T-301-REIR, REIKNIRIT
HAUST 2018
D4 - GRAPHS

Problem 1. *You have managed to acquire all the informations about friendship connections in the social network Basehook (Bh). You want to forward a message from one person to another, but have observed that the impact of a message depends a lot on along how many connections it has to travel. You therefore want to compute the following: Given a sequence of friendship connections and specific persons A and B, determine the smallest k such that there is a sequence of persons $X_0, X_1, X_2, \ldots, X_k$ where $A = X_0$, $B = X_k$, and X_i and X_{i+1} are Bh-friends, for $i = 0, 1, \ldots, k-1$. Describe briefly an efficient solution in words to this problem, using the concepts of this course, and give its time complexity.*

Problem 2. *(Continued) You have studied this further and discovered that not all friendships are equal. Strong friendship connections are special, and using such friendship connections doesn't cost anything so to speak: the message can travel along arbitrarily many strong friendships connections without its impact becoming weaker. We call other connections weak. You want to compute the fewest number of weak friendship connections needed to route a message from given person A to person B (possibly using many strong connections).*

Problem 3. *Modify `BreadthFirstPaths.java` to compute the number of shortest paths between two given vertices v and w in a given digraph. This replaces the method `boolean hasPathTo(inv v)` with `int nrOfPathsTo(inv v)`.* Hint*: Note that if vertex w is of distance k from v, and it is adjacent to vertices a, b and c of distance $k-1$ from v, then each shortest $v - w$ path runs through one of a, b, and c.*

Problem 4. *Suppose the weight of every edge in a weighted graph is decreased by one. Explain (in 20 words or less) why the MST computed by Kruskal does not change.*

Problem 5. *Consider a graph with distinct edge weights such that Prim and Kruskal select the edges of the spanning tree T in* opposite *order. How must T look like?*

Class Exercises

These questions will be addressed during exercise class. They are not to be turned in.

Problem 6. *When is the heaviest edge of a graph (with distinct edge weights) contained in an MST?*

Date: October 5, 2018.

Problem 7. *Explain why the following algorithm does not necessarily produce a topological order: Run BFS and label the vertices by increasing distance to their respective source.*

Problem 8. *Prove carefully that the following algorithm finds an MST: Start with a graph containing all the edges; repeatedly go through the edges in decreasing order of weight; for each edge, check if deleting that edge will disconnect the graph; if not, delete it.*

Problem 9. *The MST problem on the following page.*

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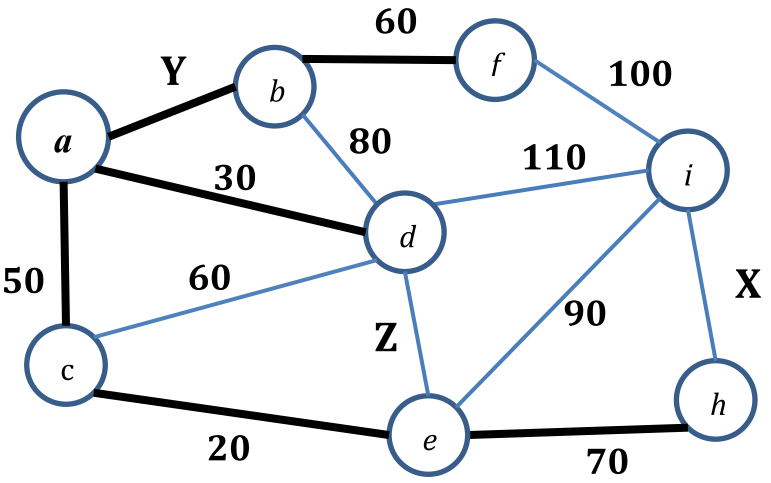
Vegin net / *Weighted graphs*



Þykku leggirnir í netinu að ofan tákna leggi sem höfðu verið valdir á <u>einhverjum tímapunkti</u> af MST reikniriti. / *The bold-faced edges in the above weighted graph represent the edges that had been selected <u>by some intermediate stage</u> of an MST algorithm.*

(a) Hvaða gildi gætu vægin X, Y, Z tekið, ef <u>reiknirit Kruskals</u> er keyrt? / *Which of the following* ***could*** *be the weights of edges X, Y, Z, respectively, if <u>Kruskal's algorithm</u> was run?*

	15	25	35	45	55	65	75	85	95	105
x	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
y	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
z	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

(c) Hvaða gildi gætu vægin X, Y, Z tekið, ef <u>reiknirit Prims</u> er keyrt, með upphafsnútinn **a**? / *Which of the following* ***could*** *be the weights of edges X, Y, Z, respectively, if <u>Prim's algorithm</u> was run with initial node* **a**?

	15	25	35	45	55	65	75	85	95	105
x	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
y	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
z	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