

ASSIGNMENT 2

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Download all python codes from

<https://github.com/Atlakeerthana/Assignment2/tree/main/Assignment2/codes>

and latex-tikz codes from

<https://github.com/Atlakeerthana/Assignment2/tree/main/Assignment2/main.tex>

1 QUESTION No 2.10

Find the intersection of the following lines

1)

$$\begin{aligned} (1 \ 1)\mathbf{x} &= 14 \\ (1 \ -1)\mathbf{x} &= 4 \end{aligned} \quad (1.0.1)$$

2)

$$\begin{aligned} (1 \ -1)\mathbf{x} &= 3 \\ \left(\frac{1}{3} \ \frac{1}{2}\right)\mathbf{x} &= 6 \end{aligned} \quad (1.0.2)$$

2 SOLUTION

1)

$$\begin{aligned} (1 \ 1)\mathbf{x} &= 14 \\ (1 \ -1)\mathbf{x} &= 4 \end{aligned} \quad (2.0.1)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 14 \\ 4 \end{pmatrix} \quad (2.0.2)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 14 \\ 0 & -2 & -10 \end{pmatrix} \quad (2.0.3)$$

$$\begin{pmatrix} 1 & 1 & 14 \\ 0 & -2 & -10 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - R_2 / -2} \begin{pmatrix} 1 & 0 & 9 \\ 0 & -2 & -10 \end{pmatrix} \quad (2.0.4)$$

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & -2 & -10 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 / -2} \begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 5 \end{pmatrix} \quad (2.0.5)$$

\therefore row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 5 \end{pmatrix} \quad (2.0.6)$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (2.0.7)$$

is also 2.

$$\begin{aligned} \therefore \text{Rank} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} &= \text{Rank} \begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \\ &= \dim \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \\ &= 2 \end{aligned} \quad (2.0.8)$$

$$\therefore \text{Rank} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \quad (2.0.9)$$

\therefore Given lines (1.0.1) have unique solution.

\therefore The given lines are intersection. PLOT OF GIVEN LINES -

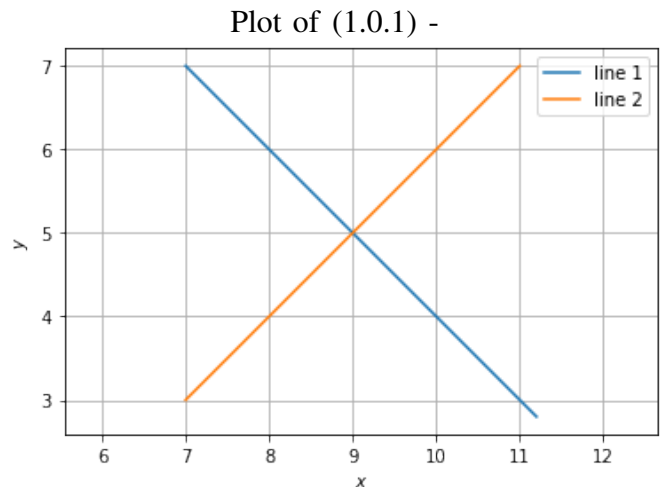


Fig. 2.1: intersecting lines

2)

$$\begin{aligned} (1 \quad -1) \mathbf{x} &= 3 \\ \left(\frac{1}{3} \quad \frac{1}{2}\right) \mathbf{x} &= 6 \end{aligned} \quad (2.0.10)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad (2.0.11)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & -1 & 3 \\ \frac{1}{3} & \frac{1}{2} & 6 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1/3} \begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{5}{6} & 5 \end{pmatrix} \quad (2.0.12)$$

$$\begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{5}{6} & 5 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2/5} \begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{1}{6} & 1 \end{pmatrix} \quad (2.0.13)$$

$$\begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{1}{6} & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow 6R_2} \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 6 \end{pmatrix} \quad (2.0.14)$$

$$\begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 6 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 6 \end{pmatrix} \quad (2.0.15)$$

\therefore row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 6 \end{pmatrix} \quad (2.0.16)$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \quad (2.0.17)$$

is also 2.

$$\therefore \text{Rank} \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 6 \end{pmatrix} \quad (2.0.18)$$

\therefore Given lines (1.0.2) have unique solution.

\therefore The given lines are intersection. PLOT OF GIVEN LINES -

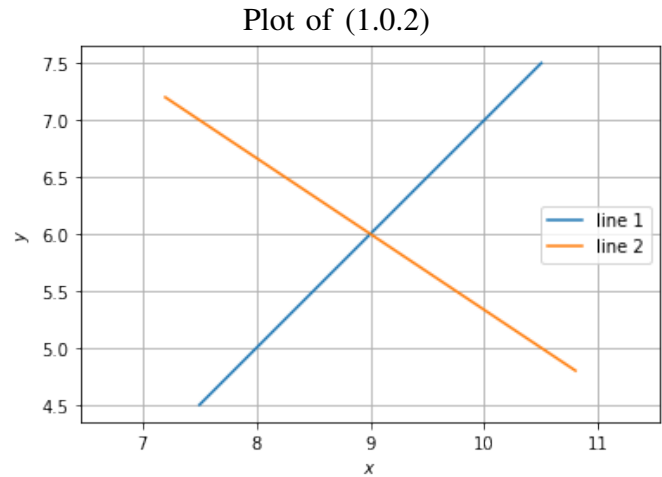


Fig. 2.2: intersecting lines