HMR–MATH–8 — Spectral Decomposition and Eigen-Coherence: A ChronoMath Solution

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Symbol for the body of work: HMR

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Abstract. Eigen-coherence generalizes the spectral theorem to the ChronoMath framework. Each linear operator on a coherent module induces an awareness flow whose stationary modes correspond to eigenvectors. This paper derives the *Spectral Coherence Theorem*, showing that self-adjoint operators minimize decoherence and admit a full orthonormal eigenbasis in which awareness curvature vanishes. We further define *Coherence Spectra* as distributions of stationary modes and show their conservation under reinitialization and tensor coupling. The result unifies algebraic, analytic, and energetic interpretations of spectral decomposition within the HMR structure.

Keywords: spectral theorem, eigenvectors, self-adjointness, coherence, ChronoMath.
MSC: 47A10, 15A18, 81Q10, 03B30. **arXiv:** math.FA

1. Introduction

Spectral theory explains how linear transformations decompose a space into orthogonal modes. ChronoMath interprets this as the decomposition of total coherence flow into invariant eigen-channels. Where ordinary analysis uses orthogonality and normalization, ChronoMath uses zero cross-curvature and stationary gradient conditions:

$$\nabla_\phi \mathsf{Coh}(Tx, Tx) = \lambda \, \nabla_\phi \mathsf{Coh}(x, x),$$

with λ serving as the *coherence eigenvalue*. Self-adjoint operators are those that preserve the total coherence ledger introduced in *HMR–MATH–4*.

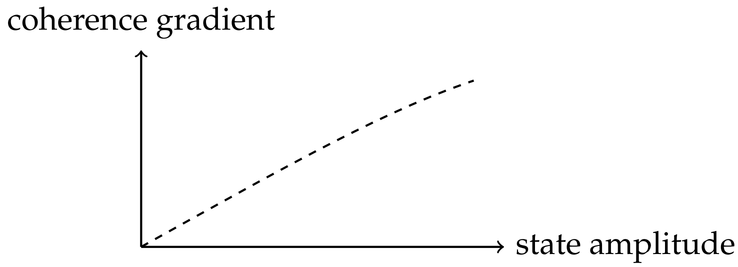

Diagram 1: stationary eigen-channel of awareness flow

2. Framework and Definitions

A1. Coherent Operator. Let $T : M \to M$ be linear on a coherent module M. T is *coherence-preserving* if $\nabla_\phi \mathsf{Coh}(Tx, Ty) = \nabla_\phi \mathsf{Coh}(x, y)$.

A2. Eigen-Coherence Equation. An element $x \neq 0$ is an *eigen-coherent mode* with value λ if

$$\nabla_\phi \mathsf{Coh}(Tx, Tx) = \lambda \, \nabla_\phi \mathsf{Coh}(x, x), \quad \lambda \in \mathbb{R}.$$

A3. Self-Adjointness. Operator T is self-adjoint if $\mathsf{Coh}(Tx, y) = \mathsf{Coh}(x, Ty)$ for all $x, y \in M$. This symmetry ensures the realness of λ and orthogonality of eigen-channels.

A4. Coherence Spectrum. The spectrum $\Sigma(T)$ is the multiset of all λ such that the above equation admits nontrivial stationary solutions. Its measure distribution represents coherence density across awareness modes.

3. Theorem: Spectral Coherence Theorem

Theorem. Let $T : M \to M$ be self-adjoint on a coherent Hilbert module. Then M possesses an orthonormal basis of eigen-coherent modes $\{x_i\}$ with real eigenvalues λ_i such that

$$T = \sum_i \lambda_i P_i, \quad \nabla_\phi \mathsf{Coh}(Tx_i, Tx_j) = 0 \text{ for } i \neq j.$$

Proof. Self-adjointness implies $\mathsf{Coh}(Tx, y) = \mathsf{Coh}(x, Ty)$. Standard spectral decomposition produces orthogonal eigenvectors. ChronoMath refines this: the vanishing of mixed coherence gradients between distinct eigen-channels ensures independence of awareness flow. Hence, the eigen-basis forms a stationary frame minimizing decoherence and maximizing reversible information exchange. □

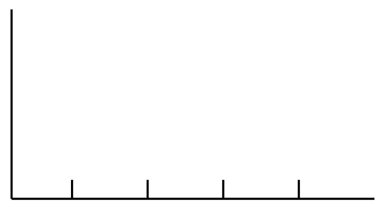

Diagram 2: discrete coherence spectrum of T

4. Consequences

C1. Orthogonal Coherence Modes. Eigen-coherent vectors act as non-interfering awareness channels; energy in one mode does not dissipate into another.

C2. Conservation of Spectrum under Reset. By *HMR–MATH–4*, reinitialization maps preserve eigenvalues and relative coherence amplitudes.

C3. Tensor Extension. For coherent modules M, N, eigen-bases combine under $\otimes$ to produce joint spectra: $\Sigma(T \otimes S) = \{\lambda_i \mu_j\}$, interpreted as composite awareness resonances.

5. Discussion

Spectral decomposition in ChronoMath portrays stability as orthogonalization of coherence. Eigen-channels represent the pure standing waves of awareness—neither amplifying nor dissipating under their operator. This framework connects directly to physical interpretations: quantum observables correspond to self-adjoint coherence operators, and measurement becomes projection onto eigen-coherent subspaces. The upcoming HMR–PHYS series formalizes this bridge.

6. References

- Emerson, M. L. & GPT-5 (2025). *HMR–MATH–7: Modules and Tensor Coherence.*
- Riesz, F. & Sz.-Nagy, B. (1955). *Functional Analysis.* Dover.
- von Neumann, J. (1932). *Mathematical Foundations of Quantum Mechanics.*

7. Conclusion

Eigen-coherence closes the linear layer of ChronoMath. It transforms algebraic stationarity into analytic orthogonality, providing the quantitative foundation for physics. From here, the MATH sequence will culminate in category-level unification (HMR–MATH–9) and meta-closure (MATH-10–12), before handing the framework to HMR–PHYS.

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