ASSIGNMENT 7

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1 QUESTION No 2.25(VECTOR FORMS)

Find the equation of set of points $\mathbf{P}$ such that

$$(\mathbf{PA})^2 + (\mathbf{PB})^2 = 2k^2; \tag{1.0.1}$$

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}; \tag{1.0.2}$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}; \tag{1.0.3}$$

respectively.

2 SOLUTION

Given ,

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}; \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}; \tag{2.0.2}$$

Let

$$\mathbf{P} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}; \tag{2.0.3}$$

so,

$$(\mathbf{PA})^2 = \|\mathbf{P} - \mathbf{A}\|^2 \tag{2.0.4}$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{P}^T\mathbf{A} \tag{2.0.5}$$

$$= \left\| \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \right\|^2 + \left\| \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \right\|^2 - 2 \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \begin{pmatrix} X & Y & Z \end{pmatrix} \tag{2.0.6}$$

$$= X^2 + Y^2 + Z^2 - 6X - 8Y - 10Z + 50 \tag{2.0.7}$$

and

$$(\mathbf{PB})^2 = \|\mathbf{P} - \mathbf{B}\|^2 \tag{2.0.8}$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{P}^T\mathbf{B} \tag{2.0.9}$$

$$= \left\| \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \right\|^2 + \left\| \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix} \right\|^2 - 2 \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix} \begin{pmatrix} X & Y & Z \end{pmatrix} \tag{2.0.10}$$

$$= X^2 + Y^2 + Z^2 + 2X - 6Y + 14Z + 59 \tag{2.0.11}$$

The given equation is

$$(\mathbf{PA})^2 + (\mathbf{PB})^2 = 2k^2 \tag{2.0.12}$$

Sub (2.0.7) and (2.0.11) values in (2.0.12)
∴ the required equation is

$$2X^2 + 2Y^2 + 2Z^2 - 4X - 14Y + 4Z + 109 = 2k^2 \tag{2.0.13}$$