$$A = \begin{bmatrix} 5 & 2 \\ -3 & 10 \end{bmatrix}$$

Eigenvalues: $\det(A - \lambda I) = 0$

$$\Rightarrow \det\left(\begin{bmatrix} 5 & 2 \\ -3 & 10 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\Rightarrow \det\left(\begin{bmatrix} 5-\lambda & 2 \\ -3 & 10-\lambda \end{bmatrix}\right) = 0$$

$$\Rightarrow (5-\lambda)(10-\lambda) - (-3)(2) = 0$$

$$\Rightarrow \lambda^2 - 15\lambda + 56 = 0$$

$$\Rightarrow (\lambda - 7)(\lambda - 8) = 0$$

$$\Rightarrow \lambda_1 = 7, \quad \lambda_2 = 8$$

Eigenvector for $\lambda_1 = 7$: $A v_1 = \lambda_1 v_1$

$$\Rightarrow \begin{bmatrix} 5 & 2 \\ -3 & 10 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 7 \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 5a + 2b &= 7a \quad \text{———} (1) \\ -3a + 10b &= 7b \quad \text{———} (2) \end{aligned}$$

$$\Rightarrow a = b$$

$$\therefore v_1 = \begin{bmatrix} a \\ a \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Eigenvector for $\lambda_2 = 8$: $A v_2 = \lambda_2 v_2$

$$\Rightarrow \begin{bmatrix} 5 & 2 \\ -3 & 10 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 8 \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 5a + 2b &= 8a \quad \text{———} (1) \\ -3a + 10b &= 8b \quad \text{———} (2) \end{aligned}$$

$$\Rightarrow b = 3/2\, a$$

$$\therefore v_2 = \begin{bmatrix} a \\ 3/2\, a \end{bmatrix} = a \begin{bmatrix} 1 \\ 3/2 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & 5 \\ 7 & 2 \end{bmatrix}$$

Eigenvalues: $\det(A - \lambda I) = 0$

$$\det\left(\begin{bmatrix} 6 & 5 \\ 7 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 6-\lambda & 5 \\ 7 & 2-\lambda \end{bmatrix}\right) = 0$$

$$(6-\lambda)(2-\lambda) - (7)(5) = 0$$

$$\lambda^2 - 8\lambda - 23 = 0$$

$$\lambda = \frac{8 \pm \sqrt{64 + 4 \times 23}}{2} \Rightarrow \begin{array}{l} \lambda_1 = 4 + \sqrt{39} \\ \lambda_2 = 4 - \sqrt{39} \end{array}$$

Eigenvector for $\lambda_1$ : $Av_1 = \lambda_1 v_1$

$$\Rightarrow \begin{bmatrix} 6 & 5 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = (4+\sqrt{39})\begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \begin{array}{ll} 6a + 5b = (4+\sqrt{39})a & \text{\_\_\_\_\_\_ (1)} \\ 7a + 2b = (4+\sqrt{39})b & \text{\_\_\_\_\_\_ (2)} \end{array}$$

$$\Rightarrow a = \frac{(2+\sqrt{39})}{7} b$$

$$v_1 = \begin{bmatrix} \frac{2+\sqrt{39}}{7} b \\ b \end{bmatrix} = b \begin{bmatrix} (2+\sqrt{39})/7 \\ 1 \end{bmatrix}$$

Eigenvector for $\lambda_2$ : $Av_2 = \lambda_2 v_2$

$$\Rightarrow \begin{bmatrix} 6 & 5 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = (4-\sqrt{39})\begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \begin{array}{ll} 6a + 5b = (4-\sqrt{39})a & \text{\_\_\_\_\_\_ (1)} \\ 7a + 2b = (4-\sqrt{39})b & \text{\_\_\_\_\_\_ (2)} \end{array}$$

$$\Rightarrow a = \frac{(2-\sqrt{39})}{7} b$$

$$v_2 = \begin{bmatrix} \frac{2-\sqrt{39}}{7} b \\ b \end{bmatrix} = b \begin{bmatrix} (2-\sqrt{39})/7 \\ 1 \end{bmatrix}$$