ASSIGNMENT 2

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Download all python codes from

https://github.com/keerthireddy.atla/ASSIGNMENT2/tree/main/CODES

and latex-tikz codes from

https://github.com/keerthireddy.atla/ASSIGNMENT2/tree/main/tex

1 Question No 2.10

Find the intersection of the following lines

1)
$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 14 \\ \begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

2)
$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 3 \\ \begin{pmatrix}\frac{1}{3} & \frac{1}{2}\end{pmatrix}\mathbf{x} = 6 \tag{1.0.2}$$

2 SOLUTION

1)
$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 14 \\ \begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 4 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}14\\4\end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}1 & 1 & 14\\1 & -1 & 4\end{pmatrix} \xleftrightarrow{R_2\leftarrow R_2-R_1} \begin{pmatrix}1 & 1 & 14\\0 & -2 & -10\end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix}1 & 1 & 14\\0 & -2 & -10\end{pmatrix} \xleftrightarrow{R_1\leftarrow R_1-R_2/-2} \begin{pmatrix}1 & 0 & 9\\0 & -2 & -10\end{pmatrix} \tag{2.0.4}$$

$$\begin{pmatrix}1 & 0 & 9\\0 & -2 & -10\end{pmatrix} \xleftrightarrow{R_2\leftarrow R_2/-2} \begin{pmatrix}1 & 0 & 9\\0 & 1 & 5\end{pmatrix} \tag{2.0.5}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}1 & 0 & 9\\0 & 1 & 5\end{pmatrix} \tag{2.0.6}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} \tag{2.0.7}$$

is also 2.

$$\begin{aligned}\because Rank\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} &= Rank\begin{pmatrix}1 & 1 & 14\\1 & -1 & 4\end{pmatrix}\\ &= dim\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\\ &= 2 \end{aligned}\tag{2.0.8}$$

$$\because Rank\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} = \begin{pmatrix}1 & 1 & 14\\1 & -1 & 4\end{pmatrix} \tag{2.0.9}$$

$\therefore$ Given lines (1.0.1) have unique solution.
$\therefore$ The given lines are intersection.

2)
$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 3 \\ \begin{pmatrix}\frac{1}{3} & \frac{1}{2}\end{pmatrix}\mathbf{x} = 6 \tag{2.0.10}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}1 & -1\\\frac{1}{3} & \frac{1}{2}\end{pmatrix}\mathbf{x} = \begin{pmatrix}3\\6\end{pmatrix} \tag{2.0.11}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}1 & -1 & 3\\\frac{1}{3} & \frac{1}{2} & 6\end{pmatrix} \xleftrightarrow{R_2\leftarrow R_2-R_1/3} \begin{pmatrix}1 & -1 & 3\\0 & \frac{5}{6} & 5\end{pmatrix} \tag{2.0.12}$$

(2.0.13)

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}1 & -1 & 3\\0 & \frac{5}{6} & 5\end{pmatrix} \tag{2.0.14}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \tag{2.0.15}$$

is also 2.

$$\because Rank \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 \\ \frac{1}{3} & \frac{1}{2} & 6 \end{pmatrix} \tag{2.0.16}$$

$\therefore$ Given lines (1.0.2) have unique solution.
$\therefore$ The givens lines are intersection. PLOT OF GIVEN LINES -

Plot of (1.0.1) -

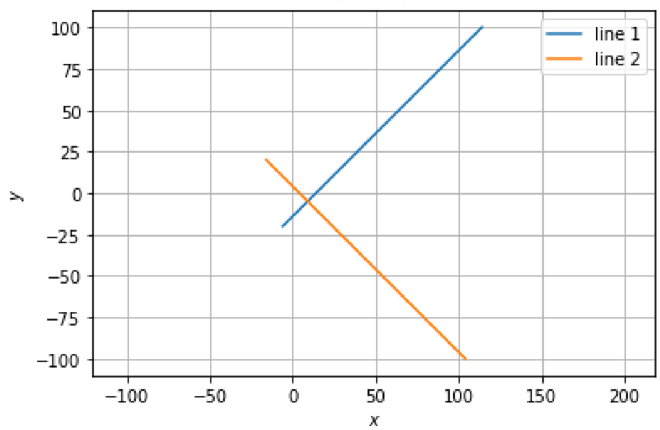

Fig. 2.1: perpendicular lines

Plot of (1.0.2)

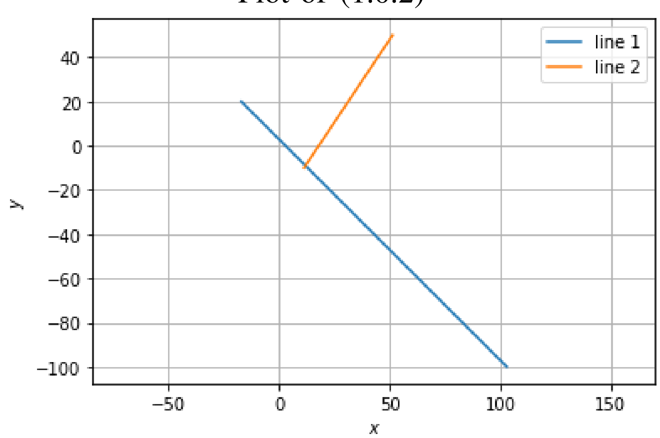

Fig. 2.2: Perpendicular lines