ASSIGNMENT 2

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1 QUESTION No 2.10

Find the intersection of the following lines

1)

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 14$$
$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

2)

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 3$$
$$\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \end{pmatrix}\mathbf{x} = 6 \tag{1.0.2}$$

2 SOLUTION

1)

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 14$$
$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 4 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 14 \\ 4 \end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 14 \\ 0 & -2 & -10 \end{pmatrix} \tag{2.0.3}$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2/-2} \begin{pmatrix} 1 & 0 & 9 \\ 0 & -2 & -10 \end{pmatrix} \tag{2.0.4}$$

$$\xrightarrow{R_2 \leftarrow R_2/-2} \begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 5 \end{pmatrix} \tag{2.0.5}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 5 \end{pmatrix} \tag{2.0.6}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{2.0.7}$$

is also 2.

$$\because Rank\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = Rank\begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix}$$
$$= dim\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$
$$= 2 \tag{2.0.8}$$

$$\because Rank\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \tag{2.0.9}$$

$\therefore$ Given lines (1.0.1) have unique solution. $\therefore$ The given lines are intersection. PLOT OF GIVEN LINES -
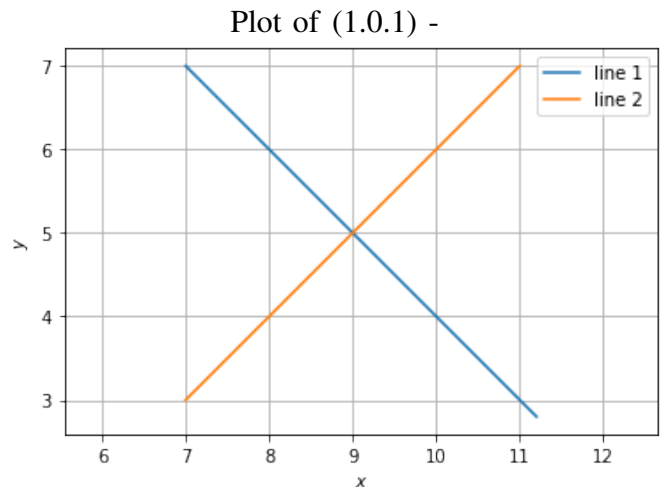

Fig. 2.1: intersecting lines

2)

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 3$$
$$\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = 6 \qquad (2.0.10)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \qquad (2.0.11)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & -1 & 3 \\ \frac{1}{3} & \frac{1}{2} & 6 \end{pmatrix} \xleftarrow{R_2 \leftarrow R_2 - R_1/3} \begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{5}{6} & 5 \end{pmatrix} \qquad (2.0.12)$$

$$\xleftarrow{R_2 \leftarrow R_2/5} \begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{1}{6} & 1 \end{pmatrix} \qquad (2.0.13)$$

$$\xleftarrow{R_2 \leftarrow 6R_2} \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 6 \end{pmatrix} \qquad (2.0.14)$$

$$\xleftarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 6 \end{pmatrix} \qquad (2.0.15)$$

∵ row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 6 \end{pmatrix} \qquad (2.0.16)$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \qquad (2.0.17)$$

is also 2.

$$\therefore Rank \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 6 \end{pmatrix}$$

$$(2.0.18)$$

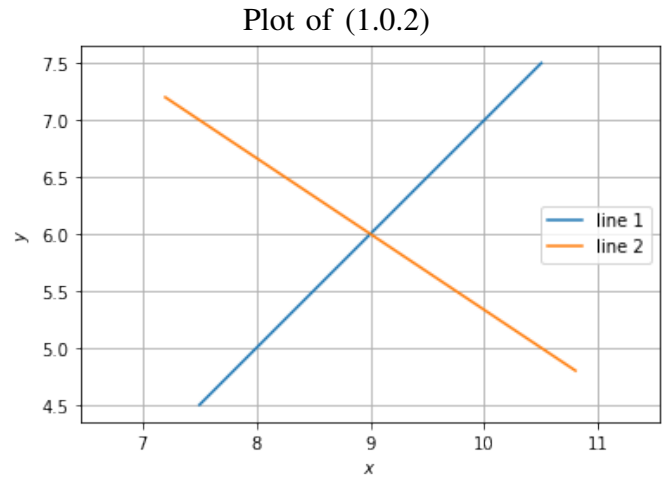∴ Given lines (1.0.2) have unique solution. ∴ The givens lines are intersection. PLOT OF GIVEN LINES -

Fig. 2.2: intersecting lines