

ASSIGNMENT 6

A.keerthana

Download all python codes from

https://github.com/tejasri3657/Assignment-6/blob/main/Assignment_6.py

Latex-tikz codes from

<https://github.com/tejasri3657/Assignment-6/new/main>

1 QUESTION No 2.74(E)

In each of the following find the equation for the ellipse that satisfies the given condition:

- 1) Conjugate axis length 24, foci $\begin{pmatrix} 0 \\ \pm 13 \end{pmatrix}$

2 SOLUTION

Let

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}, b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.1)$$

$$\text{Now, } c^2 = a^2 - b^2 \quad (2.0.2)$$

$$\Rightarrow 169 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2} \quad (2.0.3)$$

And, Conjugate axis length = 24

$$\Rightarrow 2b = 24 \Rightarrow b = 12 \quad (2.0.4)$$

$$\Rightarrow \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} = 12 \quad (2.0.5)$$

From (2.0.3) & (2.0.5), Find λ_1, λ_2 & $\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f$

(2.0.6)

Final equation is :

$$\frac{\mathbf{y}^T D \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.7)$$

$$\Rightarrow \frac{\mathbf{y}^T \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.8)$$

$$\Rightarrow \mathbf{y}^T \begin{pmatrix} 25 & 0 \\ 0 & 144 \end{pmatrix} = 1 \quad (2.0.9)$$

Plot of ellipse:

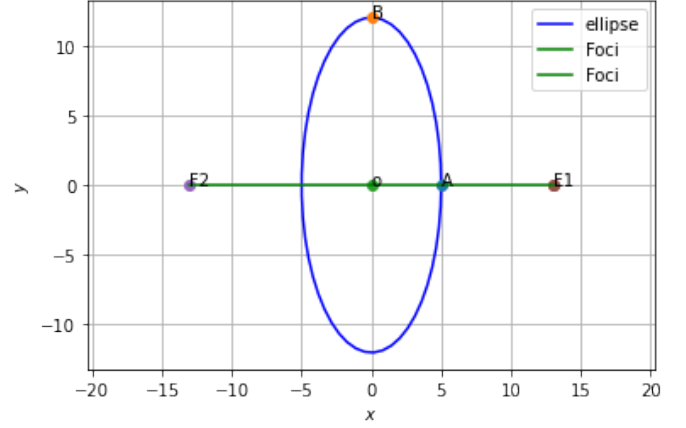


Fig. 2.1: Ellipse $\frac{x^2}{25} + \frac{y^2}{144} = 1$