

ASSIGNMENT 5

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Download all python codes from

<https://github.com/Atlakeerthana/Assignment5/tree/main/Assignment5>

and latex-tikz codes from

<https://github.com/Atlakeerthana/Assignment5/tree/main/Assignment5>

1 QUESTION No 2.23(QUAD FORMS)

Find the roots of the following quadratic equations, if they exist.

1)

$$3x^2 - 5x + 2 = 0 \quad (1.0.1)$$

2)

$$x^2 + 4x + 5 = 0 \quad (1.0.2)$$

2 SOLUTION

1) The vector form of

$$y = 3x^2 - 5x + 2 \quad (2.0.1)$$

is

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -5 & 0 \end{pmatrix} \mathbf{x} + 2 = 0 \quad (2.0.2)$$

Thus

$$y = 0 \implies 3x^2 - 5x + 2 = 0 \quad (2.0.3)$$

Compare given quadratic equation $3x^2 - 5x + 2 = 0$ with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -5, c = 2 \quad (2.0.4)$$

so,

$$b^2 - 4ac = (-5)^2 - 4(3)(2) \quad (2.0.5)$$

$$= 25 - 24 = 1 > 0 \quad (2.0.6)$$

Since the square of a real number is positive.

$$(x - 1)(3x - 2) = 0 \quad (2.0.7)$$

$$x_1 = 1, x_2 = \frac{2}{3} \quad (2.0.8)$$

The roots are 1 and 0.66

$$x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 0.66 \\ 0 \end{pmatrix} \quad (2.0.9)$$

which can be verified from the fig(1.0.1) generated by following python code.

Plot of (1.0.1) -

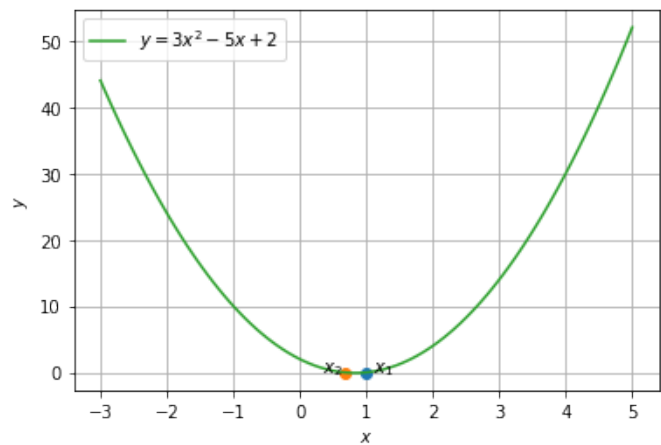


Fig. 2.1: roots of $3x^2 - 5x + 2$.

2) The vector form of

$$y = x^2 + 4x + 5 \quad (2.0.10)$$

is

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} + 5 = 0 \quad (2.0.11)$$

Thus

$$y = 0 \implies x^2 + 4x + 5 = 0 \quad (2.0.12)$$

Compare given quadratic equation $x^2 + 4x + 5 = 0$ with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 4, c = 5 \quad (2.0.13)$$

so,

$$b^2 - 4ac = (4)^2 - 4(1)(5) \quad (2.0.14)$$

$$= 16 - 20 = -4 < 0 \quad (2.0.15)$$

Since the square of a real number cannot be negative, so there are no real roots for the given

equation.

So

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (2.0.16)$$

$$x^2 + 4x + 5 = 0 \quad (2.0.17)$$

$$\left(x - \begin{pmatrix} -2 \\ 1 \end{pmatrix}\right) \left(x - \begin{pmatrix} -2 \\ -1 \end{pmatrix}\right) = 0 \quad (2.0.18)$$

$$x = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad (2.0.19)$$

So figure (1.0.2) show that the equation does not intersect the x-axis hence there are no real roots.

Plot of (1.0.2) -

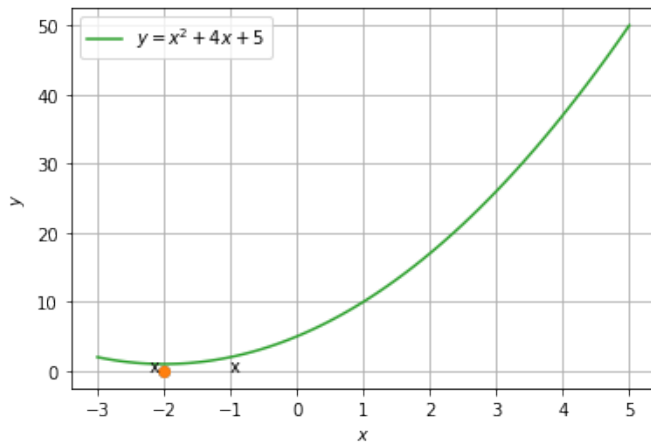


Fig. 2.2: roots of $x^2 + 4x + 5$.