METU Department of Aerospace Eng
AE305 Numerical Methods
HW#1, Fall 2020
Deadline announced on ODTUCLASS

The velocity of an aircraft accelerating under the action of the engine thrust, aerodynamic forces, and ground friction is governed by

$$\frac{W}{g}\frac{dV}{dt} = T - D - \mu(W - L)$$

where W is the aircraft weight, g is gravitational acceleration, V is the velocity with respect to ground, T is the total engine thrust, D is the aerodynamic drag, μ is the friction coefficient between the landing gear tires and the runway surface, and L is the aerodynamic lift. The aerodynamic forces are given by

$$L = C_L \frac{1}{2}\rho_\infty V_\infty^2 S,$$
$$D = C_D \frac{1}{2}\rho_\infty V_\infty^2 S$$

where C_L and C_D are the aerodynamic lift and drag coefficients, respectively; ρ_∞ is the freestream air density, V_∞ is the freestream air velocity relative to the airplane, and S is the wing planform area.

Consider a turbofan-powered airplane with the following charactersitics:

Wing span, $b = 16.25\,\mathrm{m}$

Wing planform area, $S = 29.24\,\mathrm{m}^2$

Take-off weight, $W_{\mathrm{TO}} = 88250\,\mathrm{N}$

Max. available thrust, $T_{\mathrm{A,max,SL}} = 16256\,\mathrm{N}$/per engine at sea level (SL)

Number of engines, $N_e = 2$

Drag polar, $C_D = 0.0207 + 0.0605 C_L^2$, in ground roll motion

Max. lift coeff, $C_{L,max} = 1.792$ on ground

Friction coeff. between landing gear and runway surf, $\mu = 0.02$

For this aircraft taking off with max. take-off weight,

(1) Calculate the **minimum** time needed for lift-off at airport elevations of 0 m (sea level, SL), using the **Euler's method** with 3 different time steps. Plot the velocity versus time curves for all of the time steps used. compare and discuss them.

(2) Calculate the **minimum** time needed for lift-off at airport elevations of 1000 m, and 2000 m, with the **minimum** time step used in part (1), Assume the thrust generated by the engines is simply proportional to the free-stream air density ($T_{\mathrm{A,max}} = T_{\mathrm{A,max,SL}}\frac{\rho_\infty}{\rho_{\infty,\mathrm{SL}}}$), using the **Euler's method**. Plot the velocity versus time curves for all of the three elevations, compare and discuss the effects of airport elevation.

(3) Repeat (1) using a second-order Runge-Kutta (**RK2**) algorithm where p_1 is chosen arbitrarily within the $(0 \ldots 1)$ interval and a_1 and a_2 are evaluated accordingly. Compare the velocity versus time curves yielded by the Euler's and your own RK2 methods and discuss them.

(4) (BONUS) Note the area under a velocity versus time curve gives the distance covered in time. Then, calculate the **minimum** ground roll distances for lift-off at the three elevations mentioned above, using a simple trapezoidal integration rule. Compare the results and discuss.

$$\int_0^{N_p \Delta t} V(t)\,dt \approx \sum_{i=0}^{N_p - 1} \frac{1}{2}(V_i + V_{i+1})\Delta t$$