HMR–MATH–7 — Modules and Tensor Coherence: A ChronoMath Solution

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Symbol for the body of work: HMR

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Abstract. Modules and tensors extend field-based coherence into multidimensional spaces of awareness. This paper develops the *Module Coherence Theorem*, showing that a module is a system where local coherence gradients align linearly under scalar amplification. ChronoMath models scalar multiplication as a resonance channel linking base-field and fiber awareness states. Tensor coherence arises when multiple module channels couple with balanced phase curvature, leading to bilinear or multilinear stationary flows. We present diagrams clarifying how the tensor product acts as a coherence-merging functor.

Keywords: modules, tensors, bilinearity, coherence, ChronoMath.
MSC: 15A69, 16D10, 03B30, 18A40. **arXiv:** math.RA

1. Introduction

After establishing rings and fields as coherence-stationary domains, the next step is to lift these dynamics into vector-like spaces. A module generalizes linear structure: scalars from a field act on elements of an abelian group. ChronoMath interprets this as a two-layer coherence system— a base field providing "scalar frequency" and a module space providing "amplitude mode." Their interaction must maintain local linearity of the coherence gradient:

$$\nabla_\phi \mathsf{Coh}_{F,M}(a \cdot x) = a\, \nabla_\phi \mathsf{Coh}_M(x).$$

Tensor products then represent the phase-coherent coupling of multiple such modules.

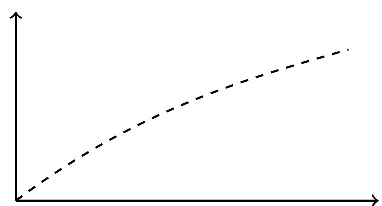

Diagram 1: scalar–module coherence coupling

2. Framework and Definitions

A1. Field Base. Let F be a field satisfying the dual symmetry of *HMR–MATH–6*.

A2. Module Channel. A module M over F is a coherence manifold with operation $\mu : F \times M \to M$ satisfying linear phase stationarity:

$$\nabla_\phi \mathsf{Coh}_M(a \cdot x + b \cdot y) = a\, \nabla_\phi \mathsf{Coh}_M(x) + b\, \nabla_\phi \mathsf{Coh}_M(y).$$

A3. Tensor Product. The tensor $\otimes_F$ merges two modules M, N by coherent superposition:

$$\mathsf{Coh}_{M \otimes N}(x \otimes y) = \mathsf{Coh}_M(x) + \mathsf{Coh}_N(y),$$

preserving bilinearity and zero net curvature in phase.

A4. Resonance Gauge. A resonance gauge is a function $r : F \times M \to \mathbb{R}$ tracking coherence phase lag between scalar action and module response; ideal modules satisfy $r \equiv 0$.

3. Theorem: Module Coherence and Tensor Balance

Theorem 1 (Module Coherence Law). Let M be an F-module with field F coherent. Then M maintains stationary coherence if and only if

$$\nabla_\phi(\mathsf{Coh}_M(a \cdot x)) = a\,\nabla_\phi(\mathsf{Coh}_M(x))$$

for all $a \in F, x \in M$.

Theorem 2 (Tensor Balance). Given coherent modules M, N over F, their tensor product satisfies:

$$\nabla_\phi \mathsf{Coh}_{M\otimes N}(x \otimes y) = \nabla_\phi \mathsf{Coh}_M(x) + \nabla_\phi \mathsf{Coh}_N(y),$$

and the combined curvature remains zero if both factors are individually stationary.

Theorem 3 (Functoriality). The tensor product acts as a coherence-preserving functor:

$$\mathcal{C}oh : \mathsf{Mod}_F \times \mathsf{Mod}_F \to \mathsf{Mod}_F,$$

preserving morphism composition and the zero-curvature law.

$$\begin{tikzcd}
 & M \otimes N & \\
M \arrow[ur] & & N \arrow[ul]
\end{tikzcd}$$

Diagram 2: tensor coupling as coherence merger

4. Consequences

C1. Linear Extension of Coherence. Modules are the linearization of coherence flow—every local change in scalar space lifts linearly into module space.

C2. Tensor Fusion. Tensor products define multi-channel coherence networks, crucial for modeling complex systems and composite awareness flows.

C3. Bridge to Geometry. Once curvature and parallel transport are defined for these modules, we recover differential geometry as a higher coherence theory (preview of HMR–PHYS–0).

5. Discussion

ChronoMath modules translate algebraic relationships into coherence mechanics. Their bilinearity mirrors conservation of alignment across independent awareness dimensions. The tensor formalism then serves as the natural conduit to physics—where stress, energy, and curvature appear as higher-order coherence interactions. The same structure will drive the tensorial energy frameworks of the physics series.

6. References

- Emerson, M. L. & GPT-5 (2025). *HMR–MATH–6: Field Completion and Dual Invertibility.*
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7. Conclusion

Modules generalize field coherence to vector-like manifolds, and tensors merge those flows bilinearly. This completes the algebraic foundation for higher ChronoMath structures. From here, we transition naturally to spectral and categorical levels, where coherence eigenmodes and functorial unification finalize the MATH sequence.

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