ASSIGNMENT 4

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Download all python codes from

https://github.com/Atlakeerthana/Assignment4/tree/main/Assignment4

and latex-tikz codes from

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1 Linear Forms-2.43 D

Determine whether the given planes are parallel or perpendicular, and in case they are neither,find the angle between them. $\begin{pmatrix}2 & -1 & 3\end{pmatrix}$ $\mathbf{x}$=1 and $\begin{pmatrix}2 & -1 & 3\end{pmatrix}$ $\mathbf{x}$=-3

2 Explanation

Given the planes,

$$P_1 : \begin{pmatrix}2 & -1 & 3\end{pmatrix}\mathbf{x} = 1 \tag{1}$$

$$P_2 : \begin{pmatrix}2 & -1 & 3\end{pmatrix}\mathbf{x} = -3 \tag{2}$$

The normal vector of P_1 and P_2 are

$$\mathbf{n}_1 = \begin{pmatrix}2\\-1\\3\end{pmatrix} \tag{3}$$

and

$$\mathbf{n}_2 = \begin{pmatrix}2\\-1\\3\end{pmatrix}, \tag{4}$$

respectively. The angle between two planes is same as the angle between their normal vectors.Let θ be the angle between $\mathbf{n}_1$ and $\mathbf{n}_2$.Then

$$\cos\theta = \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{5}$$

$$\|\mathbf{n_1}\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14} \tag{6}$$

$$\|\mathbf{n_2}\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14} \tag{7}$$

$$\mathbf{n_1}^\top \mathbf{n_2} = 2\times 2 + (-1)\times -1 + 3\times 3 = 14 \tag{8}$$

Then,

$$\cos\theta = \frac{14}{\sqrt{14}\,\sqrt{14}} = 1 \tag{9}$$

$\therefore$ the planes P_1 and P_2 are neither parallel nor perpendicular. From 9, the angle between the planes is

$$\theta = \cos^{-1}(1) = 0^\circ \tag{10}$$

Fig 2.1 shows the planes are neither parallel nor perpendicular.

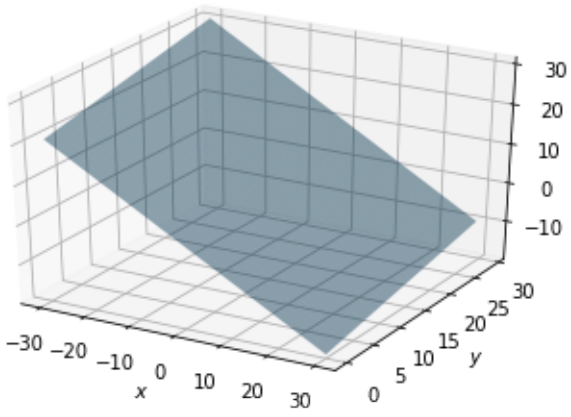

Fig. 2.1. Planes P_1 and P_2