D6 - Satisfiability

Solutions

Reading material on Satisfiability (relating to questions 1-4) is in special notes, see Modules in Canvas). Material on NP-completeness (questions 6-7) is pp.903–921 in Sedgewick-Wayne.

1. Consider the CNF formula

$$x_1x_2, \overline{x_1}\overline{x_2}, \overline{x_1}\overline{x_3}, \overline{x_2}\overline{x_3}, x_2x_3$$

   a) Find a satisfying assignment for the formula, and b) Give a falsifying 3-clause (i.e., a clause with 3 literals that, if added to the formula, makes the formula false)

2. (8%) Perform unit propagation on the CNF formula

$$x_1x_2, \overline{x_2}, x_2x_4, \overline{x_1}x_2\overline{x_4},$$

3. Perform pure literal elimination on CNF formula

$$x_1x_2\overline{x_3}, \overline{x_2}\overline{x_3}, x_2\overline{x_3}x_4, \overline{x_1}x_2\overline{x_4}, x_1x_3x_5\ .$$

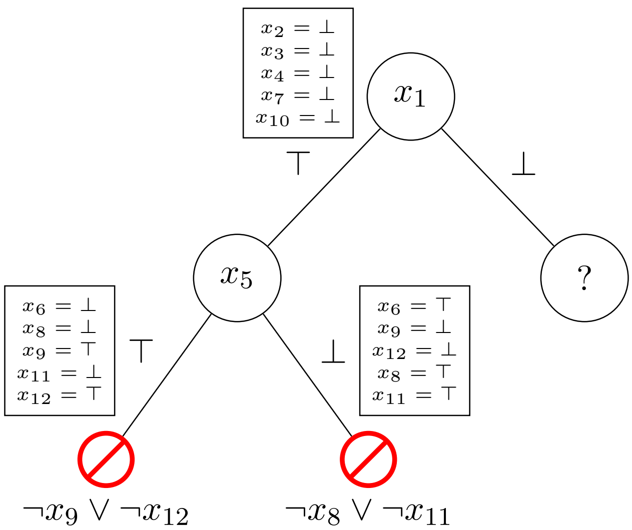

Figure 1: Partial run of the DPLL heuristic

4. (20%) The result of the initial trace of $\hat{\mathcal{F}}$ done in class (and lecture slides) is shown in Fig. . Continue the trace for the subtree given by the partial assignment $x_1 = F$. Label the edges with the unit clauses that were propagated. On "dead-end" states

(shown on slide with a red X), identify the clause that is responsible. Assume that next variable explored is always the lowest unassigned one.

5. (10%) (Problem 6.62.) Suppose that $P \neq NP$. Which of the following can we infer:

   e. If X is NP-complete, then X cannot be solved in polynomial time.

   f. If X is in NP, then X cannot be solved in polynomial time.

   g. If X is in NP but not NP-complete, then X can be solved in polynomial time.

   h. If X is in NP, then X is not NP-complete.

6. (16%) Classify the following problems as in P, as NP-complete, or neither. Identify the usual name for the (underlying abstract) problem.

   (a) Given a graph, is it possible to mark each node by "X", "Y" or "Z", such that neighbors always get a different mark?

   (b) Given a set X of numbers $x_1, x_2, \ldots, x_n$, are there two sets X_1, X_2 such that $X_1 \cup X_2 = X$ and $X_1 \cap X_2 = \emptyset$ and such that $\sum_{x_i \in X_1} x_i = \sum_{x_i \in X_2} x_i$?

   (c) Given an integer k and an virtual environment consisting of n avatars, where any two are either *friends* or *enemies*, are there k avatars that are all friends of each other?

   (d) Given a computer network and two nodes a and b, and an integer k, is it possible to forward a message from a to b using at most k intermediate nodes?

7. Implement the pure-literal elimination rule, either in a Java function or in words. State the time complexity.