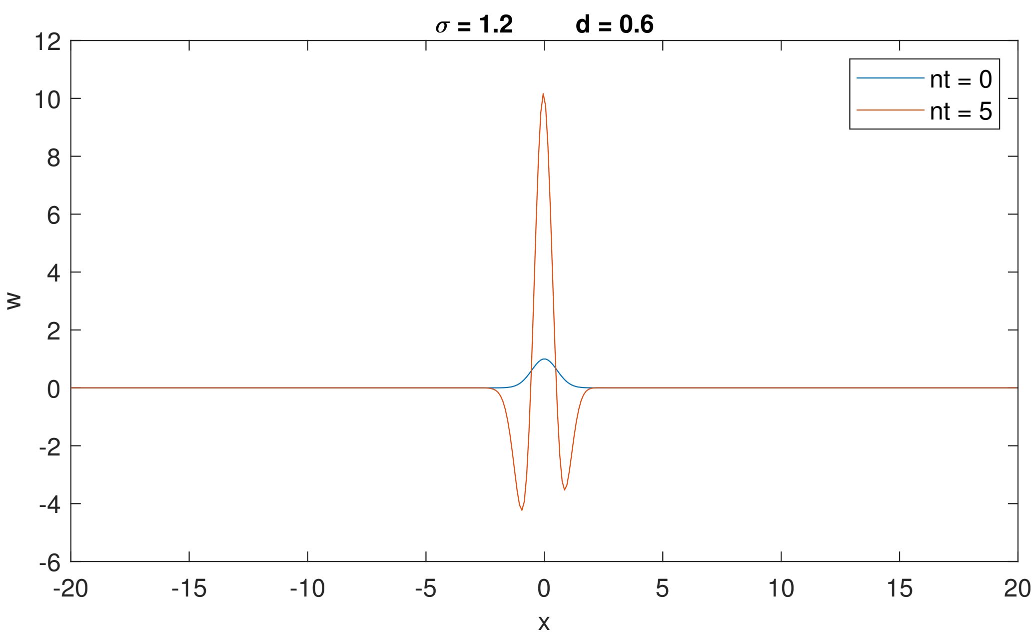

Figure 19: Solution of given FDE with $\sigma = 1.2$ and $d = 0.6$ at different time steps.

3.4 Bonus: Implicit Solution

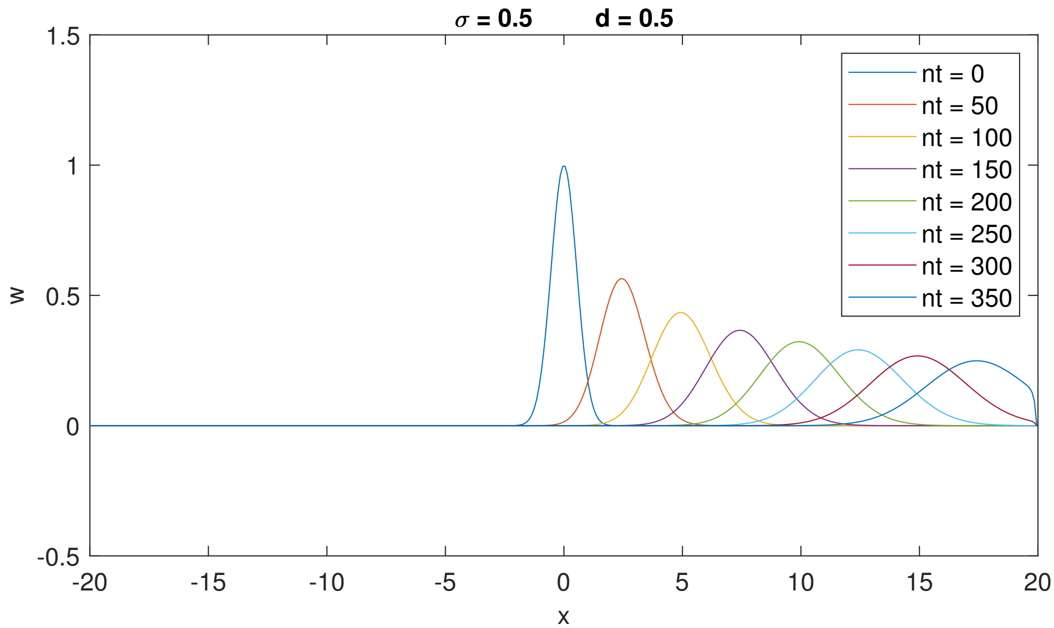

Figure 20: Implicit Solution of Convection diffusion equation with $\sigma = 0.5$ and $d = 0.5$

Implicit FDE derived in Equation 21 is solved for various σ and d values. It is found that solution converges for every σ and d combination. Therefore implicit method is unconditionally stable. Figures 20, 21 and 22 shows some of the solutions. As this method is unconditionally stable, a larger timestep can be used for calculations. But it should be noted that the accuracy of the solution decreases as timestep increases. Because of that, the time step size is still limited if one wants an accurate solution.

