Ex

$y, x = \mu_y + z_y$

- y: class label
- μ_y: signal
- z_y: zero-mean noise w/ cov Σ_y

Between-class variance (Signal power)

$$\sigma^2_{between}(\vec{w}) = (w^T \mu_1 - w^T \mu_0)^2$$

Within-class var (Noise power)

$$\sigma^2_{within}(\vec{w}) = p_0 w^T \Sigma_0 w + p_1 w^T \Sigma_1 w$$

$$= w^T (p_0 \Sigma_0 + p_1 \Sigma_1) w$$

Find $\hat{w}$ (Direction to project data) Maximize SNR

$$w^* = w_{Fisher} = \underset{w}{\operatorname{argmax}} \; SNR(\vec{w}).$$

$$= \frac{w^T S_B w}{w^T S_W w} \leftarrow \underset{w}{\operatorname{argmax}} \frac{(w^T (\mu_1 - \mu_0)^2)}{w^T (p_0 \Sigma_0 + p_1 \Sigma_1) w} \left(= \frac{\text{Signal}}{\text{Noise}}\right)$$

(class separation; from class; normalize by factor proportional to in-class var)

Solve for $\hat{w}$

$$\hat{w}_{Fisher} = (p_0 \Sigma_0 + p_1 \Sigma_1)^{-1} (\mu_1 - \mu_0)$$

Decision rule:

$$h_{Fisher}(\vec{x}) = \mathbb{I}\{\hat{w}_{Fisher}^T \vec{x} > \zeta\}$$

This is like the "bias" (negative bias)

Bias can be chosen by cross-validation

(bias)