ASSIGNMENT 4

Atla keerthana

1 LINEAR FORMS-2.43 D

Determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angle between them. $\begin{pmatrix} 2 & -1 & 3 \end{pmatrix}$ $\mathbf{x}=1$ and $\begin{pmatrix} 2 & -1 & 3 \end{pmatrix}$ $\mathbf{x}=-3$

2 EXPLANATION

Given the planes,

$$P_1 : \begin{pmatrix} 2 & -1 & 3 \end{pmatrix} \mathbf{x} = 1 \tag{1}$$

$$P_2 : \begin{pmatrix} 2 & -1 & 3 \end{pmatrix} \mathbf{x} = -3 \tag{2}$$

The normal vector of P_1 and P_2 are

$$\mathbf{n}_1 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \tag{3}$$

and

$$\mathbf{n}_2 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \tag{4}$$

respectively. The angle between two planes is same as the angle between their normal vectors. Let θ be the angle between $\mathbf{n}_1$ and $\mathbf{n}_2$. Then

$$\cos \theta = \frac{\mathbf{n}_1^\top \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \tag{5}$$

$$\|\mathbf{n}_1\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14} \tag{6}$$

$$\|\mathbf{n}_2\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14} \tag{7}$$

$$\mathbf{n}_1^\top \mathbf{n}_2 = 2 \times 2 + (-1) \times -1 + 3 \times 3 = 14 \tag{8}$$

Then,

$$\cos \theta = \frac{14}{\sqrt{14}\,\sqrt{14}} = 1 \tag{9}$$

$\therefore$ the planes P_1 and P_2 are neither parallel nor perpendicular. From 9, the angle between the planes is

$$\theta = \cos^{-1}(1) = 0° \tag{10}$$

Fig 2.1 shows the planes are neither parallel nor perpendicular.

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Fig. 2.1. Planes P_1 and P_2