ASSIGNMENT 7

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1 QUESTION No 2.25(VECTOR FORMS)

Find the equation of set of points $\mathbf{P}$ such that

$$(\mathbf{PA})^2 + (\mathbf{PB})^2 = 2k^2; \tag{1.0.1}$$

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}; \tag{1.0.2}$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}; \tag{1.0.3}$$

respectively.

2 SOLUTION

Given ,

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}; \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}; \tag{2.0.2}$$

Let,

$$\mathbf{P} = \mathbf{X}; \tag{2.0.3}$$

so,

$$(\mathbf{PA})^2 = \|\mathbf{P} - \mathbf{A}\|^2 \tag{2.0.4}$$
$$= \|\mathbf{X} - \mathbf{A}\|^2 \tag{2.0.5}$$
$$= \|\mathbf{X}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{X}^T\mathbf{A} \tag{2.0.6}$$

and

$$(\mathbf{PB})^2 = \|\mathbf{P} - \mathbf{B}\|^2 \tag{2.0.7}$$
$$= \|\mathbf{X} - \mathbf{B}\|^2 \tag{2.0.8}$$
$$= \|\mathbf{X}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{X}^T\mathbf{B} \tag{2.0.9}$$

The given equation is

$$(\mathbf{PA})^2 + (\mathbf{PB})^2 = 2k^2 \tag{2.0.10}$$

Sub (2.0.6) and (2.0.9) values in (2.0.10)

$$\|\mathbf{X}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{X}^T\mathbf{A} + \|\mathbf{X}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{X}^T\mathbf{B} = 2k^2 \tag{2.0.11}$$

$$\implies 2\|\mathbf{X}\|^2 + \|\mathbf{A}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{X}^T(\mathbf{A} + \mathbf{B}) = 2k^2 \tag{2.0.12}$$

sub $\mathbf{A},\mathbf{B}$ values in equation (2.0.12) ,we get

$$2\|\mathbf{X}\|^2 + \left\|\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}\right\|^2 + \left\|\begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}\right\|^2 - 2\mathbf{X}^T\left(\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}\right) = 2k^2 \tag{2.0.13}$$

$\therefore$ the required equation is

$$2\|\mathbf{X}\|^2 - 2\mathbf{X}^T\begin{pmatrix} 2 \\ 7 \\ -2 \end{pmatrix} + 109 - 2k^2 = 0 \tag{2.0.14}$$