Preliminary version.
Under submission to Virginia Policy Review.

Redistricting Reform in Virginia: Districting Criteria in Context

Daryl DeFord and Moon Duchin

April 16, 2019

Abstract

A constitutional amendment process is underway in Virginia to reform the state's redistricting practices through the creation of an independent commission. Here, we assess rules that are under consideration to be implemented by statute, particularly those specifying constraints and priorities for the districts themselves. We focus on potential rules to limit geographic splitting and to mandate population balance between districts.

To address the impacts of these kinds of criteria, we generate large ensembles of valid districting plans while varying constraints and priorities, using Markov chain methods. We show in particular that tighter splitting rules chiefly "cut off the tails" of the distribution of partisan outcomes, making the most skewed partisan maps into more extreme outliers. On the other hand, tightening or loosening population deviation has no effect at all on the normal range of measurable outcomes.

The lessons drawn here extend to dozens of other states that seek redistricting reform through amendment, statute, or citizen initiative: new rules have complicated effects that can nonetheless be effectively modeled on a state-by-state basis.

Contents

1 Introduction

In advance of the 2020 census and attendant redistricting process, the citizens of several states have used the ballot initiative process to institute independent redistricting commissions and new constraints on permissible plans. However, not all states permit direct citizen engagement of this form. In numerous states, changes to the redistricting process must go through the legislature and are subject to complex ratification procedures.

In Virginia, redistricting reform is underway through a delicate and multi-step constitutional amendment process: a proposed amendment must pass the legislature twice, with an intervening legislative election, before going to the voters directly in a referendum. The "First Read" occurred in February 2019, when an amendment bill passed with a 83-15 vote in the House of Delegates and a 40-0 vote in the state Senate.

As approved, the amendment is nearly entirely focused on procedural concerns relating to the creation of a new bipartisan commission that will control the approval of new congressional and legislative districts. Earlier versions, both House and Senate, included language specifying and ordering principles or criteria for the districts themselves. These were stripped out of the final bill in large part because of rumors or hunches about their impact on racial or partisan outcomes.

In this note, we construct mathematical models that provide empirical evidence to speak directly to these concerns.[1]

1.1 Background and stakeholders in Virginia redistricting

1.1.1 Legislators

The General Assembly of the State of Virginia is separated into two bodies: a 100-member House of Delegates, with two-year terms, and a 40-member Senate, with four-year terms. Both have elections in odd-numbered years, with the Senate elections on a cycle including 2015 and 2019. The Senators and Representatives elected this year will be in place when the 2020 Census data is released and the new districts are to be drawn. Under the system currently in place, the legislators themselves commission and approve the electoral boundaries in a contentious and highly partisan process, with vetoes and protracted litigation extending the redistricting intrigue well into the decade.

At the time of the last redistricting, Virginia had split political control, with Republicans leading the House and Democrats controlling the Senate. The Democratic-controlled Senate was in charge of the redrawn Senate plan, while the Republican-controlled House was in charge of the House plan, in something of a well-known gentlemen's agreement. The General Assembly maps were passed in 2011 under this agreement, after some revisions due to a Governor's veto of the first attempt.The Congressional process was more convoluted, with much of the disagreement focused on the placement and composition of the Black opportunity-to-elect districts. The House of Delegates passed a Congressional map in 2011, but the Democratically controlled Senate refused to ratify the plan. However, in the Fall 2011 election, the Republican party gained control of the Senate and when the new Assembly was seated in 2012 they passed that same plan.

Within the legislature, the Legislative Black Caucus is a force. At the time of writing, its official membership is 5 state Senators and 16 members of the House of Delegates, all Democrats.[2] In practice, they do not serve as a rubber stamp for the Democratic caucuses on redistricting, but seek assurances that electoral opportunities for African-American communities are being safeguarded, which can be a delicate question because of the balance between beneficial electoral opportunity and harmful racial packing. Indeed in the redistricting cycle that followed the 2010 Census, the district court noted that the majority of the Legislative Black Caucus signed off on the Republican-controlled plan for the House of Delegates that was later found to be an unconstitutional racial gerrymander [22, p5].

[1] A study of Virginia redistricting with similar goals can be found in Altman-McDonald [1], but without an ensemble analysis. Instead, they compare about a dozen plans for each set of districts: the plans adopted and others that were considered by the Legislature, a handful of plans made by an advisory commission to the governor, and several generated by students in the context of a competition. That paper also contains an excellent overview of the history and political context of redistricting in the state.

[2] `https://www.vablackcaucus.com`

1.1.2 Courts and special master

In the previous cycle, several maps were challenged in the courts; lawsuits about racial gerrymandering of House and Congressional maps each reached the Supreme Court of the United States two separate times.

Personhuballah v. Alcorn is a challenge to a Congressional district as an unconstitutional racial gerrymander, filed in 2013 in federal court. The three-judge district court found for the plaintiffs in 2014. But in 2015, the U.S. Supreme Court vacated the decision for remand in light of the Court's holding in another case, Alabama Legislative Black Caucus. Later in 2015, the three-judge district court once again found the challenged congressional district to be an unconstitutional racial gerrymander. On appeal, the U.S. Supreme Court held that the members of congress bringing the appeal did not have standing to appeal, effectively leaving the District Court's decision in place [12]. The District Court appointed an outside expert to construct a remedial plan in 2015 and approved the new districting plan in January 2016, effectively ending the litigation.

Vesilind v. Va. State Board of Elections was filed by a group of individual voters in state court in 2015, challenging state House and state Senate districts for not being compact enough, in violation of the state constitution. The trial court in 2017 held that the districts were constitutional under a deferential standard of review for state statutes (in this case, the act the General Assembly passed in 2011 which created the state's legislative districts). On appeal, the Virginia Supreme Court unanimously affirmed this ruling, holding in essence that the General Assembly was permitted to define their own constitutional threshold for compactness.

Finally, **Bethune-Hill v. Virginia Board of Elections** is the only ongoing litigation. Complaint filed in federal court in 2014 by residents of 12 of the 100 House of Delegates districts, alleging that the districts were racially gerrymandered. In particular, the imposition of a minimum 55% Black Voting Age Population had been used on the selected districts. In 2015, the three-judge district court ruled that, though District 75 was drawn with race as a predominant factor, the district was justified because the racial polarization in the district was so high that a 55% BVAP threshold was required to comply with the Voting Rights Act of 1965 [6]. It upheld the other 11 districts, finding that they were not racial gerrymanders. On review in 2017, the U.S. Supreme Court upheld the three-judge court's ruling on District 75 and reversed its holding on the other 11 districts, finding that the three-judge court had applied the incorrect standard of review in analyzing the evidence. The Supreme Court sent the case back to the three-judge court to re-try the case on the 11 remaining districts in contention under the correct standard. This time, in July 2018, the three-judge panel on re-review held that the 11 districts were, in fact, unconstitutional racial gerrymanders. After this ruling the court, the legislature was charged with constructing a new map, within a specified deadline. When the legislative processes stalled, the court again appointed an outside expert to redraw the districting plan and in February 2019 a new map was approved. Concurrently, the District Court ruling was appealed to the Supreme Court. Oral argument was heard in March 2019 and a ruling is expected in June 2019.

One result of this litigation is that both the current Congressional and House maps were ultimately re-drawn by the same outside expert appointed as "special master" by the courts: Bernard Grofman, a professor of political science at UC Irvine. Grofman's plans have taken nonstandard approaches to the remedial map-drawing process (such as by creating modular zones in his recent House of Delegates plan and strictly subdividing the redistricting problem into those smaller pieces [11]). We will consider some of the properties of his remedial Congressional plan in this study.

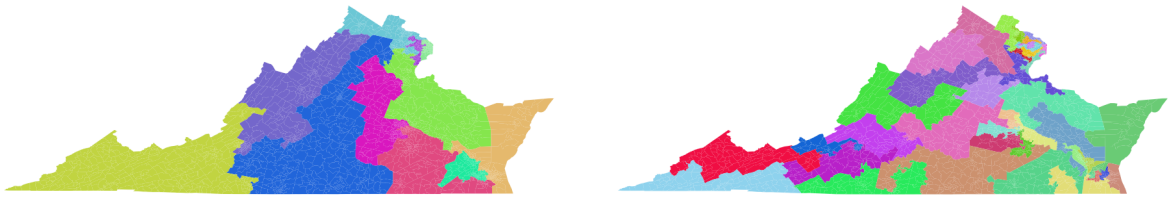

Figure 1: Current Virginia districting plans as of 2019: the 11 Congressional districts drawn by Bernard Grofman as a court-appointed "special master," and the 40 Senate districts drawn by the Democratic-controlled state Senate.

1.1.3 Reform groups and civil rights groups

National civil rights organizations—such as NAACP and its litigation wing LDF, the Lawyers' Committee for Civil Rights Under Law, and the League of Women's Voters—have traditionally been players in Virginia's redistricting battles, filing or joining lawsuits, commissioning demonstration plans, or filing amicus briefs to argue for maps with more opportunity for African-American and coalition districts.

