ASSIGNMENT 6

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Download all python codes from

https://github.com/Atlakeerthana/Assignment6/tree/main/Assignment6

Latex-tikz codes from

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1 Question No 2.74(e)

In each of the following find the equation for the ellipse that satisfies the given condition:

1) Conjugate axis length 24, foci $\begin{pmatrix} 0 \\ \pm 13 \end{pmatrix}$

2 Solution

Let

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}, b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} \tag{2.0.1}$$

$$Now, c^2 = a^2 - b^2 \tag{2.0.2}$$

$$\implies 169 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} \tag{2.0.3}$$

And,Conjugate axis length = 24

$$\implies 2b = 24 \implies b = 12 \tag{2.0.4}$$

$$\implies \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} = 12 \tag{2.0.5}$$

$$From (2.0.3) \ \& \ (2.0.5), Find \lambda_1, \lambda_2 \ \& \ \mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f \tag{2.0.6}$$

Final equation is :

$$\frac{\mathbf{y}^\top D y}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.7}$$

$$\implies \frac{\mathbf{y}^\top \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.8}$$

$$\implies \mathbf{y}^\top \begin{pmatrix} 25 & 0 \\ 0 & 144 \end{pmatrix} = 1 \tag{2.0.9}$$

Plot of ellipse:

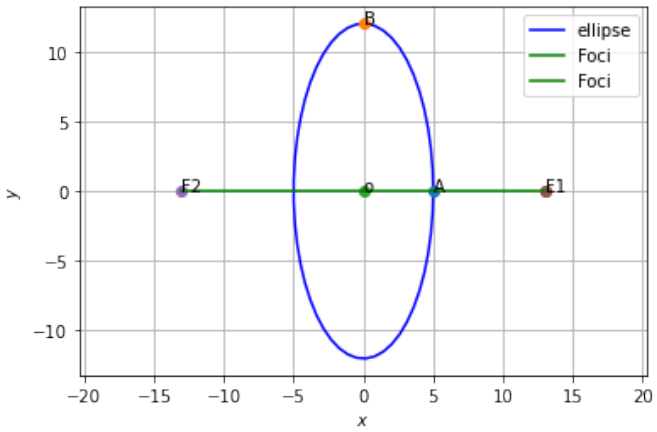

Fig. 2.1: Ellipse $\frac{x^2}{25} + \frac{y^2}{144} = 1$