

ASSIGNMENT 5

Atla keerthana

Download all python codes from

<https://github.com/Atlakeerthana/Assignment5/tree/main/Assignment5>

and latex-tikz codes from

<https://github.com/Atlakeerthana/Assignment5/tree/main/Assignment5>

∴ Vertex \mathbf{c} is given by

$$\begin{pmatrix} \frac{-5}{2} & -1 \\ 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -2 \\ \frac{5}{2} \\ 0 \end{pmatrix} \quad (2.0.7)$$

$$\Rightarrow \begin{pmatrix} \frac{-5}{2} & -1 \\ 3 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -2 \\ \frac{5}{2} \end{pmatrix} \quad (2.0.8)$$

$$\Rightarrow \mathbf{c} = \begin{pmatrix} \frac{5}{6} \\ \frac{1}{12} \end{pmatrix} \quad (2.0.9)$$

1 QUESTION No 2.23(QUAD FORMS)

Find the roots of the following quadratic equations, if they exist.

1)

$$3x^2 - 5x + 2 = 0 \quad (1.0.1)$$

2)

$$x^2 + 4x + 5 = 0 \quad (1.0.2)$$

2 SOLUTION

Given

1)

$$y = 3x^2 - 5x + 2 \quad (2.0.1)$$

$$\Rightarrow 3x^2 - 5x + 2 - y = 0 \quad (2.0.2)$$

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{-5}{2} \\ \frac{-1}{2} \end{pmatrix} \mathbf{x} + 2 = 0 \quad (2.0.3)$$

Here,

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{-5}{2} \\ \frac{-1}{2} \end{pmatrix}, f = 2 \quad (2.0.4)$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.0.5)$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.6)$$

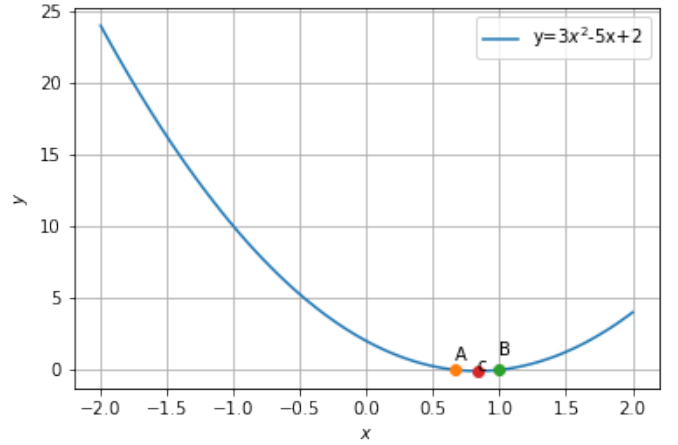


Fig. 2.1: $y = 3x^2 - 5x + 2$

Now,

$$\mathbf{p}_1^T \mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{5}{6} \\ \frac{1}{12} \end{pmatrix} \quad (2.0.10)$$

$$= \frac{-1}{12} \quad (2.0.11)$$

and,

$$\mathbf{p}_2^T \mathbf{V} \mathbf{p}_2 = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.12)$$

$$= 3 \quad (2.0.13)$$

∴

$$(\mathbf{p}_1^T \mathbf{c})(\mathbf{p}_2^T \mathbf{V} \mathbf{p}_2) = \frac{-1}{4} < 0 \quad (2.0.14)$$

Hence, the given equation has real roots.

2)

$$y = x^2 + 4x + 5 \quad (2.0.15)$$

Here,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}, f = 5 \quad (2.0.16)$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.0.17)$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.18)$$

\therefore Vertex \mathbf{c} is given by

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ 2 \\ 0 \end{pmatrix} \quad (2.0.19)$$

$$\Rightarrow \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \quad (2.0.20)$$

$$\Rightarrow \mathbf{c} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (2.0.21)$$

Now,

$$\mathbf{p}_1^T \mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (2.0.22)$$

$$= 1 \quad (2.0.23)$$

and,

$$\mathbf{p}_2^T \mathbf{V} \mathbf{p}_2 = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.24)$$

$$= 1 \quad (2.0.25)$$

\therefore

$$(\mathbf{p}_1^T \mathbf{c})(\mathbf{p}_2^T \mathbf{V} \mathbf{p}_2) = (1)(1) = 1 < 0 \quad (2.0.26)$$

Hence, the given equation does not have real roots.

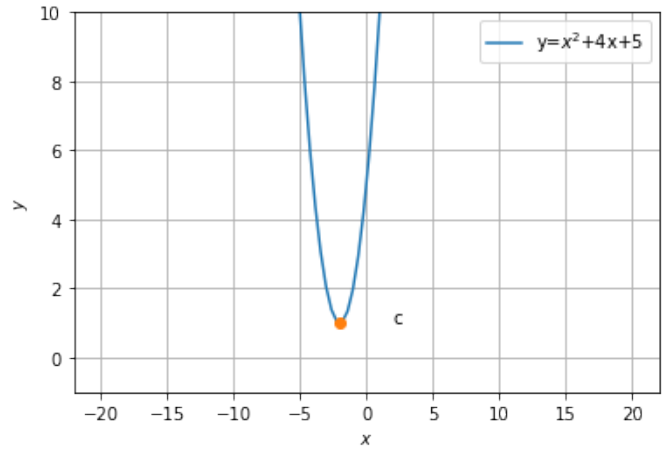


Fig. 2.2: $y = x^2 + 4x + 5$