In recent years, a fundamentally new factor in play is an enormously increased level of public attention and citizen pushback. A powerful agent in the push for redistricting reform is the nonpartisan group called OneVirginia2021, headed by Brian Cannon (JD, William and Mary). Founded in 2014, the group quickly swelled to have nearly 100,000 members and blankets the state every election cycle with poll workers raising awareness of gerrymandering. Through petitions and targeted advocacy, they have been a major force in the push for a Constitutional Amendment to reform redistricting in the state.

New Virginia Majority is another advocacy group—originally founded in 2007 to focus on organizing working class communities of color, it has adopted a voting rights mandate since 2013, which has included petitioning against certain redistricting legislation, training and endorsing candidates, registering voters, and even commissioning an alternative House of Delegates map during the Bethune-Hill litigation.

2 Amending the state constitution: SJ306

In order to amend the Virginia Constitution, proposed legislation must pass the General Assembly twice, with a House of Delegates election between considerations, before being put to a vote of the citizens. Early this year, a constitutional amendment to reform the redistricting process cleared its first hurdle with the "First Read" passage of SJ306 through both branches of the General Assembly on Saturday February 23, 2019. The amendment cannot be changed between votes, so in order to have a new commission seated for the 2021 redistricting, the identical text must pass the legislature early in 2020, followed by a referendum throwing final approval to the voters during the general election in November 2020.

The main content of SJ306 is to establish a bipartisan commission for drawing the electoral boundaries after each Census. The commission is to be made up of 16 members, including four sitting legislators from each party and eight citizens. The citizen members of the commission are selected exclusively from short lists of candidates compiled by the legislative party leaders.

The only insurance policy against partisan gridlock is that control of the process is thrown to the Supreme Court of Virginia in the case that the legislature fails to pass bills approving the new maps.

2.1 Additions, adjustments, and clarifications

Although the text of the amendment cannot be changed without restarting the legislative process from the beginning, there are still important ways for the legislature to flesh out and strengthen elements of the reform mandate. To begin with, the amendment itself specifies that the General Assembly can determine the eligibility criteria for citizens who may serve on the commission. More broadly, the Assembly has the power to adopt new constraints on permissible maps and on the data considered during the map creation process, provided they do not conflict with the constitutional text. Rules could be implemented to prevent improper partisan influences on the districting process, to clarify vague terms of art like "compactness," and more.

2.1.1 Commission composition and procedure

Many eleventh-hour changes were made as the Constitutional Amendment text neared passage on and around the tense Saturday session. One of the late casualties as the House and Senate versions were reconciled was the guarantee of nonpartisan/unaffiliated members of the redistricting commission, an omission that puts its mission gravely at risk. The current commission setup is explicitly partisan, as described above. Although gerrymandering by one party against the other looms large in the public imagination, bipartisan gerrymanders—via horse-trading that serves to protect incumbents at the expense of responsive representation—are just as much of a threat. Arguably, the situation in Virginia at the beginning of the

cycle was an example of this problem, exemplified by the handshake deal that each party would control the maps for one branch of the state legislature while ceding all but the most basic oversight of the other.

Not every self-styled independent commission actually has the independence to function with maximum effectiveness as a check on party interests. For instance in Idaho's Citizen Commission for Reapportionment, the selection rules only include the weak provision that "No member may be an elected or appointed official in the state at the time of designation" [17]. And indeed of Idaho's current six commissioners, four are former state legislators, one is a former chief of staff for the House minority leader, and the last is a longtime party activist, having served at one point as a party elector.[3]

The language of SJ306 is similarly weak, barring only current members of Congress or the General Assembly, but not former members and staff, lobbyists, fundraisers, or other heavily partisan actors. When the General Assembly addresses eligibility criteria for commission members, we urge the inclusion of language that provides not only a robust definition of an unaffiliated commissioner and a rule that ensures that some be included, but also a balance of commissioners that gives the unaffiliated contingent a strong voice.

The legislature might also consider other procedural elements, such as restricting the use of particular kinds of data in the map-drawing process or appointing map evaluators in an oversight role. Access to detailed predictive data about likely voting patterns has exacerbated the problem of gerrymandering in recent years, particularly when employed by commercial consultants. Several states forbid consideration of partisan or incumbency data in order to focus the commission only on the traditional districting criteria. Taken on their own, rules like this are inadequate to safeguard against partisan or incumbent skews. Appointing a statistical expert, particularly one versed in ensemble techniques like those profiled here, can help ensure that intentional or unintentional skews in a plan are kept in check.

2.1.2 Districting criteria

In addition to commission compositions, the legislature can also place constraints on the types of maps that are permissible and what data the commission is allowed to consider while drawing the plan. An earlier version of the amendment, SJR 274 [26], contained an ordered list of redistricting "standards and criteria," summarized as follows:

1. Reduce population deviation from ideal district size to the minimum possible, limited to 0.5% for Congressional districts and 5% for legislative districts;
2. Preserve opportunity to elect candidates of choice for "substantial racial or ethnic minority communities";
3. Bar the favoring or disfavoring of any party or incumbent;
4. Respect town, city, and county boundaries as much as possible. Attempt to keep each "political subdivision" intact within a district;
5. Form districts to be compact and contiguous.

(Full text available in Appendix A.)

Only #2 and #5 made it to the final amendment in any form. The other three—population deviation, partisan/incumbency consideration, and geographic splitting—all fell victim to debates about their effectiveness and their unintended consequences. We will consider those here.

[3]Of the states that have independent commissions already in place at the time of writing, only Arizona and California have effective provisions against partisan capture, in particular by barring legislative staff from serving and ensuring that some party-unaffiliated members be included. Five states passed measures in 2018 establishing or shoring up some form of independent redistricting body; implementation details remain to be worked out.

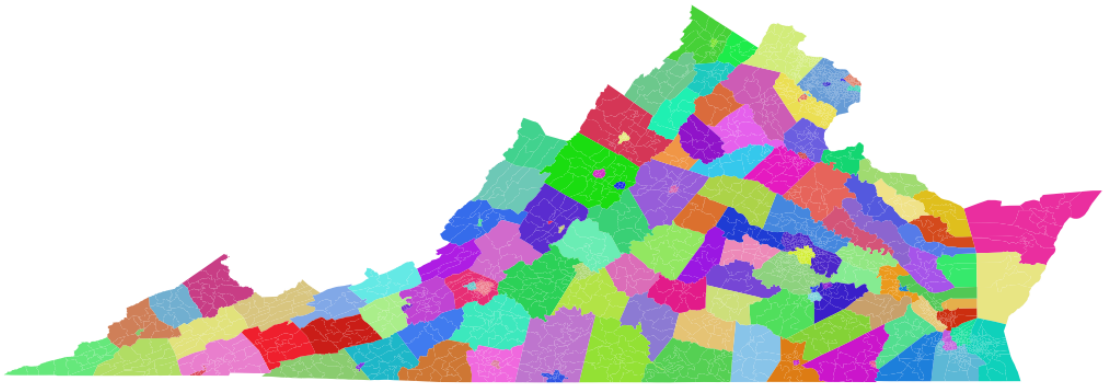

Figure 2: The 133 Virginia localities, according to the Census data on county-equivalent geographies. Notice that several of the largest cities are separate units from their surrounding counties. We will interpret rules to respect political boundaries as preferring plans that split fewer of these units between multiple districts.

2.1.3 Data transparency

The transparency of electoral data is poor in Virginia, as it is in many states. The authority to change precincts lies with local officials in the cities and counties, and up-to-date precinct shapefiles are not maintained by any state entity. This means that it is difficult to spatially locate the cast votes in any agreed-on manner.

A proposed bill in New Jersey provides a model of the kind of language that a legislature could adopt to solve this problem [18]. In particular, it requires that the precinct boundaries be reported to the state when changes are made, and that election results be reported in units that are lined up with this geography.

This type of transparency is critical to allow the newly established commission the means to create and assess plans in standard units, and to understand the political geography of the state without having to rely on the expanding array of commercial products and services.

3 Evaluating districting criteria in Virginia's political geography

In order to evaluate potential impacts of the proposed rules, we employ a technique that has been growing in popularity in the redistricting world: we generate large collections called *ensembles* of legally valid plans to understand what properties are typical and what properties are outlying. By generating ensembles under different combinations of districting criteria, we can then compare their properties to measure the effects of a change in the rules and priorities.

To do this in a meaningful way, it is fundamentally important to choose a plan generation algorithm that is powerful enough to explore the universe of possibilities robustly. We use Markov chain Monte Carlo (MCMC) methods, which have been relied on for decades and across many application domains to generate fast representative samples in settings when a complete inventory of possibilities is computationally intractable [2].

3.1 Data, methods, and algorithms

The districting ensembles were generated with the GerryChain software developed by the Metric Geometry and Gerrymandering Group (MGGG) [14]. Each ensemble variant was constructed with a Recombination (ReCom) Markov chain first introduced in our study of the Virginia House of Delegates [15]. That report contains an ensemble analysis of the House of Delegates districts, so the current study focuses on Congressional and state Senate districts.

