1

ASSIGNMENT 4

Atla keerthana

Download all python codes from

https://github.com/Atlakeerthana/Assignment4/tree/main/Assignment4

and latex-tikz codes from

https://github.com/Atlakeerthana/Assignment4/tree/main/Assignment4

1 Linear Forms-2.43 D

Determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angle between them. $\begin{pmatrix} 2 & -1 & 3 \end{pmatrix} x=1$ and $\begin{pmatrix} 2 & -1 & 3 \end{pmatrix} x=-3$

2 EXPLANATION

Given the planes,

$$P_1: (2 -1 3)\mathbf{x} = 1 \tag{1}$$

$$P_2: (2 -1 3)\mathbf{x} = -3$$
 (2)

The normal vector of P_1 and P_2 are

$$\mathbf{n}_1 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \tag{3}$$

and

$$\mathbf{n}_2 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix},\tag{4}$$

respectively. The angle between two planes is same as the angle between their normal vectors. Let θ be the angle between \mathbf{n}_1 and \mathbf{n}_2 . Then

$$\cos \theta = \frac{\mathbf{n}_1^{\mathsf{T}} \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \tag{5}$$

$$\|\mathbf{n_1}\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$$
 (6)

$$\|\mathbf{n}_2\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$$
 (7)

$$\mathbf{n}_{1}^{\mathsf{T}}\mathbf{n}_{2} = 2 \times 2 + (-1) \times -1 + 3 \times 3 = 14$$
 (8)

Then,

$$\cos \theta = \frac{14}{\sqrt{14}\sqrt{14}} = 1 \tag{9}$$

 \therefore the planes P_1 and P_2 are neither parallel nor perpendicular. From (9), the angle between the planes is

$$\theta = \cos^{-1}(1) = 0^{\circ} \tag{10}$$

Fig 2.1 shows the planes are neither parallel nor perpendicular.

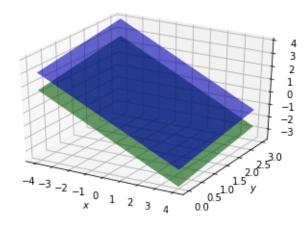


Fig. 2.1. Planes P_1 and P_2