ASSIGNMENT-3

ATLA KEERTHANA

1 QUESTION NO-2.31

Find the angle between the following pair of lines:

$$L_1 : \mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \tag{1.0.1}$$

$$L_2 : \mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ -56 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \tag{1.0.2}$$

2 SOLUTION

Looking at the directions of the lines

$$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \tag{2.0.2}$$

Clearly over here,

$$\|\mathbf{a}\| = \sqrt{(1)^2 + (-1)^2 + (-2)^2} \tag{2.0.3}$$

$$= \sqrt{6} \tag{2.0.4}$$

$$\|\mathbf{b}\| = \sqrt{(3)^2 + (-5)^2 + (-4)^2} \tag{2.0.5}$$

$$= \sqrt{50} \tag{2.0.6}$$

$$\mathbf{a}^T\mathbf{b} = \begin{pmatrix} 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \tag{2.0.7}$$

$$= ((1)(3) + (-1)(-5) + (-2)(-4)) \tag{2.0.8}$$

$$= 3 + 5 + 8 \tag{2.0.9}$$

$$= 16 \tag{2.0.10}$$

$$\cos\theta = \frac{\mathbf{a}^T\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|} \tag{2.0.11}$$

$$= \frac{16}{\sqrt{6}\,\sqrt{50}} \tag{2.0.12}$$

$$= \frac{8}{5\sqrt{3}} \tag{2.0.13}$$

$$\theta = \arccos\left(\frac{8}{5\sqrt{3}}\right) \tag{2.0.14}$$

$$\theta = 1.3930 \tag{2.0.15}$$

Plot of the lines

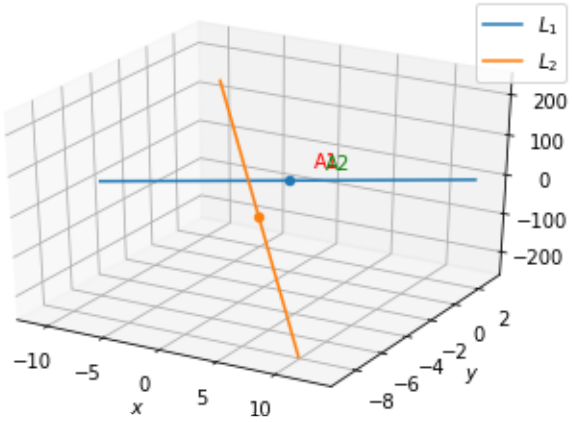

Fig. 2.1: Plot of the lines