The building blocks of these plans are *precincts*, which are administrative units with several thousand people each, on average. A validated precinct shapefile and recent electoral data is publicly available from the Princeton Gerrymandering Project [20]. Demographic data was extracted from the Census and aggregated onto the precincts using MGGG preprocessing tools [13]. We note that precincts are the best choice of building block for analytic purposes when partisan results are a primary focus, because that is the level at which votes are reported. Our prior House of Delegates study focused on racial balance, so its ensembles were made from census blocks. Also note that plans that are made from a different set of units must be "rounded off" into a precinct assignment, which is also accomplished with the preprocessing suite [13]. The actual plans considered and enacted by the state are defined using *census blocks* which are much smaller than precincts [24].

3.1.1 Ensemble generation

Here are brief technical specifications of the ensemble generation. The Recombination walk begins with a plan, then considers a possible new plan formed by merging two adjacent districts and re-splitting them with a spanning tree method. This new plan will be rejected if it is invalid according to the rules, and if it is valid it will be accepted with some probability based on the priorities set for the random walk. The neutral walk accepts all valid plans that are proposed. By default, population of a district is allowed to vary to a level 2% above or below ideal population for Congressional districts and 5% above or below for Senate districts. All districts are required to be (rook) contiguous, and water is handled by the precinct shapefile itself, which partitions the water area as well as the land area. We do not impose a numerical compactness constraint, because the Recombination walk itself tends to form districts that are considerably more compact under the discrete measure described in Section 3.1.3 than the enacted plans.

We conduct four different styles of random walk.

- **Neutral ensemble**: default settings, as described above.
- **Splits ensemble**: begin by requiring that steps do not increase the number of locality splits until a threshold is passed, then impose exponential weighting prioritizing fewer splits;
- **Population ensemble**: Congressional district generation with tighter (0.5%) or looser (5%) population constraint, Senate district generation with tighter (2%) or looser population (10%) constraint;
- **BVAP ensemble**: order districts by Black Voting Age Population as a share of total population, or BVAP%. Begin by requiring that steps do not decrease top two districts' BVAP% until levels pass 40%, then no constraint for Congressional maps, and until the top five districts are above 50% for the Senate maps, in line with the enacted plans.

Both the splits and the BVAP ensemble use a weighting rather than a hard constraint. We found that they were quite successful at reaching the target levels of splits and BVAP% in a modest number of steps (around 2000 steps sufficed). We also attempted Markov chain searches to seek to minimize splitting through weighted walks (rather than to construct a normal range) and were able to construct valid Congressional plans that split just 12 localities and Senate plans splitting just 18.

For each variant, we perform 50,000 ReCom steps and then discard the first 2000, leaving ensembles of 48,000 plans. Generating ensembles of this size can done overnight (though speeds vary depending on the variant). Independence of seed, stability when length is increased, and slack in the hard constraints all provide strong heuristic evidence that 10,000 steps is enough for the statistics of the full draw to converge to a steady state, as in [15].

3.1.2 Electoral data

After the 2015 election, the VA Senate moved to a 21-19 split in favor of the Republican party, a 52.5% Republican seat share. It is hard to measure the vote share that secured that outcome, because 20 out of 40 seats were uncontested by one of the two major parties: 10 seats had no Democrat file for election, and 10 seats had no Republican.[4]

[4]If votes from all 40 races are counted, then Republicans have a narrow edge with 50.5% of two-way vote share; if only the contested races are counted, then Republican share rises to 53.5% [5].

Since then, Virginia has seen a marked statewide Democratic shift in voter preferences, reflected in the 2016 Presidential and 2018 Senate results (where Democrats had a majority in both cases) and the 2017 House of Delegates elections (where the Democratic Party gained 15 seats). The U.S. Congressional delegation from Virginia also switched from a 7-4 Republican advantage after the 2014 election to a 7-4 Democratic advantage after the 2016 election, reflecting both the pro-Democratic shift and the switch to a new map, as federal courts had put the special master's plan in place in the intervening time.

Our shapefile has five elections matched to the precincts: President 2016, Attorney General and Lieutenant General 2017, and Governor and U.S. Senate 2018. We rely on these exogenous electoral data because statewide elections allow us to avoid confounding effects of varying incumbency status around the state, uncontested races, and other heterogeneities introduced by using elections where the candidates vary by district.[5]

In this article we have chosen to focus on the vote distribution from the Attorney General race, won by Mark Herring with approximately 53% of the two-party vote share and a total of 2,597,214 votes cast. This is close to the vote total of 2,680,818 for state Senate races (which are on the same ballot). Congressional vote totals are higher, with some variation by whether it is a presidential year: there were 3,781,569 votes cast for House races in Virginia in 2016 and 3,312,971 in 2018. For comparison, the 2018 U.S. Senate race had 3,351,373 votes.

Figure 3: R vs. D votes by precinct in the 2017 Attorney General and 2018 Senate elections. They are visually indistinguishable, but the Senate voting pattern would earn two extra Congressional seats for Democrats.

One of the subtlest parts of map evaluation on partisan axes is to use appropriate vote data, since the same map may show quite different outcomes when laid over different vote patterns. To illustrate this, consider the partisan outcomes in the special master's Congressional map (SM-Cong) and the Democratic-controlled legislature's state Senate map (DLeg-Sen) varies against the five elections in our dataset. A value of 5-6 means that five districts had D>R, so for instance Clinton votes outnumbered Trump votes in only 5/11 Congressional districts, even though Clinton won by more than 200,000 votes statewide.

	Pres 16	ATG 17	LTG 17	Gov 18	Sen 18
Special Master's Cong. Plan (SM-Cong)	5-6	5-6	5-6	6-5	7-4
Legislature's Senate Plan (DLeg-Sen)	23-17	23-17	23-17	24-16	27-13

If the goal were to analyze the fairness properties of the state Senate and the Congressional map, then the Attorney General race would provide good vote data to assess the partisan properties of the state Senate plan, while the U.S. Senate race would have vote data better suited to understanding the Congressional plan. Since the primary goal is to understand the effects of altering the districting criteria, will use one set of vote data—Attorney General 2017—for all of the main figures, to facilitate comparisons.

3.1.3 Metrics

Once an ensemble has been generated and the data has been compiled, we need an evaluation metric (also known as a summary statistic) to measure the properties of the plans.

[5]Besides the uncontested state Senate races noted above, Virginia's 11th congressional district was uncontested by Republicans in 2016, and the 3rd was uncontested in 2018. Many political scientists use regressions to control for these heterogeneities, but these techniques raise as many modeling questions as they address.

First, we will employ the simplest available metrics of compactness (or how reasonably shaped the districts are) and of locality splits. The 2,439 precincts of Virginia are arranged in a network with 6,859 edges (in other words, 6,859 pairs of precincts share a segment of boundary). To make a districting plan, some of these adjacent pairs must be separated from each other because the precincts are assigned to different districts. For instance, the left-hand plan employs tame shapes, while the right-hand plan uses wilder shapes; this is reflected in the number of cuts needed to separate the districts. The fewer cut edges, the more compact the plan. We regard this simple metric as far superior to area-based measures of compactness, which take electorally irrelevant factors heavily into account. (See [9] for further discussion.)

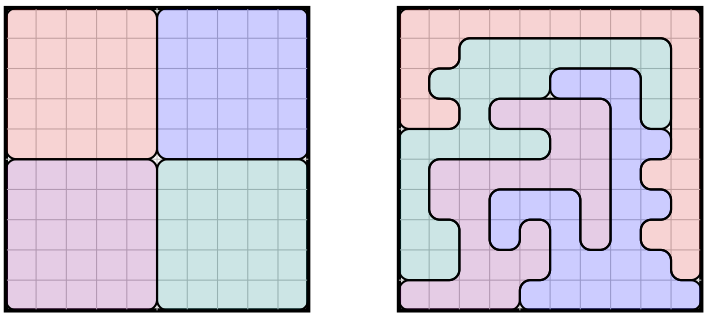

Figure 4: Cut edges as a measure of compactness. The plan on the left has 20 cut edges, while the plan on the right cuts 73 out of the 180 edges in the grid.

For geographic splitting, we will use the localities shown in Figure 2, which are derived from the Census-provided county-equivalent geographies. In Virginia the county-equivalent units include the larger cities in the state, separate from their surrounding areas. This impacts rules that prioritize preserving municipal boundaries: if the city area is not large enough to form a district on its own, then typically the surrounding county must be split so that it can provide the extra population needed to reach ideal district size.

We note that there is not yet a universally agreed-upon metric for evaluating splitting of municipal units with districting plans. A natural choice to begin is to simply count the number of units that are not kept whole (i.e., that overlap multiple districts). However, this choice is not sensitive to whether a county is split 2, 3, or ten ways, when clearly the spirit of the rule should encourage that number to be the smallest possible. It also does not distinguish between a situation where a large county is split in half and one where a small nibble is taken out of a corner. In a legal filing for the Pennsylvania supreme court, Duchin proposed an entropy-based measure of splitting that takes these other factors into account [8].[6] In this study, however, we will adopt the simplest measure, counting the number of localities that are not kept whole, and call that the number of splits.

Our primary racial statistic will be the Black Voting Age Population percentage (BVAP%) in each district.[7] This is the value that was use to evaluate the constitutionality of the maps that underwent court challenges this cycle.

