

ASSIGNMENT 6

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Download all python codes from

https://github.com/tejasri3657/Assignment-6/blob/main/Assignment_6.py

Latex-tikz codes from

<https://github.com/tejasri3657/Assignment-6/new/main>

Final equation is :

$$\frac{\mathbf{y}^T D \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.7)$$

$$\Rightarrow \frac{\mathbf{y}^T \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.8)$$

$$\Rightarrow \mathbf{y}^T \begin{pmatrix} 81 & 0 \\ 0 & 36 \end{pmatrix} = 1 \quad (2.0.9)$$

Plot of ellipse:

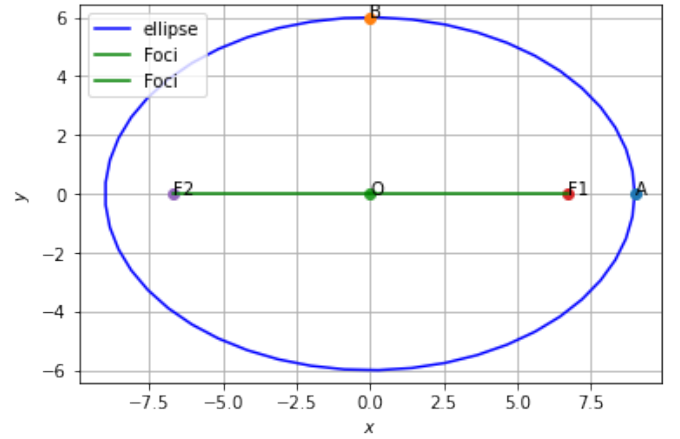


Fig. 2.1: Ellipse $\frac{x^2}{81} + \frac{y^2}{36} = 1$

1 QUESTION No 2.74(F)

In each of the following find the equation for the ellipse that satisfies the given condition:

- 1) Latus rectum length 8, foci $\begin{pmatrix} \pm 3\sqrt{5} \\ 0 \end{pmatrix}$

2 SOLUTION

Let

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}, b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.1)$$

$$\text{Now, } c^2 = a^2 - b^2 \quad (2.0.2)$$

$$\Rightarrow 45 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} \quad (2.0.3)$$

And, length of latus rectum = 8

$$\Rightarrow \frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a \quad (2.0.4)$$

$$\Rightarrow \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} = 4 \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \quad (2.0.5)$$

From (2.0.3) & (2.0.5), Find λ_1, λ_2 & $\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f$ (2.0.6)