METU Department of Aerospace Eng
AE305 Numerical Methods
HW#3, Fall 2020

Consider potential (inviscid, incompressible and irrotational) flow fields around 2-D objects. For such flows the velocity field is governed by the Laplace's equation:

$$\vec{\nabla} \cdot \vec{\nabla}\phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$

where

$$\vec{V} = \vec{\nabla}\phi$$

This equation may be solved by various numerical discretization methods, such as finite volume or finite difference. The latter requires a structured grid, while the former has no limitation on the grid type. A direct numerical solution may generate a system of linear algebratic equations, which may be solved using linear algebra tools. We will avoid the solution of system of equations by introducing a pseudo time derivative, $\partial\phi/\partial t$ such that

$$\frac{\partial \phi}{\partial t} = \nu\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right),$$

where ν is an artificial diffusion coefficient.

This PDE is similar to the heat condution equation and may be solved by the Finite Volume method discussed. If steady boundary conditions are specified, a steady state solution is obtained as the derivative goes to zero during the integration, and the Laplace's equation given above is satisfied. Note that the corresponding integral equation is given by

$$\frac{\partial}{\partial t}\int_{\Omega} \phi \, d\Omega + \oint_S \vec{F} \cdot d\vec{S} = 0 \tag{1}$$

where

$$\vec{F} = -\nu\frac{\partial \phi}{\partial x}\vec{i} - \nu\frac{\partial \phi}{\partial y}\vec{j}$$

For an external flow problem, the far-field boundary condition is

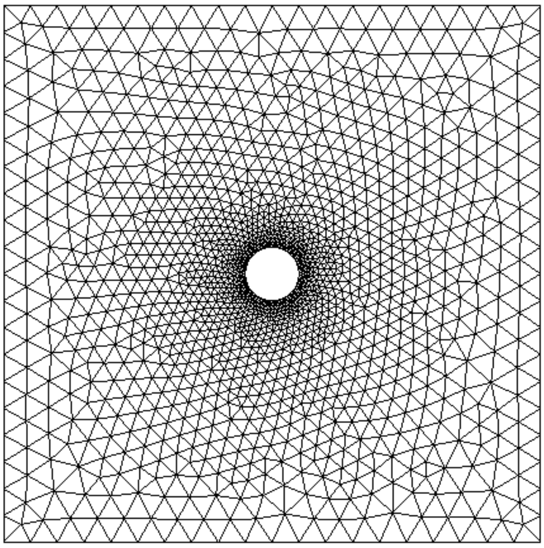
$$\begin{aligned} \vec{\nabla}\phi|_{\text{far BC}} &= \vec{V}_\infty = u_\infty\vec{i} + v_\infty\vec{j} = V_\infty(\cos\alpha\vec{i} + \sin\alpha\vec{j}) \\ &= V_\infty\cos\alpha\,\vec{i} + V_\infty\cos\alpha\,\vec{j} \end{aligned} \tag{2}$$

The boundary condition on the solid surface (wall) is simply no flow through condition. That is,

$$(\vec{n}\cdot\vec{V})_{\text{wall}} = 0 \quad \text{or} \quad (\vec{F}\cdot\vec{S})_{\text{wall}} = 0. \tag{3}$$

TASK

1. Complete the provided incomplete FV code in Fortran to solve Eqs(1)-(3), and obtain the flowfield around a cylinder (circle in 2D). Use the unstructured grid provided.



2. Plot the velocity vectors and streamlines at various values of the angle of attack, α.
3. Using the Bernoulli equation find the pressure coefficient distribution over the cylinder, and compare it with the analytical solution that is available in most aerodynamics text books.
4. Replace the cylinder with a symmetric NACA airfoil profile of your choice, and generate a triangular unstructured grid.
5. Calculate the flowfield around the airfoil at zero angle of attack, and the corresponding pressure coefficient distribution. Plot the pressure contours around the airfoil, and the pressure coefficient over the airfoild surface.
6. Obtain solutions with course, medium and and fine grids and discuss the results.