Finally, we will discuss three kinds of partisan statistic: the number of Democratic seats, the efficiency gap, and the mean-median score. We'll take the arbitrary convention of setting the direction in all three scores so that higher scores favor Democrats. Suppose the Republican vote totals were $(R_1, R_2, \ldots, R_k)$ in the districts in a particular plan, while the Democratic vote totals were $(D_1, D_2, \ldots, D_k)$. Let V be the Democratic vote share statewide, $V = (\sum D_i)/(\sum R_i + \sum D_i)$.

[6] If the localitites are $\{L_j\}$ with respective populations ℓ_j and the districts in a plan are $\{D_i\}$ with populations d_i, we can write $p_i^{(L_j)}$ for the fraction of locality j population that is contained in district i, and likewise $p_j^{(D_i)}$ for the fraction of district i population that is contained in locality j. Then a splitting score $\mathsf{Split} = \sum_i d_i \left[\sum_j \sqrt{p_j^{(D_i)}}\right] + \sum_j \ell_j \left[\sum_i \sqrt{p_i^{(L_j)}}\right]$ takes into account both the number and the type of splittings.

[7] In the Bethune-Hill litigation, there was disagreement about the correct way to count BVAP. We use the Department of Justice convention: BVAP is the number of Census-identified Black residents at least 18 years of age, and BVAP% divides that value by the total number of residents at least 18 years of age.

Then

$$\begin{aligned} \text{Dem seat share} \quad & S = \#\{i : D_i > R_i\}/k \\ \text{Efficiency gap} \quad & EG = S - 2V + \tfrac{1}{2} + \tfrac{S(1-S)(1-\rho)}{S(1-\rho)+\rho} \\ \text{Mean-median gap} \quad & MM = \text{median}(\{V_i\}) - \text{mean}(\{V_i\}) \end{aligned}$$

In the efficiency gap formula ρ is the turnout ratio: the average number of votes cast in Democratic-won districts divided by the average in Republican-won districts. This expression for EG, due to Veomett [23], makes it visible that using fixed vote data, the efficiency gap records essentially the same information as the number of seats as the districts vary. This is because V is determined by the vote data and not the districts, so if $\rho = 1$ then EG is just S plus a constant. In reality, different districting plans will yield somewhat different turnout ratios (usually values of $\rho \approx 1$),[8] introducing a small amount of noise to the simple relationship between S and V. This is why an EG histogram is simply a noisy version of the seats histogram (see Figure 5).

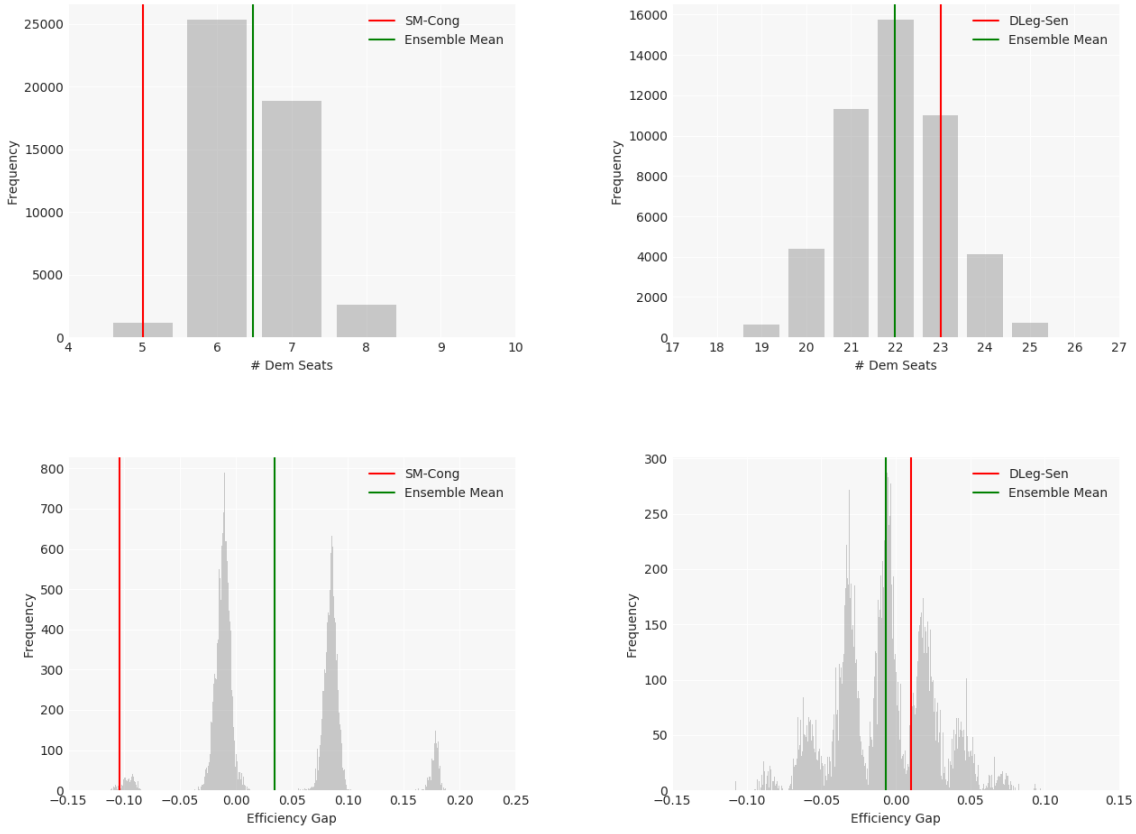

Figure 5: Neutral ensembles shown with respect to the 2017 Attorney General vote. The top histograms show the number of districts in each plan with D>R votes, out of 11 and 40 respectively. The next row shows the efficiency gap (with positive scores favoring Democrats), which is seen to be essentially identical information. For this voting pattern, Grofman's map favors Republicans by about 1.5 seats out of 11 relative to the neutral baseline, while the Democratic Legislature's plan favors Democrats by 1 seat out of 40.

[8] In Virginia, Veomett calculated ρ values of 1.09 and 1.11 in recent Congressional elections, so the noise term is not large enough to cause the spikes to interact very much. In Texas, the turnout ratio in Congressional elections has been as high as 1.74 recently.

The mean-median gap is discussed in the political science literature as a skew that measures whether securing half of the representation can be accomplished with less than half of the vote. This is because, if voter preference were to swing by the same number of percentage points in every district, a swing of $50\% - \text{median}\{V_i\}$ would push the median district to 50% vote share for each party. Thus the balance point for an evenly split delegation occurs with a mean vote level of $50\% - MM$. Others have disputed the validity of this kind of uniform swing (among other issues with the setup, including the way its logic is tied to the simplifying assumption of equal turnout), but this remains the most popular of the partisan symmetry family of scores for measuring each party's structural advantage in a districting plan.

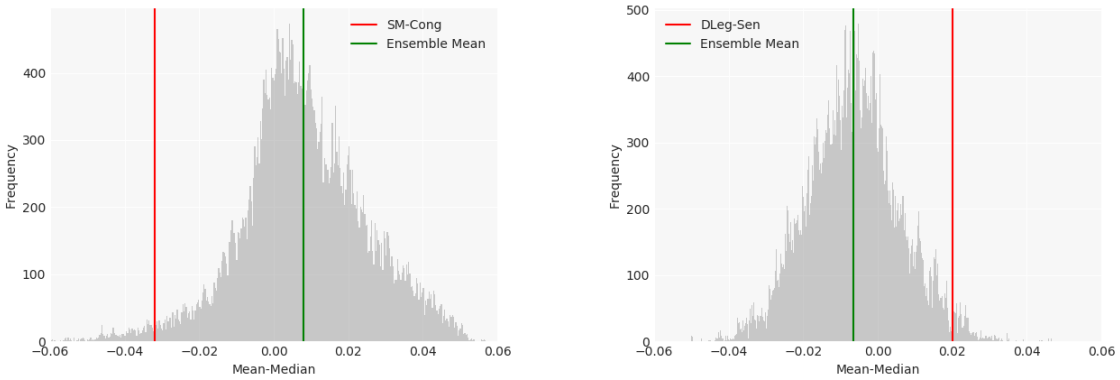

Figure 6: Neutral ensembles based on the mean-median gap tell roughly the same story as seats or EG.

3.2 Virginia's neutral baseline

The chief advantage of the ensemble method is that it allows us to establish a *baseline* for each districting metric in the state that fully incorporates the relevant rules and guidelines, as well as the underlying political geography—the physical, demographic, and electoral data from Virginia itself forms the population network from which districting plans are cut. This is extremely important, because the normal behavior of districts varies quite a bit from state to state and from one time period and legal context to another.

We have already seen the normal range of partisan metrics (seats, efficiency gap, and mean-median gap) above. Compactness and geographic splitting over the ensemble are recorded in Figure 7. Finally, district-level boxplots are shown in Figure 8: each party's share of the vote and the Black Voting Age Population percentage (BVAP%).