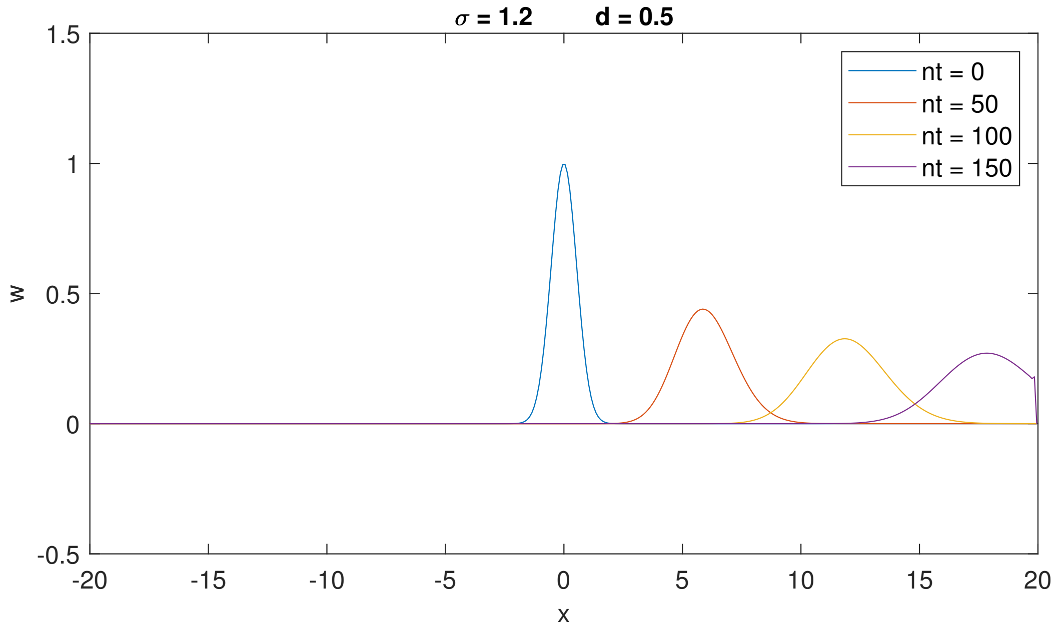

Figure 21: Implicit Solution of Convection diffusion equation with $\sigma = 1.2$ and $d = 0.5$

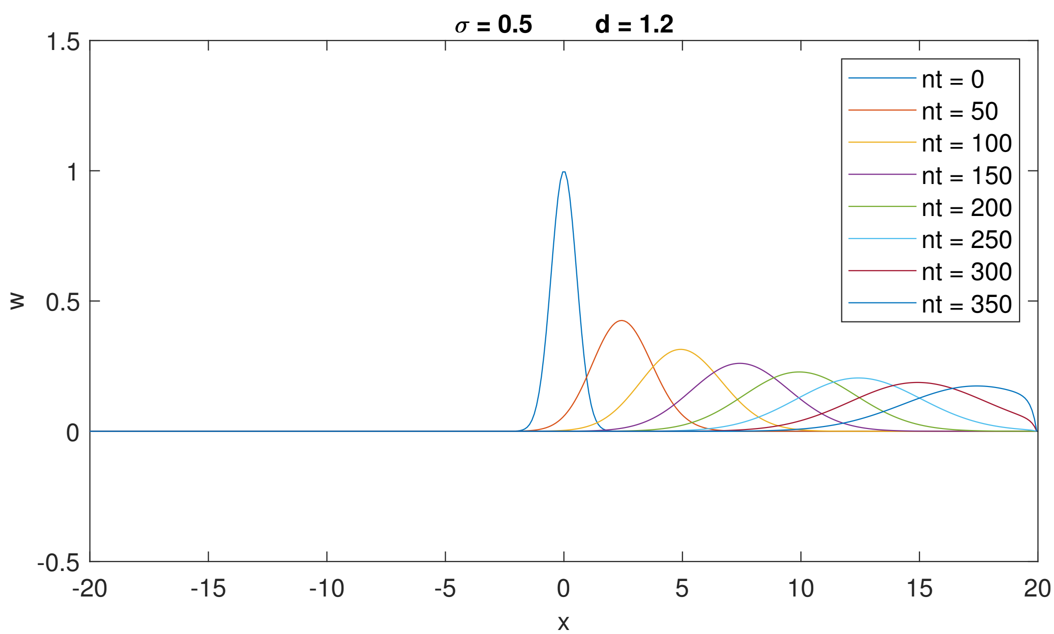

Figure 22: Implicit Solution of Convection diffusion equation with $\sigma = 0.5$ and $d = 1.2$

4 Conclusion

In this homework, the convection-diffusion equation was solved by using different finite-difference equations and convection equation was solved with the explicit FTBS equation. First, in the solution of the convection-diffusion equation, the explicit FTCS equation was used and it was shown that the solution was conditionally stable. The reason of is that for some combination of σ and d values, convergent, hence stable solutions were determined whereas solutions were unstable for other combinations, such as $\sigma = 1.2$ and $d = 0.5$ or $\sigma = 0.5$ and $d = 0.75$. Then, the explicit FTBS equation was used to solve the convection equation. The results showed that as the value of σ increased up to some value, the solutions were stable and the shrinkage in the waves of solution curves is less significant. Also, the solution was distorted and unstable for $\sigma = 1.2$. Afterwards, the solution of the convection-diffusion equation was calculated with the explicit FTBS method. It was seen that with constant d value ($d = 0.5$), unstable and disrupted solution curves were observed for some σ values such as 0.25, 0.5, and 0.75. On the other hand, for constant $\sigma = 1.2$, with increased d value up to some value, the change in the waves of solutions, which were convergent, became more insignificant. When d were increased to 0.75, a divergent solution was observed. At the next part of the homework, consistency analysis was made, and it was determined that Equation 24 is consistent. However, it was seen that from the solutions of the Equation 24 with different σ and d values, it is divergent and unstable. In the last part of the homework, the solution of the convection-diffusion equation was calculated with the BTCS Euler implicit method. All solutions obtained in Subsection 3.6 were convergent with different σ and d values. Therefore, it may be more reasonable to solve the convection-diffusion equation with the implicit FDE since it provided convergent solutions in a more large range of σ and d values than the explicit FDEs.