

ASSIGNMENT 6

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Download all python codes from

<https://github.com/Atlakeerthana/Assignment6/tree/main/Assignment6>

and latex-tikz codes from

<https://github.com/Atlakeerthana/Assignment6/tree/main/Assignment6>

1 QUESTION No 2.74(E)

In each of the following find the equation for the ellipse that satisfies the given conditions:

- a. Conjugate axis length= 24, foci= $\begin{pmatrix} 0 \\ \pm 13 \end{pmatrix}$

2 SOLUTION

Given that,

$$\text{Conjugate axis length} = 2b = 24 \quad (2.0.1)$$

$$\therefore \text{Length of semi major axis, } b = 12 \quad (2.0.2)$$

$$\text{Foci} = \mathbf{F} = \begin{pmatrix} 0 \\ \pm 13 \end{pmatrix} \quad (2.0.3)$$

Lemma 2.1. The standard equation of an ellipse is given by:

$$\frac{\mathbf{y}^T \mathbf{D} \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.4)$$

$$\text{where, } D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (2.0.5)$$

Lemma 2.2. The coordinates of foci \mathbf{F} of ellipse with y-axis as major axis are:

$$\mathbf{F} = \begin{pmatrix} 0 \\ \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \end{pmatrix} \quad (2.0.6)$$

Also, the length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \quad (2.0.7)$$

and the length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.8)$$

- 1) From (2.0.8) length of semi-minor axis is:

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = b \quad (2.0.9)$$

$$\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} = b^2 \quad (2.0.10)$$

$$\Rightarrow \lambda_2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{b^2} \quad (2.0.11)$$

- 2) From (2.0.6), the focus of ellipse is given as:

$$\mathbf{F} = \begin{pmatrix} 0 \\ \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \end{pmatrix} \quad (2.0.12)$$

or

$$\|\mathbf{F}\|^2 = \frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} \quad (2.0.13)$$

$$\|\mathbf{F}\|^2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} \quad (2.0.14)$$

- 3) Putting value of λ_2 from (2.0.11) in above equation, we get:

$$\|\mathbf{F}\|^2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - b^2 \quad (2.0.15)$$

$$\|\mathbf{F}\|^2 + b^2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} \quad (2.0.16)$$

$$\Rightarrow \lambda_1 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\|\mathbf{F}\|^2 + b^2} \quad (2.0.17)$$

- 4) For finding λ_2 :

- From (2.0.11) we have:

$$\lambda_2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{b^2} \quad (2.0.18)$$

$$\Rightarrow \lambda_2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{144} (\because b = 12) \quad (2.0.19)$$

- 5) For finding λ_1 :

- Putting value of b from (2.0.2) and \mathbf{F} from

(2.0.3) in equation (2.0.17), we get:

$$\lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\left(\sqrt{0^2 + 13^2}\right)^2 + 12^2} \quad (2.0.20)$$

$$\lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{169 + 144} \quad (2.0.21)$$

$$\lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{313} \quad (2.0.22)$$

6) Using lemma (2.1), the standard equation of ellipse is given by :

$$\frac{\mathbf{y}^\top \mathbf{D} \mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.23)$$

$$\Rightarrow \frac{\mathbf{y}^\top \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.24)$$

7) Putting (2.0.19) and (2.0.22) in above equation we get:

$$\Rightarrow \mathbf{y}^\top \begin{pmatrix} \frac{1}{313} & 0 \\ 0 & \frac{1}{144} \end{pmatrix} \mathbf{y} = 1 \quad (2.0.25)$$

which is the required equation of ellipse.

8) The Plot of ellipse is:

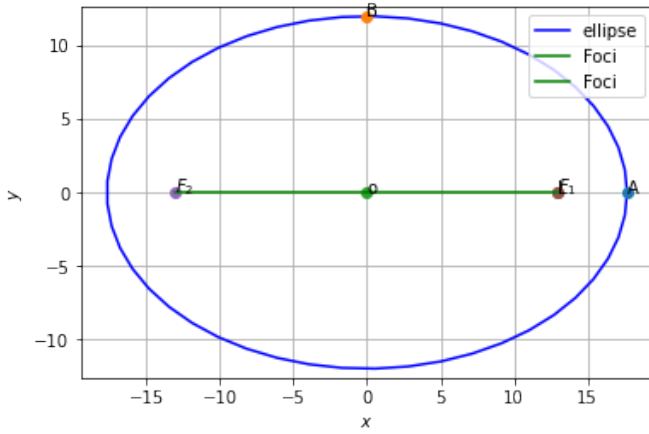


Fig. 2.1: Ellipse $\frac{x^2}{313} + \frac{y^2}{144} = 1$