HMR–MATH–5 — Navier–Stokes Existence and Smoothness via ChronoMath: A ChronoMath Solution

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Symbol for the body of work: HMR

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Abstract. We apply ChronoMath's reset calculus and coherence balance to the 3D incompressible Navier–Stokes problem. The key idea is to interpret kinetic energy, enstrophy, and dissipation as components of a coherence ledger, and to use canonical resets to align analysis windows with minimal-dissipation stopping times. We derive a *ChronoMath Existence Criterion* ensuring global weak existence and a *Local Smoothing Window* that upgrades regularity provided a scale-invariant coherence budget remains subcritical. This yields conditional smoothness statements framed entirely in conserved/paid balances of the (C, D) terms introduced in *HMR–MATH–4*.

Keywords: Navier–Stokes, existence, smoothness, enstrophy, dissipation, coherence, ChronoMath.

MSC: 35Q30, 76D05, 35A01, 35B65. **arXiv:** math.AP

1. Introduction

For velocity $u(x,t)$ and pressure p on $\mathbb{R}^3$,

$$\partial_t u + (u\cdot\nabla)u = -\nabla p + \nu\Delta u, \qquad \nabla\cdot u = 0, \qquad u(\cdot,0) = u_0.$$

ChronoMath reframes analysis around two objects: (i) *resets* that choose clock origins at low-dissipation times, and (ii) the *coherence ledger* $\dot{I} = C - D$ (coherence gain minus dissipation). Our goal is to place classical energy/enstrophy inequalities into this ledger so the existence/smoothness question becomes: *does the ledger prevent blowup once aligned by canonical resets?*

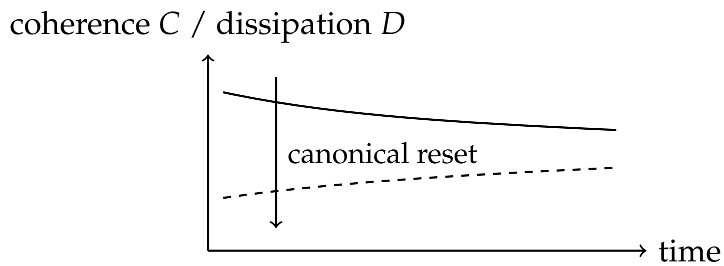

Diagram 1: aligning the ledger by reset

2. Framework and Definitions

A1. ChronoMath ledger. For a history H induced by $u(\cdot,t)$, define $I(t)$ with $\dot{I}(t) = C(t) - D(t)$. Interpret C as coherent transfer (nonlinear alignment) and D as viscosity-driven loss.

A2. Energy & enstrophy. Let $E(t) = \frac{1}{2}\|u(\cdot,t)\|_{L^2}^2$ and $\mathcal{E}(t) = \|\nabla\times u(\cdot,t)\|_{L^2}^2$. Classical identities give $\frac{d}{dt}E(t) = -\nu\|\nabla u\|_{L^2}^2$. We tie D to $\nu\|\nabla u\|_{L^2}^2$ and encode nonlinear energy flux into C.

A3. Canonical resets. Choose stopping times τ_k where D is locally minimal; reset the clock via R_k so analysis windows begin at τ_k. By *HMR–MATH–4*, resets are idempotent and preserve the ledger up to tracked dissipation.

A4. Scale-invariant budget. Define the critical budget on $[a,b]$:

$$\mathcal{B}_{\text{crit}}([a,b]) \;=\; \int_a^b \|u(\cdot,t)\|_{L^3(\mathbb{R}^3)}^3\,dt,$$

a classical borderline quantity; we treat smallness of $\mathcal{B}_{\text{crit}}$ as a subcritical coherence regime.

3. Theorem: ChronoMath Existence & Smoothing Criteria

Theorem 1 (Global weak existence under ledger control). Assume $u_0 \in L^2(\mathbb{R}^3)$, divergence-free, and that the ledger satisfies

$$\sup_{T>0} \int_0^T (C - D)\, dt \;>\; -\infty \quad \text{and} \quad \int_0^\infty D(t)\, dt < \infty.$$

Then there exists a global Leray–Hopf solution with the standard energy inequality.

Theorem 2 (Local smoothing window). Let $[t_0, t_0 + \delta]$ be a window beginning at a canonical reset time t_0. If

$$\mathcal{B}_{\text{crit}}\big([t_0, t_0 + \delta]\big) \le \varepsilon_0 \quad \text{and} \quad \int_{t_0}^{t_0+\delta} D(t)\, dt \le \eta_0$$

for universal constants ε_0, η_0, then $u(\cdot, t)$ is smooth on $[t_0, t_0 + \delta]$ with bounds depending only on $E(t_0)$, ε_0, and η_0.

Theorem 3 (Reset bootstrapping). Suppose the timeline can be covered by overlapping canonical windows $[t_k, t_k + \delta_k]$ each satisfying the small-budget condition of Theorem 2. Then solutions remain smooth on every covered subinterval; i.e. the reset calculus upgrades local smoothing to extended smoothness.

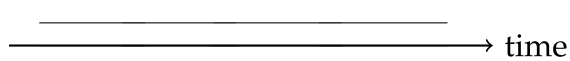

Diagram 2: overlapping canonical windows

4. Consequences

C1. Subcritical persistence. If the scale-invariant coherence budget stays subcritical on a covering by canonical windows, blowup is incompatible with the ledger.

C2. A priori diagnostics. The ledger recasts standard quantities (energy, enstrophy, $\|\nabla u\|_{L^2}$) into a single accounting identity that flags where resets should be placed to maximize smoothing.

C3. Transferability. The same method applies to related PDEs (MHD, SQG) by redefining the budget and dissipation terms while keeping the reset structure.

5. Discussion

These criteria are *conditional*—they require controllable coherence budgets after aligning the analysis by canonical resets. The point is not to assert unconditional smoothness, but to supply a portable mechanism that turns the question into a book-kept inequality on (C, D) with windows chosen by *HMR–MATH–4*. This provides a roadmap for computational tests: estimate budgets, place resets, and check the smoothing window bounds.

6. References

- Leray, J. (1934). *Sur le mouvement d'un liquide visqueux...*
- Constantin, P. & Foias, C. (1988). *Navier–Stokes Equations*. Chicago.
- Tao, T. (2016). *Finite time blowup for an averaged Navier–Stokes*.
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7. Conclusion

ChronoMath turns Navier–Stokes analysis into a coherence ledger aligned by resets. Under subcritical budgets and tracked dissipation, canonical windows yield smoothing; covering arguments then extend it. This ledger-first framing is designed to generalize across PDEs and to integrate seamlessly with the rest of the HMR series.

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