

Problem 1 (30 pts)

30pts
24pts (a) Write the result as a product of eight matrices.
6pts (b) Write it again as a product ABC (same B) of three matrices.

Problem 1

a.
$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} B \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

#5 #4 #3 #1 #2 #6 #7

b.
$$\underbrace{\begin{bmatrix} 1/2 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_B \underbrace{\begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_C \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_D \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_E \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_F$$

$C = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Problem 2 (20 pts)

Problem 2

20pts

$$(w^*, b^*) = \underset{(w, b)}{\operatorname{argmin}} \sum_{i=1}^n \alpha_i (y_i - x_i w - b)^2$$

$$= \underset{(w, b)}{\operatorname{argmin}} A(Y - Xw - b)^2$$

$$= \underset{(w_{\text{ext}})}{\operatorname{argmin}} \underbrace{A(Y - X_{\text{ext}} w_{\text{ext}})^2}_Q$$

+5 for re-expressing problem in matrix form

+5 for setting up 1st order optimality condition to find w^* by $\frac{dQ}{dw_{\text{ext}}} = 0$

+5 for recognizing needed expansion of X & w .
2 carrying it out correctly.

$$X_{\text{ext}} = \begin{bmatrix} X & \mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$w_{\text{ext}} = \begin{bmatrix} w \\ b \end{bmatrix}$$

$$\frac{dQ}{dw_{\text{ext}}} = [-X_{\text{ext}}^T] 2(A(Y - X_{\text{ext}} w_{\text{ext}})) = 0$$

$$-2X_{\text{ext}}^T A(Y - X_{\text{ext}} w_{\text{ext}}) = 0$$

$$-2X_{\text{ext}}^T AY + 2X_{\text{ext}}^T AX_{\text{ext}} w_{\text{ext}} = 0$$

$$2X_{\text{ext}}^T AX_{\text{ext}} w_{\text{ext}} = 2X_{\text{ext}}^T AY$$

$$X_{\text{ext}}^T AX_{\text{ext}} w_{\text{ext}} = X_{\text{ext}}^T AY$$

$$w_{\text{ext}}^* = (X_{\text{ext}}^T AX_{\text{ext}})^{-1} X_{\text{ext}}^T AY$$

+5 for obtaining correct answer through correctness.

Problem 3 (30 pts)

- a. Visualization of given data ← 3 pts
- b. Calculating OLS fit (coefficients) ← 20 pts
 - i. If you used scikit-learn or any other built-in OLS function for the OLS model, 15 pts were deducted.
 - ii. 10 pts deducted for incorrect calculation of coefficient and/or intercept (5 for each)
- c. Prediction of 2018 value ← 3 pts
 - i. 2 pts deducted if prediction is incorrect
- d. Visualization of fit and data on same plot ← 4 pts

Note: 3 pts deducted if all values (coeff, intercept, prediction) were just slightly off (by <10)

Problem 4 (20 pts)

- a. Correct ridge & lasso implementation and graphs ← 18 pts (9 pts each, 5 for correct training and 4 for graph)
 - i. 2 pts deducted for missing labels on features
 - ii. 1 pt deducted for missing axis labels
- b. Comparison of methods ← 2pts
 - i. 1 pt deducted for incorrect or incomplete comparison

Partial credit was deducted if the version of your code uploaded to Blackboard did not run (i.e. small indexing/iterating errors, variable misnaming, or incorrect data loading that required small tweaks to run correctly). Every effort was made by the graders to give as much credit as possible.