ASSIGNMENT 6

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1 Question No 2.74(f)

In each of the following find the equation for the ellipse that satisfies the given condition:

1) Latus rectum length 8, foci $\begin{pmatrix} \pm 3\sqrt{5} \\ 0 \end{pmatrix}$

2 Solution

Given
Latus rectum length = 8

$$Foci = \begin{pmatrix} \pm 3\sqrt{5} \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{c} = 3\sqrt{5} \tag{2.0.2}$$

since,foci on x-axis.
Equation of ellipse can be written as

$$\frac{x^2}{81} + \frac{y^2}{36} = 1 \tag{2.0.3}$$

we know,

$$c^2 = a^2 - b^2 \tag{2.0.4}$$

$$(3\sqrt{5})^2 = a^2 - b^2 \tag{2.0.5}$$

$$\therefore a^2 - b^2 = 45 \tag{2.0.6}$$

Latus rectum length=8

$$\frac{2(b^2)}{a} = 8 \tag{2.0.7}$$

$$b^2 = 4a \tag{2.0.8}$$

$$a^2 + 4a = 45 \tag{2.0.9}$$

$$a^2 - 4a - 45 = 0 \tag{2.0.10}$$

$$(a - 9)(a + 5) = 0 \tag{2.0.11}$$

$$a = -5(or)a = 9 \tag{2.0.12}$$

Since, a is distance,it can't be negative.

$$\therefore \mathbf{a} = 9 \tag{2.0.13}$$

$$b^2 = 4a = 36 \tag{2.0.14}$$

$$\implies b = 6 \tag{2.0.15}$$

Now,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2.0.16}$$

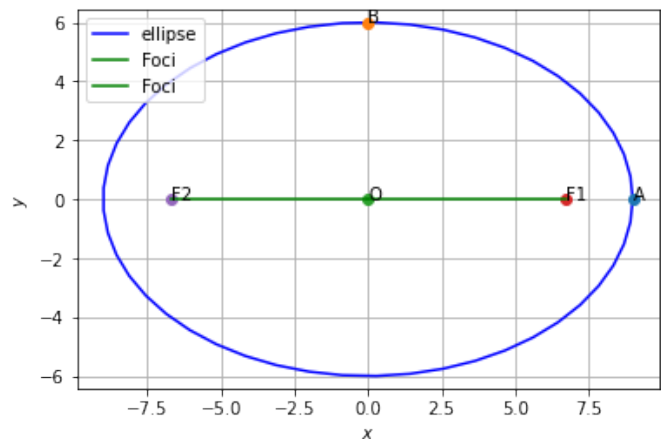putting values,

$$\frac{x^2}{81} + \frac{y^2}{36} = 1 \tag{2.0.17}$$

Plot of ellipse:

Fig. 2.1: Ellipse $\frac{x^2}{81} + \frac{y^2}{36} = 1$