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ASSIGNMENT 7

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Download all python codes from

https://github.com/Atlakeerthana/Assignment7/tree/main/Assignment7

and latex-tikz codes from

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1 Question No 2.25(vector forms)

Find the equation of set of points \mathbf{P} such that

$$(\mathbf{PA})^2 + (\mathbf{PB})^2 = 2k^2;$$
 (1.0.1)

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}; \tag{1.0.2}$$

$$\mathbf{B} = \begin{pmatrix} -1\\3\\-7 \end{pmatrix}; \tag{1.0.3}$$

respectively.

2 SOLUTION

Given,

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}; \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} -1\\3\\-7 \end{pmatrix}; \tag{2.0.2}$$

Let,

$$\mathbf{P} = \mathbf{X}; \tag{2.0.3}$$

so,

$$(\mathbf{P}\mathbf{A})^2 = ||\mathbf{P} - \mathbf{A}||^2 \tag{2.0.4}$$

$$= ||\mathbf{X} - \mathbf{A}||^2 \tag{2.0.5}$$

$$= ||\mathbf{X}||^2 + ||\mathbf{A}||^2 - 2\mathbf{X}^T\mathbf{A}$$
 (2.0.6)

and

$$(\mathbf{PB})^2 = \|\mathbf{P} - \mathbf{B}\|^2 \tag{2.0.7}$$

$$= \|\mathbf{X} - \mathbf{B}\|^2 \tag{2.0.8}$$

$$= ||\mathbf{X}||^2 + ||\mathbf{B}||^2 - 2\mathbf{X}^T\mathbf{B}$$
 (2.0.9)

The given equation is

$$(\mathbf{PA})^2 + (\mathbf{PB})^2 = 2k^2$$
 (2.0.10)

Sub (2.0.6) and (2.0.9) values in (2.0.10)

$$\|\mathbf{X}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{X}^T\mathbf{A} + \|\mathbf{X}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{X}^T\mathbf{B} = 2k^2$$
(2.0.11)

$$\implies 2 \|\mathbf{X}\|^2 + \|\mathbf{A}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{X}^T(\mathbf{A} + \mathbf{B}) = 2k^2$$
(2.0.12)

sub A,B values in equation (2.0.12), we get

$$2\|\mathbf{X}\|^{2} + \left\| \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \right\|^{2} + \left\| \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix} \right\|^{2} - 2\mathbf{X}^{T} \left(\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix} \right) = 2k^{2}$$
(2.0.13)

: the required equation is

$$2\|\mathbf{X}\|^2 - 2\mathbf{X}^T \begin{pmatrix} 2\\7\\-2 \end{pmatrix} + 109 - 2k^2 = 0 \qquad (2.0.14)$$

The Plot of Sphere is:

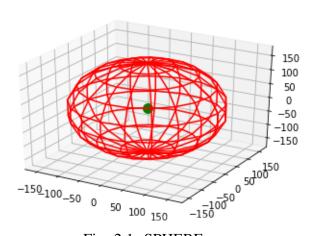


Fig. 2.1: SPHERE