Assignment 3

Atla keerthana

Download all python codes from

```
https://github.com/Gayathri1729/SRFP/tree/main/
    Assignment3
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and latex-tikz codes from

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1 CONSTR-2.31

Construct a Quadrilateral ABCD such that $BC = 4.5, AC = 5.5, CD = 5, BD = 7, AD = 5.5$.

2 Explanation

1) the given quadrilateral:-
Let the vertices of the quadrilateral $ABCD$ be $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$ and $\mathbf{D}$.
2) List out given data in form of vectors:-
Given:
$BC = 4.5, AC = 5.5, CD = 5, BD = 7, AD = 5.5$. In vector form,

$$\|\mathbf{B}-\mathbf{C}\| = 4.5 \tag{2.0.1}$$
$$\|\mathbf{A}-\mathbf{C}\| = 5.5 \tag{2.0.2}$$
$$\|\mathbf{C}-\mathbf{D}\| = 5 \tag{2.0.3}$$
$$\|\mathbf{B}-\mathbf{D}\| = 7 \tag{2.0.4}$$
$$\|\mathbf{A}-\mathbf{D}\| = 5.5 \tag{2.0.5}$$

3) Find out two triangles of given quadrilateral having same base:
Quadrilateral $ABCD$ is made up of two triangles $\triangle ACD$ and $\triangle BCD$ placed on base CD .
4) Verify that construction of both triangles,is possible or not by using the fact that ”sum of any two sides of a triangle is greater than the third side”:-

(a) Consider $\triangle ACD$,

$$||A-C|| + ||C-D|| = 6.5 > ||A-D|| \tag{2.0.6}$$
$$||A-D|| + ||C-D|| = 6.5 > ||A-C|| \tag{2.0.7}$$
$$||A-C|| + ||A-D|| = 11 > ||C-D|| \tag{2.0.8}$$

Sum of any two sides is greater than the third side in $\triangle ACD$. $\therefore$ Construction of $\triangle ACD$ is possible.

(b) Similarly in $\triangle BCD$,

$$||B-C|| + ||C-D|| = 9.5 > ||B-D|| \tag{2.0.9}$$
$$||B-C|| + ||B-D|| = 11.5 > ||C-D|| \tag{2.0.10}$$
$$||B-D|| + ||C-D|| = 12 > ||B-C|| \tag{2.0.11}$$

Sum of any two sides is greater than the third side in $\triangle BCD$. $\therefore$ Construction of $\triangle BCD$ is possible.

5) Conclude that construction of quadrilateral is possible if both triangles can be constructed otherwise not possible:- $\because$ both the triangles can be constructed,we can construct the quadrilateral with the given sides.
6) To find the coordinates of the vertices of the given quadrilateral: Let the sides of the triangles be denoted by $BC = bc, AC = ac, CD = cd, BD = bd, AD = ad$ Then,

$$bc = 4.5, ac = 5.5, cd = 5, bd = 7, ad = 5.5 \tag{2.0.12}$$

Suppose $\angle ACD$=N and $\angle BCD$=M Now,let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.13}$$
$$\mathbf{B} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix} \tag{2.0.14}$$
$$\mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} i\cos N \\ i\sin N \end{pmatrix} \tag{2.0.15}$$
$$\mathbf{D} = \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} t\cos M \\ t\sin M \end{pmatrix} \tag{2.0.16}$$

Then we know that,

$$\cos N = \frac{bc^2 + bd^2 - ac^2}{2bci} \tag{2.0.17}$$

$$p = i \cos N = \frac{bc^2 + bd^2 - ac^2}{2f} \tag{2.0.18}$$

$$= \frac{4.5^2 + 7^2 - 5.5^2}{2 \times 4.5} = 4.33 \tag{2.0.19}$$

$$\sin N = \pm \sqrt{1 - \cos^2 N} \tag{2.0.20}$$

$$q = i \sin N = \pm \sqrt{bd^2 - bd^2 \cos^2 N} \tag{2.0.21}$$

$$= \pm \sqrt{7^2 - 4.33^2} = \pm 5.500 \tag{2.0.22}$$

$$\cos M = \frac{cd^2 + bc^2 - ad^2}{2bct} \tag{2.0.23}$$

$$r = t \cos M = \frac{cd^2 + bc^2 - ad^2}{2bc} \tag{2.0.24}$$

$$= \frac{5^2 + 4.5^2 - 5.5^2}{2 \times 4.5} = 1.66 \tag{2.0.25}$$

$$\sin M = \pm \sqrt{1 - \cos^2 M} \tag{2.0.26}$$

$$s = t \sin M = \pm \sqrt{t^2 - t^2 \cos^2 M} \tag{2.0.27}$$

$$= \pm \sqrt{5^2 - 1.66^2} = \pm 4.71 \tag{2.0.28}$$

Consider q and s to be positive.Then the co-ordinates of the quadrilateral can be obtained from 2.0.13, 2.0.14, 2.0.15 and 2.0.16.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4.33 \\ 5.50 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1.66 \\ 4.71 \end{pmatrix} \tag{2.0.29}$$

7) Knowing all the coordinates, now we can construct the quadrilateral.

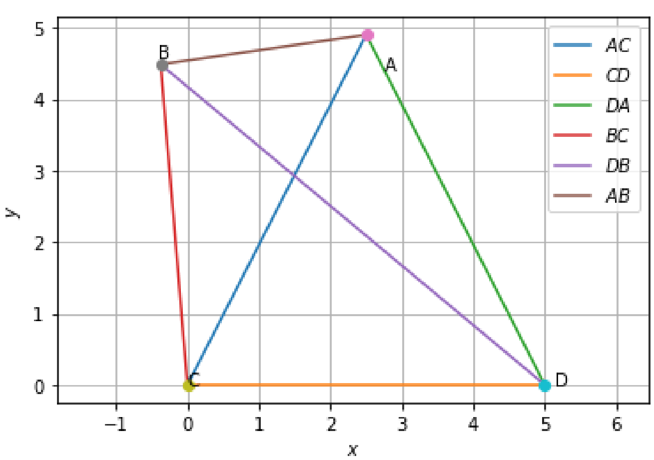

Fig. 2.1: Quadrilateral $ABCD$