ASSIGNMENT 2

A.keerthana

1 Question No 2.10

Find the intersection of the following lines

1)

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 14$$
$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4 \qquad (1.0.1)$$

2)

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 3$$
$$\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = 6 \qquad (1.0.2)$$

2 SOLUTION

1)

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 14$$
$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4 \qquad (2.0.1)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 14 \\ 4 \end{pmatrix} \qquad (2.0.2)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \xleftarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 14 \\ 0 & -2 & -10 \end{pmatrix} \qquad (2.0.3)$$

$$\begin{pmatrix} 1 & 1 & 14 \\ 0 & -2 & -10 \end{pmatrix} \xleftarrow{R_1 \leftarrow R_1 - R_2/-2} \begin{pmatrix} 1 & 0 & 9 \\ 0 & -2 & -10 \end{pmatrix} \qquad (2.0.4)$$

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & -2 & -10 \end{pmatrix} \xleftarrow{R_2 \leftarrow R_2/-2} \begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 5 \end{pmatrix} \qquad (2.0.5)$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 5 \end{pmatrix} \qquad (2.0.6)$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad (2.0.7)$$

is also 2.

$$\because Rank \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = Rank \begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix}$$
$$= dim \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$
$$= 2 \qquad (2.0.8)$$

$$\because Rank \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \qquad (2.0.9)$$

$\therefore$ Given lines (1.0.1) have unique solution.
$\therefore$ The given lines are intersection.

2)

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 3$$
$$\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = 6 \qquad (2.0.10)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \qquad (2.0.11)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & -1 & 3 \\ \frac{1}{3} & \frac{1}{2} & 6 \end{pmatrix} \xleftarrow{R_2 \leftarrow R_2 - R_1/3} \begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{5}{6} & 5 \end{pmatrix} \qquad (2.0.12)$$

$$(2.0.13)$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{5}{6} & 5 \end{pmatrix} \qquad (2.0.14)$$

results in a matrix with 2 nonzero rows, its rank

is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \quad\quad (2.0.15)$$

is also 2.

$$\therefore Rank\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 \\ \frac{1}{3} & \frac{1}{2} & 6 \end{pmatrix}$$

$$(2.0.16)$$

$\therefore$ Given lines (1.0.2) have unique solution.
$\therefore$ The givens lines are intersection. PLOT OF GIVEN LINES -
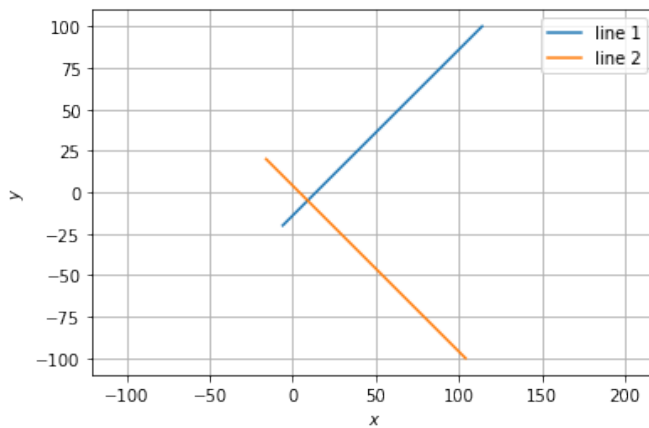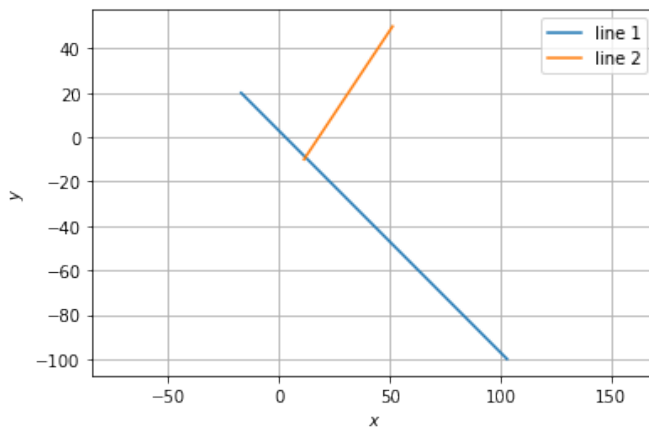
Plot of (1.0.1) -

Fig. 2.1: perpendicular lines

Plot of (1.0.2)

Fig. 2.2: Perpendicular lines