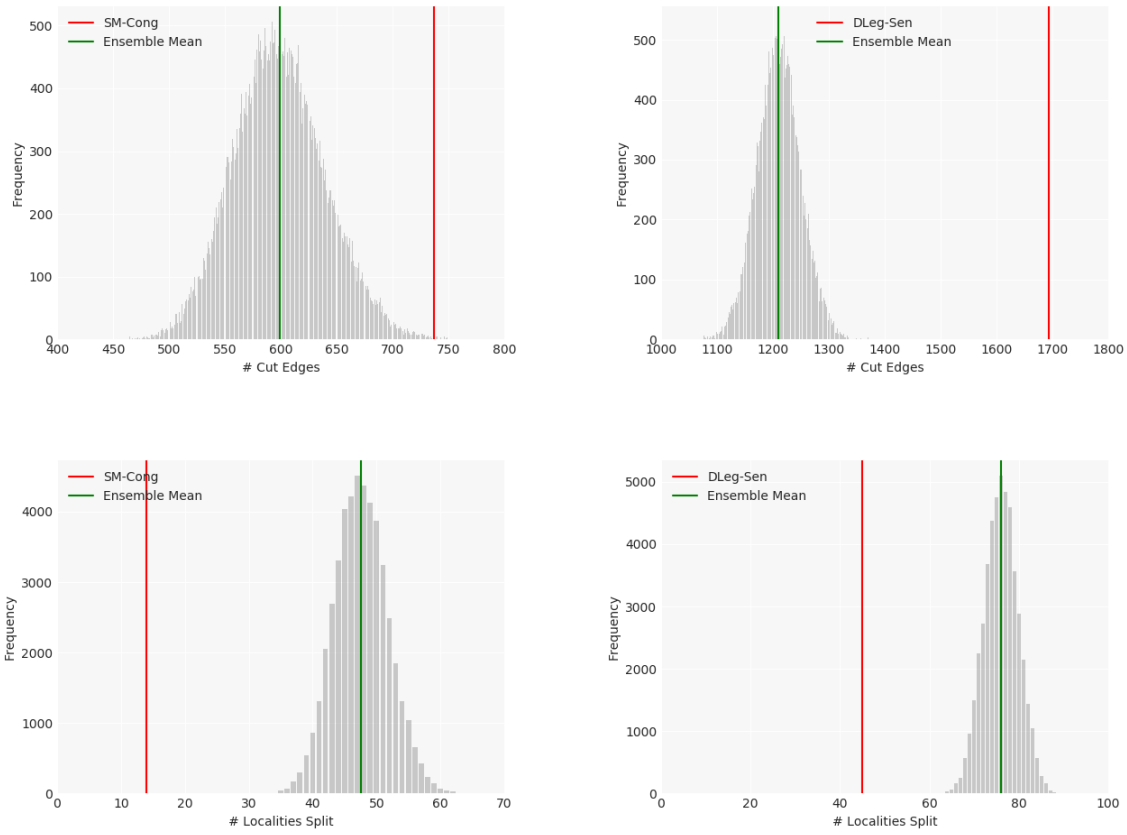

Figure 7: The enacted maps both split fewer localities, but on the other hand are a great deal less compact, than the bulk of the neutral ensembles. (Maps with more cut edges are more wildly shaped and less compact.) This provides corroboration that splitting was taken into account when making the maps. (Compare Figure 12, which suggests that a non-splitting priority *does not* explain the observed non-compactness.)

In a state like Virginia, with roughly 20% Black population and a fraught history of racism (that much in the news in 2019), safeguarding electoral opportunity for African-Americans in particular is a major component of every redistricting process, with the Voting Rights Act as a backstop. The assessment of Voting Rights Act compliance is a multi-factor matter that never simplifies to only an appropriate BVAP%, but requires a holistic assessment of "the totality of the circumstances," including histories of discrimination, primary election dynamics, and more. Nonetheless, BVAP% is certainly a significant element in Black communities' opportunity to elect candidates of choice. In Virginia, Maxwell Palmer's 2017 expert report in Bethune-Hill [19] establishes that a BVAP% of .45 statistically suffices to elect a preferred candidate in

all but one House district in the state, indeed with so much slack that performance in the low 40s would be expected. In addition, two congressional districts (VA-3 and VA-4) currently comfortably elect Black representatives with BVAP in the low 40s: our data show Bobby Scott's 3rd district and Don McEachin's 4th district both in the .4–.45 range (visible in Fig 8). For this reason, we have marked the BVAP plots with both the 50% line (indicating majority-Black VAP) and the 40% line (which has been demonstrated to be relevant for opportunity to elect).

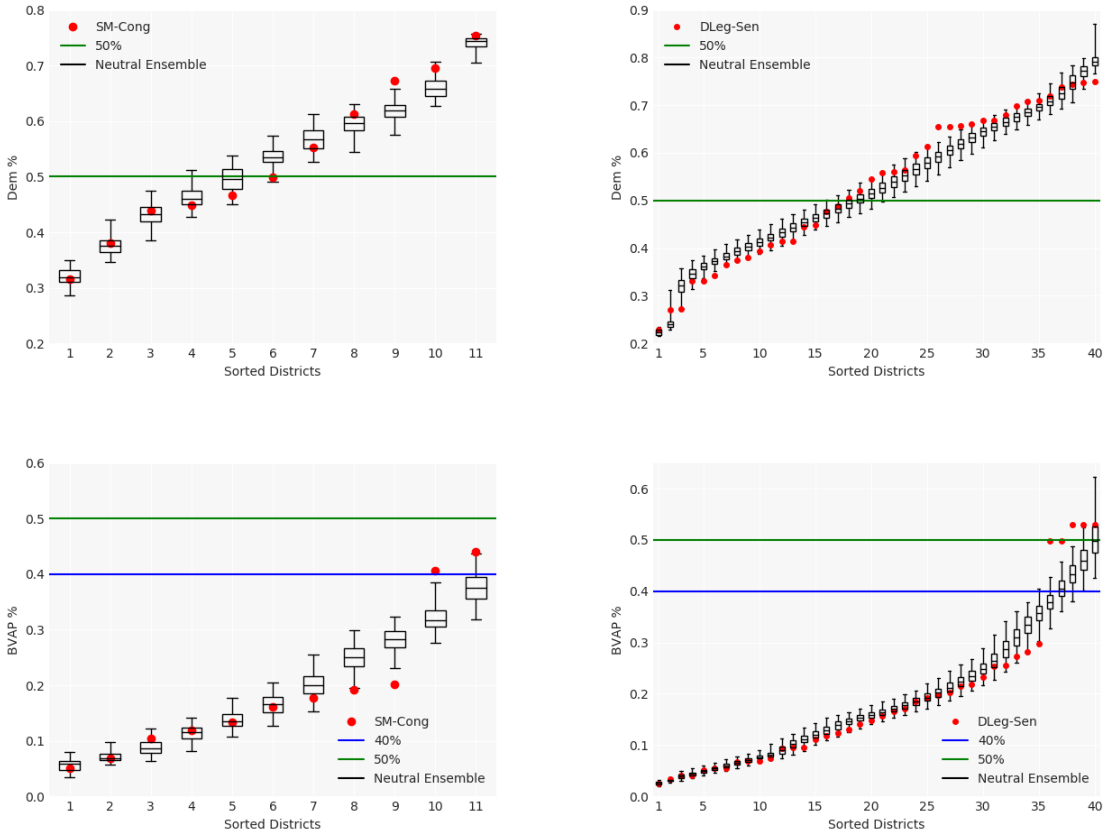

Figure 8: The Democratic share of the two-party vote (with respect to ATG 17) and the Black Voting Age Population of the enacted plans are plotted in red, compared to the baseline behavior of the neutral ensemble. Districts are sorted from lowest to highest Dem % or BVAP % in each plan to make them comparable across the ensemble. Boxes show 25th-75th percentile of scores found in the ensemble, and whiskers show 1st-99th. The plots make it clear that some efforts were made to comply with the Voting Rights Act.

Together, these plots illuminate some facts about the neutral tendency of compact, contiguous, population-balanced plans in Virginia. One finding is that the neutral baseline shows much closer parity between the parties than is the case in many other states. This is apparent no matter whether we consider deviation from seats-to-votes proportionality, efficiency gap, or mean-median score. Consider an election like ATG 17 in which one party has 53% of the votes. A proportionality norm dictates that 53% of the vote should secure 53% of the seats, while efficiency gap operates on a double-bonus norm, in which 53% of the votes should earn 56% of the seats. Mean-median deems a plan fair in this situation when the median district (the one that appears at the center of a sorted-district party share plot like the ones at the top of Fig 8) has 53% share. For our vote data, the vote-proportional share of seats would have been 5.8 in the Congressional case and 21.2 in the Senate, while the double-bonus share would have been 6.2 and 22.4. Across our ensemble, the average seats for Democrats in a neutral plan comes out close to 6.5 and 22. That means that the structural demographics of the state produce a very modest benefit for Democrats in Congressional districting and very

near parity in Senate plans.

For example, in efficiency gap terms, the neutral ensemble for Virginia's Congressional maps has a mean EG of roughly +.035, while the Senate ensemble averages to +.007. (To assess what counts as a large EG, recall that the authors present $EG = 0$ as ideal, while proposing a magnitude of over .08 as part of a legal test for detecting gerrymanders [21].) In other states, much larger efficiency gaps are observed on average: Pennsylvania's *typical* efficiency gap is over .1 in the Republican direction, while Massachusetts's Democratic advantage is as large in the other direction, even in the total absence of intentional gerrymandering [7, 10].

