

ASSIGNMENT 6

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Download all python codes from

<https://github.com/Atlakeerthana/Assignment6/tree/main/Assignment6>

and latex-tikz codes from

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1 QUESTION No 2.74(E)

In each of the following find the equation for the ellipse that satisfies the given conditions:

- a. Conjugate axis length= 24, foci= $\begin{pmatrix} 0 \\ \pm 13 \end{pmatrix}$

2 SOLUTION

Given that,

$$\text{Conjugate axis length} = 2b = 24 \quad (2.0.1)$$

$$\text{Foci} = \mathbf{F} = \begin{pmatrix} 0 \\ \pm 13 \end{pmatrix} \quad (2.0.2)$$

Lemma 2.1. The standard equation of an ellipse is given by:

$$\frac{\mathbf{y}^T D \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.3)$$

$$\text{where, } D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (2.0.4)$$

The focus of ellipse, \mathbf{F} is

$$\mathbf{F} = \sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \quad (2.0.5)$$

Also, the length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \quad (2.0.6)$$

and the length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.7)$$

- 1) Using (2.0.3) length of conjugate axis is:

$$2b = 24 \quad (2.0.8)$$

$$\Rightarrow b = 12 \quad (2.0.9)$$

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = 12 \quad (2.0.10)$$

$$\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} = 144 \quad (2.0.11)$$

$$\Rightarrow \lambda_2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{144} \quad (2.0.12)$$

- 2) Using (2.0.3), the focus ellipse is given as:

$$\mathbf{F} = \sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \quad (2.0.13)$$

$$\mathbf{F}^2 = \frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} \quad (2.0.14)$$

$$\mathbf{F}^2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} \quad (2.0.15)$$

$$\Rightarrow 169 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} \quad (2.0.16)$$

- 3) Putting (2.0.11) in above equation we get:

$$169 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - 144 \quad (2.0.17)$$

$$25 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} \quad (2.0.18)$$

$$\Rightarrow \lambda_1 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{25} \quad (2.0.19)$$

- 4) Using lemma (2.1), the standard equation of ellipse is given by :

$$\frac{\mathbf{y}^T D \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.20)$$

- 5) Putting (2.0.12) and (2.0.19) in above equation

we get:

$$\Rightarrow \frac{\mathbf{y}^\top \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.21)$$

$$\Rightarrow \mathbf{y}^\top \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{144} \end{pmatrix} \mathbf{y} = 1 \quad (2.0.22)$$

6) The Plot of ellipse is:

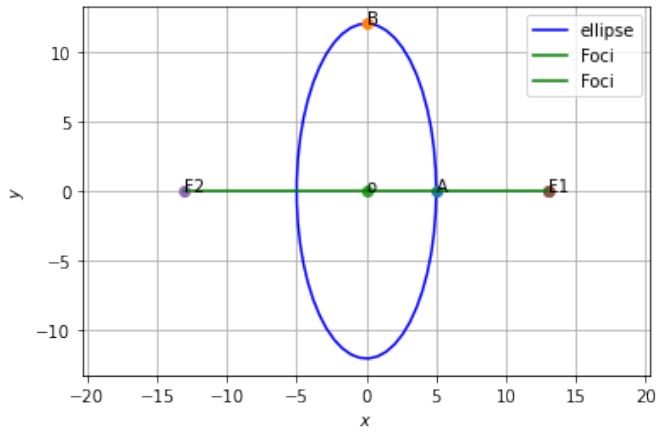


Fig. 2.1: Ellipse $\frac{x^2}{25} + \frac{y^2}{144} = 1$