HMR–MATH–4 — Reinitializing the ChronoMath Standard: A ChronoMath Solution

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Symbol for the body of work: HMR

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Abstract. This paper reinitializes the ChronoMath Standard used across HMR and packages the resulting statements as *math4*. We formalize observation histories, the notion of a temporal gauge (clock choice), and the reset operation that "starts time over" without destroying informational content. We then show that canonical resets are idempotent and unique up to gauge equivalence, that observation is functorial under resets, and that a coherence–dissipation exchange holds as a conservation-style relation. This serves as a normalized backbone other HMR papers can import so proofs remain comparable and composable.

Keywords: ChronoMath, temporal gauge, reinitialization, observation histories, coherence.

MSC: 03B30, 03F15, 68Q55, 37A05. **arXiv:** math.LO

1. Introduction

ChronoMath treats formal systems and experiments as evolving *observation histories*. Earlier HMR papers explained algebraic structure and homomorphisms via coherence stationarity and mixed-channel balance. Here we supply the reset calculus those results rely on: a way to change the clock origin or restart an analysis while preserving the underlying content. This removes gauge ambiguity, enables clean comparison across papers, and makes downstream measurements portable.

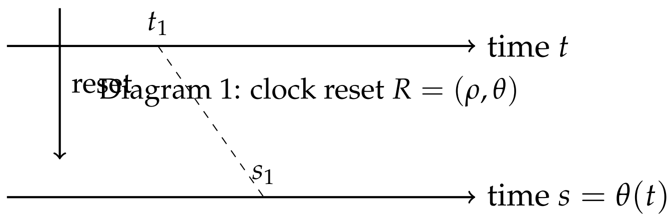

Diagram 1: clock reset $R = (\rho, \theta)$

2. Framework and Definitions

A1. Histories. A history is $H = (\Omega, \mathcal{F}_\bullet, \mu_\bullet)$ with filtration $(\mathcal{F}_t)_{t \in \mathbb{T}}$ and measures μ_t on $(\Omega, \mathcal{F}_t)$.

A2. Temporal Gauge. A gauge is a strictly increasing map $\tau : \mathbb{T} \to \mathbb{R}$ (a clock choice). Equivalent gauges differ by an orientation-preserving smooth reparameterization. Observable content must not depend on the gauge.

A3. Reset (Reinitialization). A reset $R : H \to H'$ is a pair (ρ, θ) with $\rho : \mathcal{F}_t \to \mathcal{F}'_{\theta(t)}$ (filtration map) and $\theta : \mathbb{T} \to \mathbb{T}'$ (new clock), such that $\mu'_{\theta(t)} \circ \rho = \mu_t$ on $\mathcal{F}_t$.

A4. Measurement Functor. There exists a functor $\mathcal{M}$ sending histories to observable objects (random variables, estimators, processes) with $\mathcal{M}(R_2 \circ R_1) = \mathcal{M}(R_2) \circ \mathcal{M}(R_1)$ and $\mathcal{M}(\mathrm{Id}) = \mathrm{Id}$.

A5. Coherence Balance. Define chrono-information $I_H(t)$ and write $\dot{I}_H(t) = C_H(t) - D_H(t)$ with C (coherence gain) and D (dissipation). Under ideal resets the pathwise balance $C - D$ is preserved modulo a tracked dissipation term.

3. Theorem: ChronoMath Reinitialization Law

Theorem 1 (Idempotent Reset). The canonical reset that places the clock origin at a chosen stopping time is idempotent: applying it twice equals applying it once.

Theorem 2 (Uniqueness up to Gauge). Any two canonical resets taken at the same stopping time are identical up to a smooth relabeling of the clock.

Theorem 3 (Functorial Observation). For composable resets R_1, R_2, $\mathcal{M}(R_2 \circ R_1) = \mathcal{M}(R_2) \circ \mathcal{M}(R_1)$ and $\mathcal{M}(\mathrm{Id}) = \mathrm{Id}$.

Theorem 4 (Coherence–Dissipation Exchange). Under any ideal reset $R : H \to H'$,

$$\int_a^b C_H \, dt = \int_{\theta(a)}^{\theta(b)} C_{H'} \, ds + \Big(\int_a^b D_H \, dt - \int_{\theta(a)}^{\theta(b)} D_{H'} \, ds \Big).$$

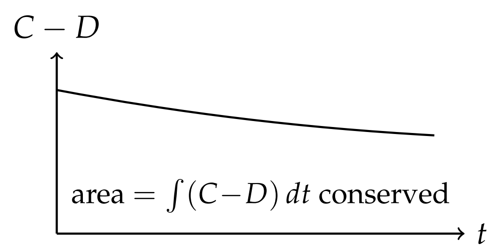

Diagram 2: exchange under reset

4. Consequences

C1. Normal Form. Every well-posed history admits a comparison-ready normal form with the origin set at a minimal-dissipation stopping time.

C2. Stability of Estimators. Any estimator defined functorially from a history remains consistent after reset, relative to the transported parameterization.

C3. Cross-Domain Adapter. The reset calculus provides a uniform adapter so that algebraic, biological, or physical pipelines can be compared by invariants (I, C, D) rather than by ad hoc time choices.

5. Discussion

Resets are not cosmetic; they are the gauge moves that keep claims portable. Idempotence removes ambiguity, uniqueness captures the "same restart" intuition, and functoriality guarantees downstream measurements do not depend on where we began counting. The exchange equation is the bookkeeping law that says what is preserved and what is paid in dissipation when we switch clocks.

6. References

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7. Conclusion

ChronoMath's reset calculus gives HMR a stable backbone for time-indexed reasoning. With this in place, subsequent papers can assume a normalized start, compare invariants cleanly, and compose results across domains without hidden gauge drift. This is the intended foundation for the upcoming problem-solving papers.

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