In other contexts, the primary application of ensembles would be to detect gerrymandering by comparing proposed plans against the neutral baseline. Our primary focus here is different: we seek to observe changes to that baseline as other conditions are layered in. To this end, we will compare the neutral ensembles to alternative ensembles modeling the potential effects of enforcing other rules and priorities on the map-drawing process. We will consider in turn modifying the rules for population deviation, prioritizing BVAP% over .4, and prioritizing fewer locality splits.

3.3 Population deviation

Before this amendment process began, Virginia has had one of the most restrictive rules in the country for population deviation in legislative maps, requiring deviation of no more than 1% from ideal size in House of Delegates districts and 2% for Senate districts. By contrast, the legal standard and the common practice is that the difference between largest and smallest districts be within 10% of ideal district size, with more deviation permitted to further legitimate state aims.[9] The new amendment text brings Virginia in line with actual current national practices, which are documented in [16].

By contrast, current practice takes Congressional population balance to the level of a fetish—only 13 states have a Congressional top-to-bottom deviation of more than *one person*, and only six states by more than 100 people. Only West Virginia exceeds one half of one percent.

It is important to remember that overly tight population constraints have frequently been used as excuses for poor behavior in the redistricting process. Consider the following passage from a University of Maryland reported story on the state's contested congressional district from the 2000 redistricting cycle:

> Former Secretary of State John T. Willis, who was in charge of the redistricting as chairman of the Governor's Redistricting Advisory Committee, said the committee did not mean for the 3rd District to look like it does. That's just how the numbers worked out, he said.
>
> "It's a very complex situation, and population is the No. 1 driving characteristic," Willis said. ..."All of our congressional districts don't deviate by more than one person." [3]

A seemingly related concern has been raised by legislators and observers: some worry that weakening the population deviation limit will lead to malapportionment for Virginia's citizens, particularly harming urban areas. That is because of the essentially universal trend that urban areas grow in population much faster than rural areas, which can lead to some districts having population twice as high as others by the last election conducted in their lifespan. This malapportionment drift is real; however, it is unrelated to loosening population deviation rules, except if there is some reason to believe that districts over ideal size would be systematically more likely to be created in urban areas. We find no evidence that any combination of rules (including a high priority on geographic splitting) would create this tendency automatically.

[9]The courts have traditionally considered a measure of deviation by difference from highest to lowest as a proportion of ideal size. Following the new Virginia convention, we instead bound the allowed largest and smallest size. These are of course related, because districts than can be no more than 5% above or below ideal will have maximum top to bottom deviation no more than 10%.

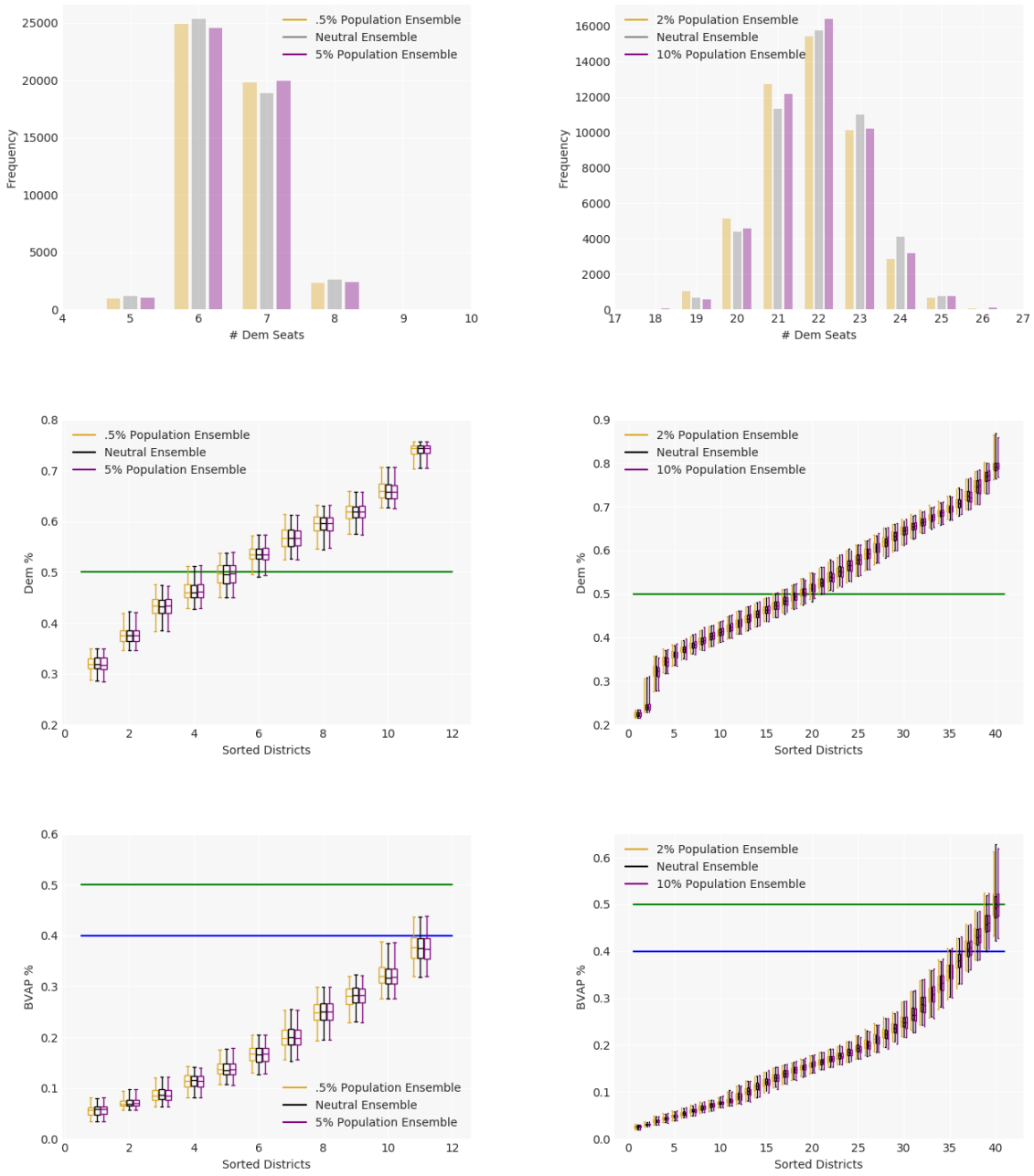

Figure 9: Here the neutral ensemble is compared to alternative ensembles generated with significantly tighter and weaker population bounds. There is strikingly little difference in partisan or racial outcomes.

The story told by these population plots is striking: even massively loosening the deviation does not allow the algorithm to find significant collections of plans that favor one party or the other. And the average tendency is remarkably robust: the Democratic seats mean changes by less than .02 seats even as the deviation threshold changes by a factor of ten. Differences in the BVAP bottom line are even more undetectable.

3.4 Racial balance

As described above in §3.1.1, our BVAP ensembles were constructed to preferentially adopt plans in which a fixed number of districts were pushed over a fixed BVAP%. It is fundamentally important to note that we do not use this setup to model best practices under the Voting Rights Act, but rather as a hypothesis of the principles followed in actually producing the enacted plans: two districts over .4 in the Congressional plan, and five districts over .5 in the Senate plan. The results of these runs are shown in Figure 10.

