HW 6: due Thursday, April 15 at 9pm

Problem 1

Consider the following functions

- $a(n) = 3n^2 + 7$
- $b(n) = 7n \log n$
- $c(n) = 3^n$
- $d(n) = 2\sqrt{n} + 400,000$
- $e(n) = 10n^2 - 40$
- $f(n) = \log \log n$
- $g(n) = 2n^{2/3} \log^2 n$
- $h(n) = 2^{\sqrt{n}}$
- $i(n) = n!$
- $j(n) = e^n * n^{23}$
- $k(n) = 2^{2^n}$
- $l(n) = n^{\log n}$

1. Order these functions from left to right so that $a \leq b$ means $O(a(n)) \leq O(b(n))$. For example, you might write

   $a \leq b \leq c \leq d \leq e \leq f \leq g \leq h \leq i \leq j \leq k \leq l$

   but of course that's not the right answer. What is?

2. A function is called *super-polynomial* if it is not $O(n^k)$ for some constant k. Which of the functions listed above are *super-polynomial*?

3. A function is called *sub-exponential* if it runs in time $2^{o(n)}$. Which of the functions would you describe as *sub-exponential?*

Problem 2

Let $S = \{x_1, \ldots x_n\}$ be a set of n distinct positive integers, and let t also be a

positive integer. Define
SUBSET-SUM = $\{(S, t) \mid$ some subset of S sums to exactly$t\}$
Show that SUBSET-SUM is in NP.

Problem 3

Suppose we had a Turing Machine X that could decide SUBSET-SUM (from Problem 2) in polynomial time. Explain how to use X to construct a new polynomial time Turing Machine that not only says YES/NO when given an instance (S, t), but also, when the answer is YES, explicitly returns a subset of S that sums to t.

Problem 4

A 2-vertex coloring is a function that assigns the colors *blue* and *red* to vertices of a graph. A 2-vertex coloring is called *proper* if n o two vertices that are connected by an edge receive the same color. Let
2COLOR $= \{G \mid G$ has a proper 2-vertex coloring$\}$

Show 2COLOR is in NP.

Problem 5

Show 2COLOR (from Problem 4) is in P.