ASSIGNMENT 5

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Download all python codes from

https://github.com/Atlakeerthana/Assignment5/tree/main/Assignment5

and latex-tikz codes from

https://github.com/Atlakeerthana/Assignment5/tree/main/Assignment5

1 Question No 2.23(Quad forms)

Find the roots of the following quadratic equations, if they exist.

1)
$$3x^2 - 5x + 2 = 0 \tag{1.0.1}$$

2)
$$x^2 + 4x + 5 = 0 \tag{1.0.2}$$

2 Solution

Given

1)
$$y = 3x^2 - 5x + 2 \tag{2.0.1}$$
$$\implies 3x^2 - 5x + 2 - y = 0 \tag{2.0.2}$$

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{-5}{2} \\ \frac{-1}{2} \end{pmatrix} \mathbf{x} + 2 = 0 \tag{2.0.3}$$

Here,

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = -\begin{pmatrix} \frac{-5}{2} \\ \frac{-1}{2} \end{pmatrix}, f = 2 \tag{2.0.4}$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.5}$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p_1}^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.6}$$

$\therefore$Vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} \frac{-5}{2} & -1 \\ 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -2 \\ \frac{5}{2} \\ 0 \end{pmatrix} \tag{2.0.7}$$
$$\implies \begin{pmatrix} \frac{-5}{2} & -1 \\ 3 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -2 \\ \frac{5}{2} \end{pmatrix} \tag{2.0.8}$$
$$\implies \mathbf{c} = \begin{pmatrix} \frac{5}{6} \\ \frac{-1}{12} \end{pmatrix} \tag{2.0.9}$$

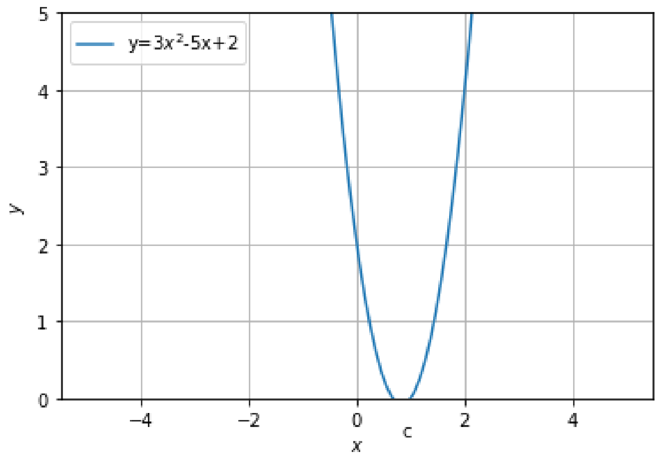

Fig. 2.1: $y = 3x^2 - 5x + 2$

Now,

$$\mathbf{p_1}^T \mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{5}{6} \\ \frac{-1}{12} \end{pmatrix} \tag{2.0.10}$$
$$= \frac{-1}{12} \tag{2.0.11}$$

and,

$$\mathbf{p_2}^T \mathbf{V} \mathbf{p_2} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.12}$$
$$= 3 \tag{2.0.13}$$

$\because$

$$(\mathbf{p_1}^T \mathbf{c})(\mathbf{p_2}^T \mathbf{V} \mathbf{p_2}) = \frac{-1}{4} < 0 \tag{2.0.14}$$

Hence,the given equation has real roots.

2)

$$y = x^2 + 4x + 5 \tag{2.0.15}$$

Here,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 2 \\ \frac{-1}{2} \end{pmatrix}, f = 5 \tag{2.0.16}$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.17}$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p_1}^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.18}$$

$\therefore$Vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ 2 \\ 0 \end{pmatrix} \tag{2.0.19}$$

$$\implies \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \tag{2.0.20}$$

$$\implies \mathbf{c} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.21}$$

Now,

$$\mathbf{p_1}^T \mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.22}$$

$$= 1 \tag{2.0.23}$$

and,

$$\mathbf{p_2}^T \mathbf{V} \mathbf{p_2} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.24}$$

$$= 1 \tag{2.0.25}$$

$\because$

$$(\mathbf{p_1}^T \mathbf{c})(\mathbf{p_2}^T \mathbf{V} \mathbf{p_2}) = (1)(1) = 1 < 0 \tag{2.0.26}$$

Hence,the given equation does not have real roots.

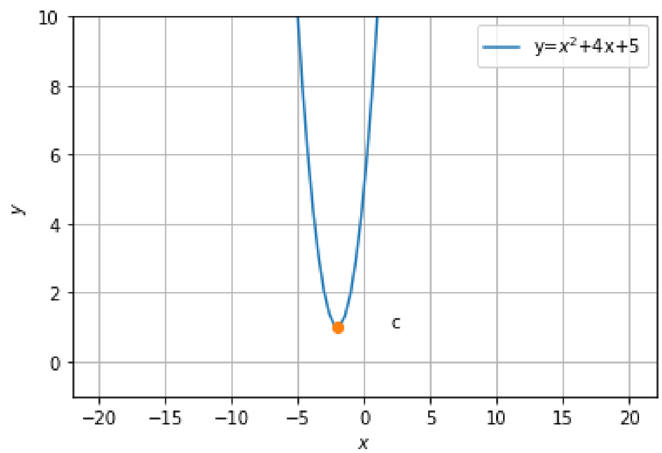

Fig. 2.2: $y = x^2 + 4x + 5$