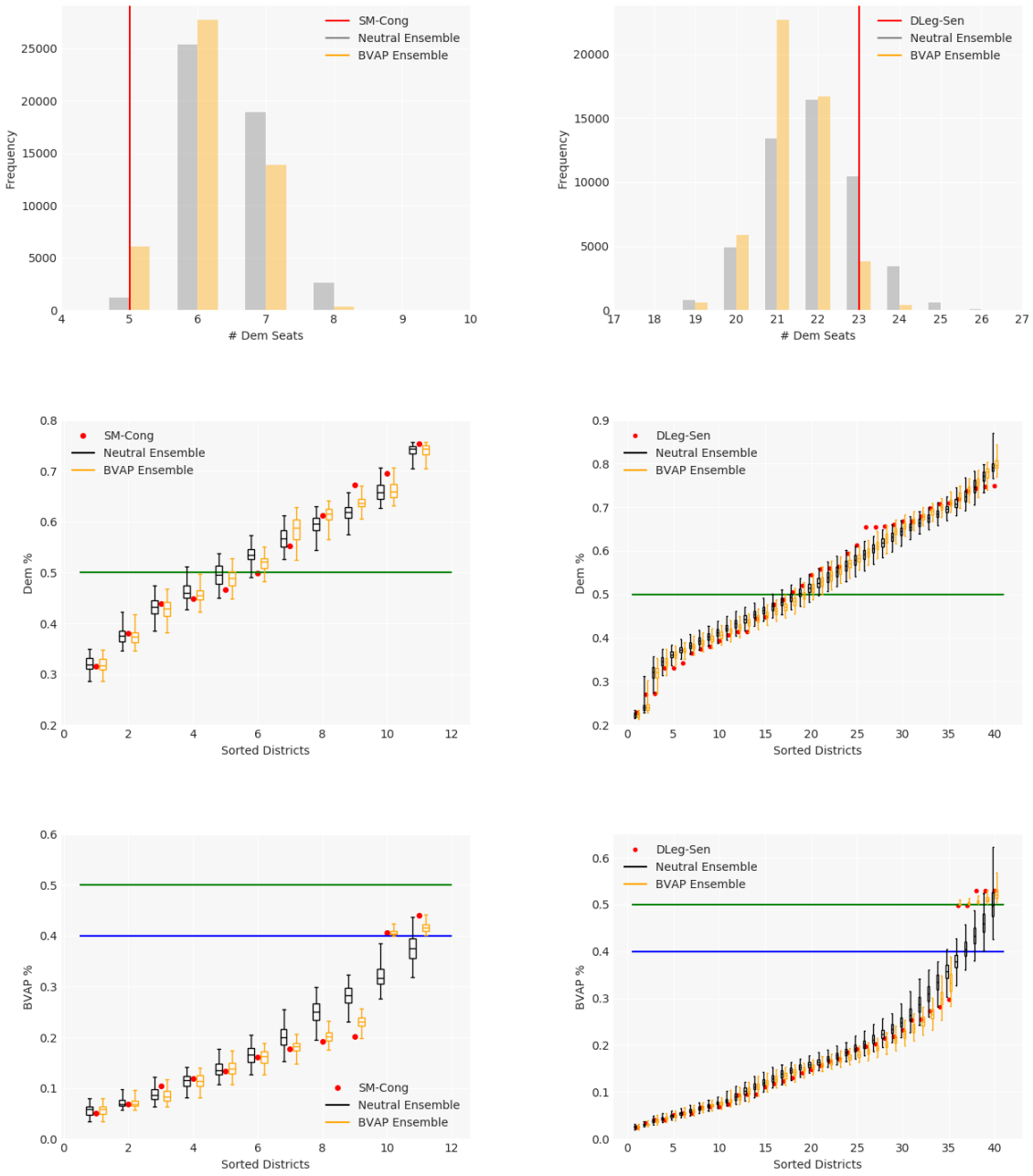

Figure 10: Statistics for an ensemble that preserves minority representation.

The findings here are, on one hand, that these models do very successfully explain the outlying racial

demographics in both plans. The BVAP statistics of the enacted Congressional and Senate plan (Fig 10, bottom row) look like outliers to the neutral ensemble, but reflect normal behavior in our BVAP ensemble.

On the other hand, these practices did not impose significant changes to the partisan bottom line. The seats averages do shift modestly, but they move in the direction of better proportionality of seats to votes.

3.5 Geographic splitting

One of the most important debates around criteria for the new commission is a possible priority on preservation of political boundaries, or minimizing county and city (locality) splits. One motivation for a splitting constraint is that cities and counties can be powerful communities of shared interest and sometimes identity, and that they will fare best when kept whole as a significant bloc of a representative's constituency. Another motivation is to put some limits on the seemingly boundless power of redistricters to carve up populations at will.

The question of county splitting played a significant role in the special master's plan in the House of Delegates as a result of the Bethune-Hill case, in the form of a self-imposed "potentially implicated county" constraint on plans [11, p3-4]. Consequently, Grofman's Congressional plan has only 14 locality splits, while a Congressional plan made without attention to splitting might have 40-60. A Senate plan has more districts, so we should expect more splits, and indeed the range of 70-85 splits is typical without any attempt to track them, while the legislature's Senate plan a number of splits in the low 40s.[10] These numbers are reflected in Figure 8. The fact that both plans have far fewer splits than the neutral ensembles is completely to be expected, because the neutral algorithm is locality-blind. This provides a good reminder of the general principle that algorithms do only what we tell them to, and a warning that leaving a relevant consideration out of the model can produce not just middling but poor results.

When we run an alternative Markov chain that prefers plans with ≤ 20 splits, the results are informative. One ancillary finding, as observed elsewhere, is that the splitting priority does not impose a cost on compactness, but instead improves the compactness expected in a typical plan (Fig 11).

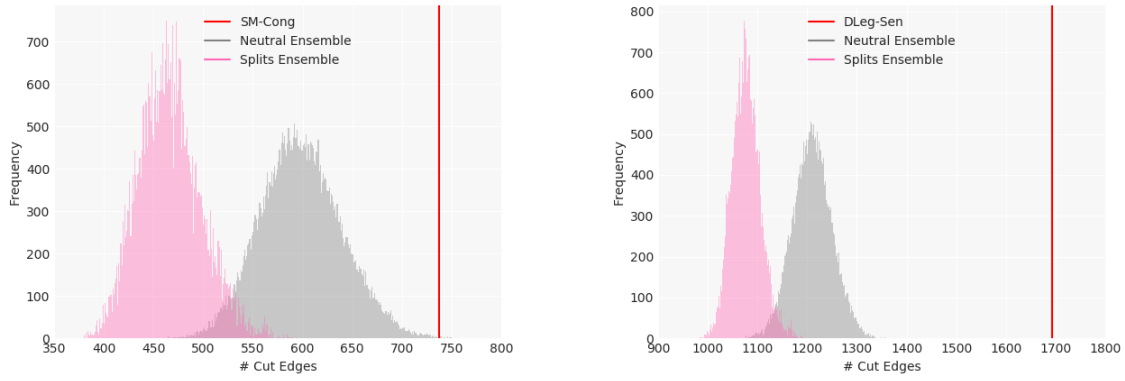

Figure 11: Compactness comparison. Note that the ensemble preserving locality boundaries scores *better* on the compactness measure without any change in the compactness settings.

Our main finding is that in partisan terms the splits ensemble is centered on the same baseline as the neutral ensemble, but with markedly lower variance. In other words, a priority on preserving county and city boundaries "chops off the tails of the bell curve" of party outcomes, making maps that structurally favor either party more statistically anomalous. This is seen most clearly in the seats plots from the top row of Figure 12.

[10] This is hard to count exactly because of the roundoff into precinct units during preprocessing.

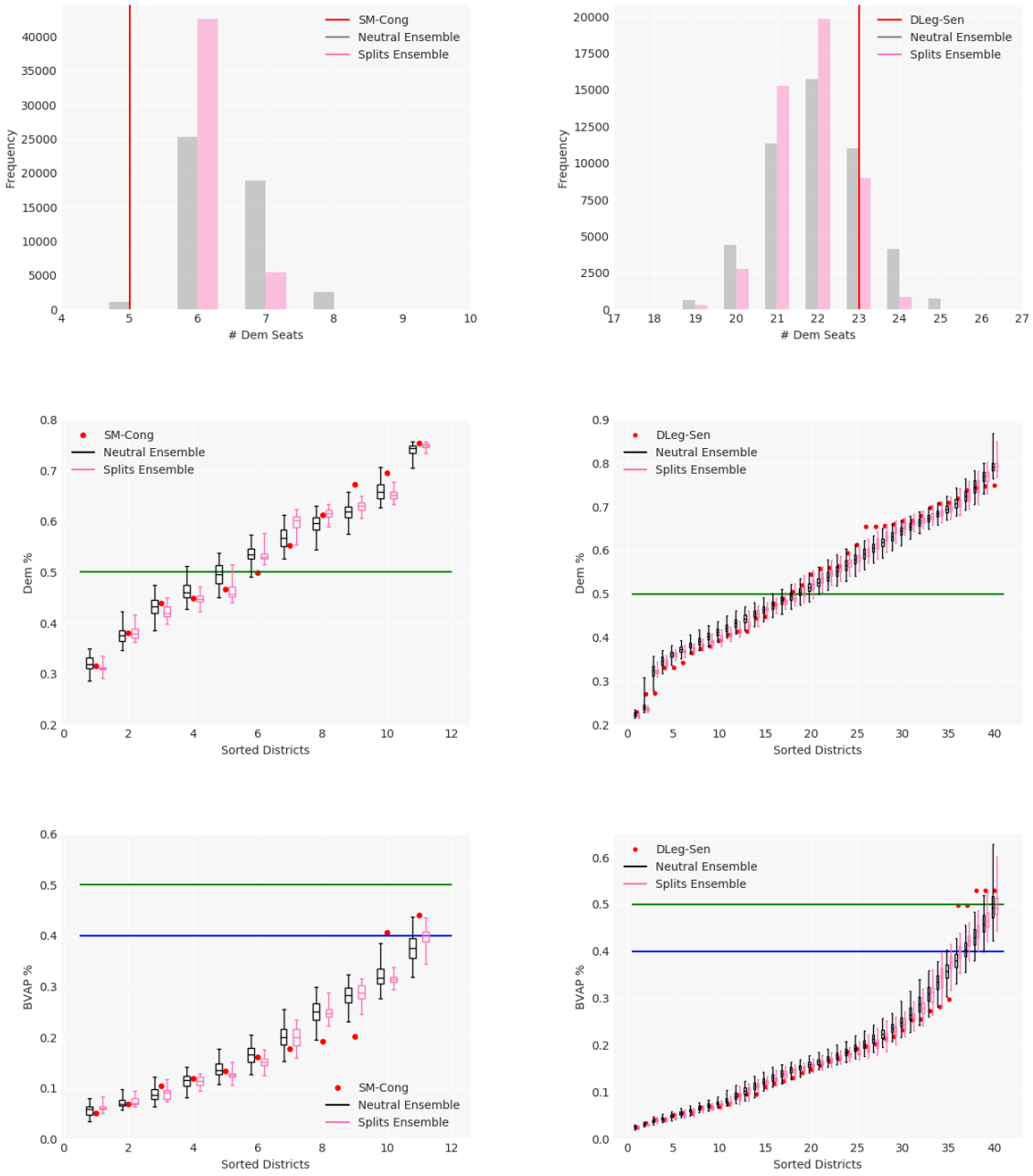

Figure 12: Baselines for a splits ensemble favoring ≤ 20 locality splits.

