

Assignment 8

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Download all python codes from

<https://github.com/ooharapolu/ASSIGNMENT8/Assignment8.py>

and latex-tikz codes from

<https://github.com/ooharapolu/ASSIGNMENT8/main.tex>

1 QUESTION No-2.69(MATRICES)

Obtain the inverse of the following matrix Using elementary operations

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} \quad (1.0.1)$$

2 SOLUTION

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} \quad (2.0.1)$$

The augmented matrix $[A|I]$ is as given below:-

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \quad (2.0.2)$$

We apply the elementary row operations on $[A|I]$ as follows :-

$$[A|I] = \left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \quad (2.0.3)$$

$$\xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \quad (2.0.4)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array} \right) \quad (2.0.5)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array} \right) \quad (2.0.6)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + 5R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array} \right) \quad (2.0.7)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 / 2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array} \right) \quad (2.0.8)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array} \right) \quad (2.0.9)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array} \right) \quad (2.0.10)$$

By performing elementary transformations on augmented matrix $[A|I]$, we obtained the augmented matrix in the form $[I|A]$. Hence we can conclude that the matrix A is invertible and inverse of the matrix is:-

$$\therefore \mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{pmatrix} \quad (2.0.11)$$