ASSIGNMENT 2

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Download all python codes from

https://github.com/Atlakeerthana/Assignment2/tree/main/Assignment2/codes

and latex-tikz codes from

https://github.com/Atlakeerthana/Assignment2/blob/main/Assignment2/main.tex

1 Question No 2.10

Find the intersection of the following lines

1)

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 14 \\ \begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

2)

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 3 \\ \begin{pmatrix} \frac{1}{3} & \frac{1}{2} \end{pmatrix}\mathbf{x} = 6 \tag{1.0.2}$$

2 SOLUTION

1)

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 14 \\ \begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 4 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 14 \\ 4 \end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 14 \\ 0 & -2 & -10 \end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix} 1 & 1 & 14 \\ 0 & -2 & -10 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - R_2/-2} \begin{pmatrix} 1 & 0 & 9 \\ 0 & -2 & -10 \end{pmatrix} \tag{2.0.4}$$

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & -2 & -10 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2/-2} \begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 5 \end{pmatrix} \tag{2.0.5}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & 0 & 9 \\ 0 & 1 & 5 \end{pmatrix} \tag{2.0.6}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{2.0.7}$$

is also 2.

$$\begin{aligned} \because Rank\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} &= Rank\begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \\ &= dim\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \\ &= 2 \end{aligned} \tag{2.0.8}$$

$$\because Rank\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 14 \\ 1 & -1 & 4 \end{pmatrix} \tag{2.0.9}$$

$\therefore$ Given lines (1.0.1) have unique solution.
$\therefore$ The given lines are intersection. PLOT OF GIVEN LINES -

Plot of (1.0.1) -

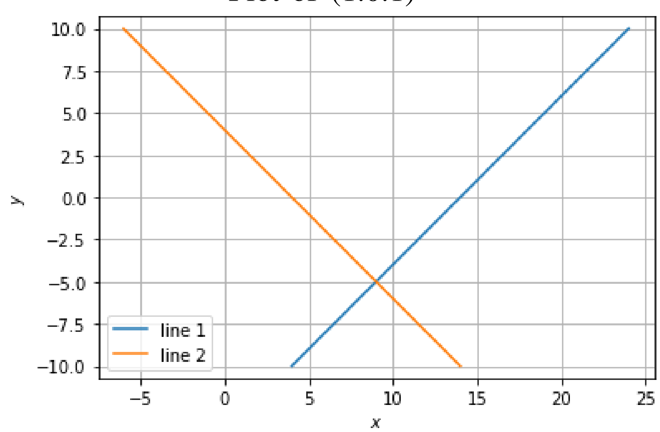

Fig. 2.1: perpendicular lines

2)

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 3 \\ \begin{pmatrix} \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = 6 \tag{2.0.10}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \tag{2.0.11}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & -1 & 3 \\ \frac{1}{3} & \frac{1}{2} & 6 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1/3} \begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{5}{6} & 5 \end{pmatrix} \tag{2.0.12}$$

$$\tag{2.0.13}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & -1 & 3 \\ 0 & \frac{5}{6} & 5 \end{pmatrix} \tag{2.0.14}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \tag{2.0.15}$$

is also 2.

$$\because Rank \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 \\ \frac{1}{3} & \frac{1}{2} & 6 \end{pmatrix} \tag{2.0.16}$$

$\therefore$ Given lines (1.0.2) have unique solution.
$\therefore$ The givens lines are intersection. PLOT OF GIVEN LINES -

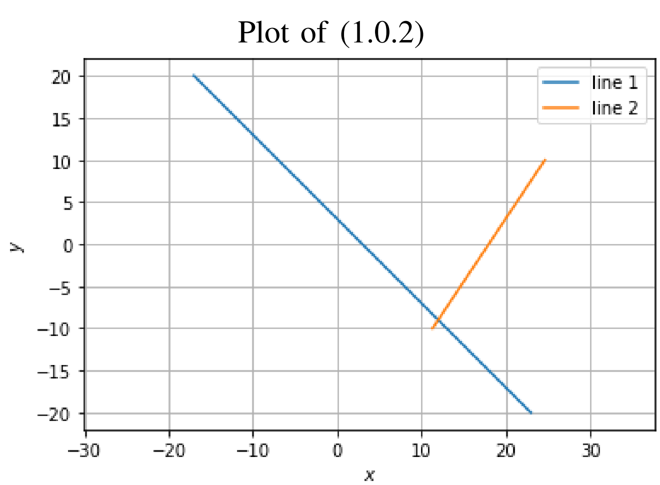

Fig. 2.2: Perpendicular lines