

Assignment 3

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Download all python codes from

<https://github.com/Atlakeerthana/Assignment3/blob/main/Assignment3/assignment3.py>

and latex-tikz codes from

<https://github.com/Atlakeerthana/Assignment3/blob/main/Assignment3/main.tex>

1 CONSTR-2.31

Construct a Quadrilateral $ABCD$ such that $BC = 4.5, AC = 5.5, CD = 5, BD = 7, AD = 5.5$.

2 EXPLANATION

1) the given quadrilateral:-

Let the vertices of the quadrilateral $ABCD$ be **A,B,C** and **D**.

2) List out given data in form of vectors:-

Given:

$BC = 4.5, AC = 5.5, CD = 5, BD = 7, AD = 5.5$. In vector form,

$$\|\mathbf{B} - \mathbf{C}\| = 4.5 \quad (2.0.1)$$

$$\|\mathbf{A} - \mathbf{C}\| = 5.5 \quad (2.0.2)$$

$$\|\mathbf{C} - \mathbf{D}\| = 5 \quad (2.0.3)$$

$$\|\mathbf{B} - \mathbf{D}\| = 7 \quad (2.0.4)$$

$$\|\mathbf{A} - \mathbf{D}\| = 5.5 \quad (2.0.5)$$

3) Find out two triangles of given quadrilateral having same base:

Quadrilateral $ABCD$ is made up of two triangles $\triangle ACD$ and $\triangle BCD$ placed on base CD .

4) Verify that construction of both triangles, is possible or not by using the fact that "sum of any two sides of a triangle is greater than the third side":-

(a) Consider $\triangle ACD$,

$$\|\mathbf{A} - \mathbf{C}\| + \|\mathbf{C} - \mathbf{D}\| = 6.5 > \|\mathbf{A} - \mathbf{D}\| \quad (2.0.6)$$

$$\|\mathbf{A} - \mathbf{D}\| + \|\mathbf{C} - \mathbf{D}\| = 6.5 > \|\mathbf{A} - \mathbf{C}\| \quad (2.0.7)$$

$$\|\mathbf{A} - \mathbf{C}\| + \|\mathbf{A} - \mathbf{D}\| = 11 > \|\mathbf{C} - \mathbf{D}\| \quad (2.0.8)$$

Sum of any two sides is greater than the third side in $\triangle ACD$. \therefore Construction of $\triangle ACD$ is possible.

(b) Similarly in $\triangle BCD$,

$$\|\mathbf{B} - \mathbf{C}\| + \|\mathbf{C} - \mathbf{D}\| = 9.5 > \|\mathbf{B} - \mathbf{D}\| \quad (2.0.9)$$

$$\|\mathbf{B} - \mathbf{C}\| + \|\mathbf{B} - \mathbf{D}\| = 11.5 > \|\mathbf{C} - \mathbf{D}\| \quad (2.0.10)$$

$$\|\mathbf{B} - \mathbf{D}\| + \|\mathbf{C} - \mathbf{D}\| = 12 > \|\mathbf{B} - \mathbf{C}\| \quad (2.0.11)$$

Sum of any two sides is greater than the third side in $\triangle BCD$. \therefore Construction of $\triangle BCD$ is possible.

5) Conclude that construction of quadrilateral is possible if both triangles can be constructed otherwise not possible:- \therefore both the triangles can be constructed, we can construct the quadrilateral with the given sides.

6) To find the coordinates of the vertices of the given quadrilateral: Let the sides of the triangles be denoted by $BC = bc, AC = ac, CD = cd, BD = bd, AD = ad$. Then,

$$bc = 4.5, ac = 5.5, cd = 5, bd = 7, ad = 5.5 \quad (2.0.12)$$

Suppose $\angle ACD = N$ and $\angle BCD = M$. Now, let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.13)$$

$$\mathbf{B} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} i \cos N \\ i \sin N \end{pmatrix} \quad (2.0.15)$$

$$\mathbf{D} = \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} t \cos M \\ t \sin M \end{pmatrix} \quad (2.0.16)$$

Then we know that,

$$\cos N = \frac{bc^2 + bd^2 - ac^2}{2bci} \quad (2.0.17)$$

$$p = i \cos N = \frac{bc^2 + bd^2 - ac^2}{2f} \quad (2.0.18)$$

$$= \frac{4.5^2 + 7^2 - 5.5^2}{2 \times 4.5} = 4.33 \quad (2.0.19)$$

$$\sin N = \pm \sqrt{1 - \cos^2 N} \quad (2.0.20)$$

$$q = i \sin N = \pm \sqrt{bd^2 - bd^2 \cos^2 N} \quad (2.0.21)$$

$$= \pm \sqrt{7^2 - 4.33^2} = \pm 5.500 \quad (2.0.22)$$

$$\cos M = \frac{cd^2 + bc^2 - ad^2}{2bct} \quad (2.0.23)$$

$$r = t \cos M = \frac{cd^2 + bc^2 - ad^2}{2bc} \quad (2.0.24)$$

$$= \frac{5^2 + 4.5^2 - 5.5^2}{2 \times 4.5} = 1.66 \quad (2.0.25)$$

$$\sin M = \pm \sqrt{1 - \cos^2 M} \quad (2.0.26)$$

$$s = t \sin M = \pm \sqrt{t^2 - t^2 \cos^2 M} \quad (2.0.27)$$

$$= \pm \sqrt{5^2 - 1.66^2} = \pm 4.71 \quad (2.0.28)$$

Consider q and s to be positive. Then the coordinates of the quadrilateral can be obtained from 2.0.13, 2.0.14, 2.0.15 and 2.0.16.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4.33 \\ 5.50 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1.66 \\ 4.71 \end{pmatrix} \quad (2.0.29)$$

7) Knowing all the coordinates, now we can construct the quadrilateral.

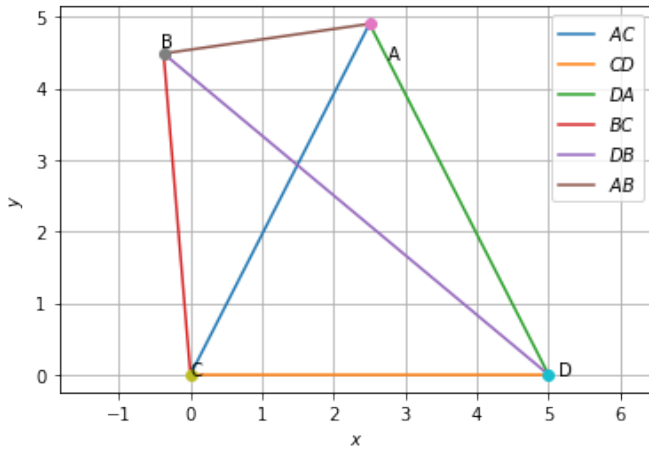


Fig. 2.1: Quadrilateral $ABCD$