Notably, where there is a partisan difference in representation predicted by a priority on boundary preservation, it is both modest in size and mixed in direction. The expected Congressional representation moves by about .3 seats in favor of Republicans, whereas the expected Senate shift benefits Democrats by about .5 seats.

4 Conclusions

As [1] put it in their study of Virginia redistricting: "In the absence of legal alternatives against which to benchmark a redistricting authority's plan, the public cannot assess if alternatives exist that can better achieve various goals." We propose that robust ensemble analysis supplies this value and more, providing not only the existence of (many) plans which meet redistricting criteria at least as well as proposed plans, but a baseline for the properties of compliant plans. Furthermore, the use of ensembles makes it plain that there are not dozens, not thousands, but trillions upon trillions of different districting plans that satisfy the requirements and even uphold the priorities enumerated in law.

In Section 2, we reviewed possible compositional and procedural rules for the new independent commission. The current text of the amendment leaves too much room for partisan gamesmanship by proxy, and a clear role for unaffiliated members should be created. Another recommendation is the use of an outside statistical expert to offer guarantees that a proposed plan is not an extreme outlier in any salient metric. This kind of role would leave room for the commission to offer a justification for a plan's deviation from expected properties of plans created pursuant to state and federal rules.

In Section 3, we conduct experiments rooted in the particular political geography of Virginia to understand the impact and interactions of different potential districting criteria. For population deviation, we find remarkably little measurable effects of tighter or looser limits, within the scope of national practices. This leads us to recommend the adoption of a looser population balance rule (say, .05% for Congress and 5% for General Assembly), because it leaves more room for prioritizing other considerations such as minority opportunity to elect and the preservation of relevant boundaries.

In considering VRA concerns, we found that the statistical behavior of the enacted plans comports remarkably well with an ensemble designed to hit BVAP targets. These BVAP ensembles do not impose a cost on partisan fairness, by any of the standard measures.

Finally, a high priority on preserving localities intact does change the partisan picture, but it does so by virtually eliminating the most extreme partisan outcomes (in both directions) from occurring.[11] To the extent that the median partisan outcome budges at all, it moves toward better proportionality of seats to statewide votes.

The use of computer-generated ensembles—by algorithms that offer indications of representative sampling—is a very powerful tool for assessment of proposed plans as well as for studying the impact of possible rules as we do here. In closing, however, we emphasize that the role of human judgment in redistricting will never be supplanted by an algorithm. Strong knowledge of the state's terrain and communities is fundamental, and stubbornly hard to model. By setting baselines and bounds rather than seeking to select an optimal plan, these techniques acknowledge that reality.

Ongoing debate and deliberation about fairness is essential to a healthy democracy. In our view, ensembles of districting plans should be used to assess and perhaps to propose plans for consideration because computers may locate substantially different valid plans than humans ever found by hand. This puts computing in a crucial but supporting role in the search for democratic best practices.

[11] In fact, in the Congressional plan, the 5- and 8-seat Democratic outcomes are completely eliminated from the splits ensemble.

References

[1] Micah Altman and Michael McDonald, *A Half-Century of Virginia Redistricting Battles: Shifting from Rural Malapportionment to Voting Rights to Public Participation*. University of Richmond Law Review 771, 792–93 (2013).

[2] Amicus Brief of Mathematicians, Legal Scholars, and Students. https://mggg.org/amicus/

[3] Elysa Batista, Politics Makes Strange Bedfellows, Even Stranger Congressional Boundaries. (Feb 2004). http://www.newsline.umd.edu/politics/specialreports/elections04/fineline022004.htm

[4] Ballotpedia, *Redistricting in Virginia after the 2010 census*. https://ballotpedia.org/Redistricting_in_Virginia_after_the_2010_census

[5] Ballotpedia, Virginia State Senate elections, 2015. https://ballotpedia.org/Virginia_State_Senate_elections,_2015

[6] Brennan Center for Justice, *Legal filings in Bethune-Hill v. Virginia Board of Elections*. https://www.brennancenter.org/legal-work/bethune-hill-v-virginia-board-elections

[7] Daryl DeFord and Moon Duchin, *Political Geography and Pennsylvania Redistricting*, preprint.

[8] Moon Duchin, *Outlier analysis for Pennsylvania congressional redistricting*. https://mggg.org/uploads/md-report.pdf

[9] Moon Duchin and Bridget Tenner, *Discrete geometry for electoral geography*, arXiv:1808.05860, (2018).

[10] Moon Duchin, Taissa Gladkova, Eugene Henninger-Voss, Ben Klingensmith, Heather Newman, and Hannah Wheelen, Locating the representational baseline: Republicans in Massachusetts, arXiv:1810.09051, (2018).

[11] Bernard Grofman, Report of the Special Master, Bethune-Hill vs. Virginia State Board of Elections, December 2018. http://redistricting.dls.virginia.gov/2010/Data/Court%20Ordered%20Redistricting%20House/Grofman-Special%20Master%20Report%20in%20Golden%20Bethune-Hill-12-7-2018.pdf

[12] Justin Levitt, *Legal filings in Page v. Virginia Board of Elections*. http://redistricting.lls.edu/cases-VA.php#VA

[13] MGGG Preprocessing software. https://github.com/mggg/maup

[14] MGGG GerryChain software. https://github.com/gerrychain

[15] MGGG, *Comparison of Districting Plans for the Virginia House of Delegates*. https://mggg.org/VA-report.pdf

[16] National Conference of State Legislatures, *2010 Redistricting Deviation Table*. http://www.ncsl.org/research/redistricting/2010-ncsl-redistricting-deviation-table.aspx

[17] National Conference of State Legislatures, *Redistricting Commissions: Congressional Plans*. http://www.ncsl.org/research/redistricting/redistricting-commissions-congressional-plans.aspx

[18] New Jersey Voting Precinct Transparency Act. https://www.njleg.state.nj.us/2018/Bills/A5000/4564_I1.HTM

[19] Expert Report of Maxwell Palmer, Bethune-Hill v. Virginia State Board of Elections, August 2017.

[20] Princeton Gerrymandering Project Virginia Precincts. https://github.com/PrincetonUniversity/VA-gerrymander

[21] Nicholas O. Stephanopoulos and Eric M. McGhee, Partisan Gerrymandering and the Efficiency Gap, University of Chicago Law Review. 82, 831-–900, (2014).

[22] U.S. District Court for the Eastern District of Virginia, Memorandum Opinion: Bethune-Hill v. Virginia State Board of Elections. https://www.brennancenter.org/sites/default/files/legal-work/BH_v_VABoardofElections_Opinion_06.26.18.pdf

[23] Ellen Veomett, *Efficiency Gap, Voter Turnout, and the Efficiency Principle*. Election Law Journal Volume 17, Number 4 (2018), 249–263.

[24] Virginia Department of Legislative Services Districting Plans. https://redistricting.dls.virginia.gov/2010/RedistrictingPlans.aspx

[25] Virginia Final Amendment Bill SJ306 https://lis.virginia.gov/cgi-bin/legp604.exe?191+ful+SJ306ER+pdf

[26] Virginia Bill SJ274 Full Text http://lis.virginia.gov/cgi-bin/legp604.exe?191+ful+SJ274+pdf

A Text of criteria from Senate Joint Resolution No. 274

1. Legislative and congressional districts shall have a total population as nearly equal as practicable to the ideal population for such districts. Deviations from equal population may be permitted to achieve other redistricting criteria specified in this subsection, but in no case shall a congressional district have a total population that varies by more than one half of one percent from the ideal congressional district population nor shall an electoral district for the Senate or the House of Delegates have a total population that varies by more than five percent from the ideal district population for a Senate or House of Delegates district, respectively.

2. Legislative and congressional districts shall not be drawn to abridge or deny the ability of substantial racial or ethnic minority communities to elect representatives of their choice.

3. Legislative and congressional districts shall not be drawn to favor or disfavor any political party, incumbent legislator or member of Congress, or individual or entity.

4. Legislative and congressional districts shall be drawn to respect existing political boundaries of counties, cities, and towns. To the extent practicable, and except in cases where the political subdivision is larger than an electoral district, each political subdivision shall be wholly contained within a single electoral district. Where a departure from existing political boundaries is necessary to comply with one or more of the redistricting criteria specified in this subsection, the district lines shall be drawn giving consideration to natural geographic boundaries, such as mountains and rivers; physical boundaries, such as major roads and residential subdivisions; and communities of interest.

5. Legislative and congressional districts shall be composed of compact and contiguous territory. Districts divided by water may be deemed contiguous but connections by water running downstream or upriver are not permissible. Districts shall not have irregular or contorted perimeters, unless justified by one or more of the redistricting criteria specified in this subsection, including adherence to existing political boundaries or natural geographic boundaries.