ASSIGNMENT-8

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https://github.com/Atlakeerthana/Assignment8/tree/main/Assignment8

and latex-tikz codes from

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1 QUESTION No-2.69(Matrices)

Obtain the inverse of the following matrix Using elementary operations

1) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

2 Solution

1) Given that

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} \tag{2.0.1}$$

The augmented matrix $[A|I]$ is as given below:-

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.0.2}$$

We apply the elementary row operations on $[A|I]$ as follows :-

$$[A|I] = \left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.0.3}$$

$$\xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2.0.4}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right) \tag{2.0.5}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right) \tag{2.0.6}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + 5R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right) \tag{2.0.7}$$

$$\xleftrightarrow{R_3 \leftarrow R_3/2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{array}\right) \tag{2.0.8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{array}\right) \tag{2.0.9}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{array}\right) \tag{2.0.10}$$

By performing elementary transformations on augmented matrix$[A|I]$, we obtained the augmented matrix in the form $[I|A]$. Hence we can conclude that the matrix A is invertible and inverse of the matrix is:-

$$\therefore \mathbf{A^{-1}} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{pmatrix} \tag{2.0.11}$$

2) QR decomposition of $\begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

Let us use the Gram-schmidt approach to obtain QR decomposition of$\mathbf{A}$. Consider rows vectors say $\mathbf{a_1}$,$\mathbf{a_2}$ and $\mathbf{a_3}$ of $\mathbf{A}$ which is given by

$$\mathbf{a_1} = \begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \tag{2.0.12}$$

$$\mathbf{a_2} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \tag{2.0.13}$$

$$\mathbf{a_3} = \begin{pmatrix} 3 & 1 & 1 \end{pmatrix} \tag{2.0.14}$$

we can express these as

$$\mathbf{u_1} = \mathbf{a_1} = \begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{e_1} = \frac{\mathbf{u_1}}{\|\mathbf{u_1}\|} \tag{2.0.16}$$

$$\mathbf{e_1} = \frac{\begin{pmatrix} 0 & 1 & 2 \end{pmatrix}}{\sqrt{0+1+4}} \tag{2.0.17}$$

$$\mathbf{e_1} = \begin{pmatrix} 0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \tag{2.0.18}$$

$$\mathbf{u_2} = \mathbf{a_2} - (\mathbf{a_2 e_1})\mathbf{e_1} \tag{2.0.19}$$

$$= \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} - (\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix})\begin{pmatrix} 0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \tag{2.0.20}$$

$$= \begin{pmatrix} 1 & \frac{2}{5} & \frac{-1}{5} \end{pmatrix} \tag{2.0.21}$$

$$\mathbf{e_2} = \frac{\mathbf{u_2}}{\|\mathbf{u_2}\|} = \frac{\begin{pmatrix} 1 & \frac{2}{5} & \frac{-1}{5} \end{pmatrix}}{\sqrt{1 + \frac{4}{25} + \frac{1}{25}}} \tag{2.0.22}$$

$$\mathbf{e_2} = \begin{pmatrix} \frac{5}{\sqrt{30}} & \frac{2}{\sqrt{30}} & \frac{-1}{\sqrt{30}} \end{pmatrix} \tag{2.0.23}$$

$$\mathbf{u_3} = \mathbf{a_3} - (\mathbf{a_3 e_1})\mathbf{e_1} - (\mathbf{a_3 e_2})\mathbf{e_2} \tag{2.0.24}$$

$$\mathbf{u_3} = \begin{pmatrix} \frac{1}{3} & \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \tag{2.0.25}$$

$$\mathbf{e_3} = \frac{\mathbf{u_3}}{\|\mathbf{u_3}\|} \tag{2.0.26}$$

$$= \frac{\begin{pmatrix} \frac{1}{3} & \frac{-2}{3} & \frac{1}{4} \end{pmatrix}}{\sqrt{\frac{1}{9} + \frac{4}{9} + \frac{1}{9}}} \tag{2.0.27}$$

$$\mathbf{e_3} = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix} \tag{2.0.28}$$

Thus,

$$\mathbf{Q} = (e_1|e_2|----|e_n) \tag{2.0.29}$$

$$= \begin{pmatrix} 0 & \frac{5}{\sqrt{30}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{30}} & \frac{-2}{\sqrt{6}} \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{30}} & \frac{1}{\sqrt{6}} \end{pmatrix} \tag{2.0.30}$$

Then

$$\mathbf{R} = \begin{pmatrix} a_1e_1 & a_2e_1 & a_3e_1 \\ 0 & a_2e_2 & a_3e_2 \\ 0 & 0 & a_3e_3 \end{pmatrix} \tag{2.0.31}$$

$$= \begin{pmatrix} \frac{5}{\sqrt{5}} & \frac{8}{\sqrt{5}} & \frac{3}{\sqrt{5}} \\ 0 & \frac{6}{\sqrt{30}} & \frac{16}{\sqrt{30}} \\ 0 & 0 & \frac{2}{\sqrt{6}} \end{pmatrix} \tag{2.0.32}$$

From equations (2.0.30) and (2.0.32) the obtained **QR** Decomposition is

$$\begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & \frac{5}{\sqrt{30}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{30}} & \frac{-2}{\sqrt{6}} \\ \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{30}} & \frac{1}{\sqrt{6}} \end{pmatrix}\begin{pmatrix} \frac{5}{\sqrt{5}} & \frac{8}{\sqrt{5}} & \frac{3}{\sqrt{5}} \\ 0 & \frac{6}{\sqrt{30}} & \frac{16}{\sqrt{30}} \\ 0 & 0 & \frac{2}{\sqrt{6}} \end{pmatrix} \tag{2.0.33}$$