Stellar Astrophysics Homework 5 Solutions

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Question 1:

a) The Jeans mass we discussed in class is generally defined by:

$$J_m = (\frac{5kT}{G\mu m_H})^{\frac{3}{2}}(\frac{3}{4\pi\rho_0})^{1/2}$$

This process, however, is not as simple as plugging in values into this equation and getting a mass for any gas cloud. In reality there are a number of extra effects that must be accounted for to get a more accurate Jeans Mass. These effects/properties are listed below along with how they affect the calculated mass:

i. Angular Momentum

As the cloud begins to collapse, it can begin to rotate quickly. This process would generate a centrifugal force that wants to pull parts of the cloud away from the center. This process is fighting against the gravitational collapse happening with the cloud which would increase the Jeans mass.

ii. Magnetic Fields

If the cloud has a high enough total magnetic field, as the cloud collapses this field can create magnetic pressure which would fight against the collapse. This leads to an increase in Jeans Mass

iii. Metallicity

If the cloud of dust has a high enough metallicity, the abundance of metals can make it easier for the cloud to cool down as the temperature can be more easily dissipate throughout the cloud. This would decrease Jeans Mass.

iv. Cloud Fragmentation

As the cloud collapses, it is possible that the cloud will begin to fragment into smaller clumps with their own centers of gravity. This would decrease the Jeans mass and lead to a high number of smaller mass stars forming.

v. Nearby Stars

Nearby Stars can either increase or decrease the Jeans mass depending on their evolutionary stage. For a star that is formed, its energy can dissipate through a nearby cloud which would increase the Jeans mass as the cloud would be at a higher temperature and more stable to collapse. If the nearby star goes supernova, the resultant shockwave from the event can quickly smash together clumps of material in the cloud which would onset collapse. This senecio would decrease Jeans mass.

b) The drivers of star formation efficiency are the same processes as listed above. All the mentioned processes help to determine how much gas would get converted into stars. Most of the interactions of the gas cloud hinder the formation of stars and would decrease the formation of stars. We would expect the efficiency of star formation to be less than 1 as if it was equal to 1 all the gas and dust would be turned into stars. We know it can never be one because of all of the processes hindering stellar formation.

c) To create an Initial Mass Function, the measurements required are the mass of a large sample of stars. In particular, stars that formed from the same could. This would require knowing the distance to the forming stars, the age of the cluster, and the number of stars in the cluster. The issue when calculating the IMF for a certain cloud is that higher mass stars form and die much quicker than lower mass stars, so they cannot be observed at the same time which would be necessary to find the IMF (PDMF does not have this limitation). Another issue is for stars of very low mass (~0.1M_sun). These low mass stars are hard to see based on their very low luminosity. Another issue is the formation of binary systems as it is hard to examine each star in the system separately without the other star interfering with the measurements.

Question 2:

a) To find the proportionality constant for the IMF distribution I solved the IMF function using the given variables in the problem:

$$\Phi = \frac{dN}{dm} = am^{-2.35}$$

$$m = \frac{M_{produced}}{M_{sun}} = \int mdN$$

$$\int dN = a \int_{m\ low}^{m\ high} m^{-2.35} dm$$

$$\frac{M_{produced}}{M_{sun}} = a \int_{0.1}^{20} m^{-1.35} dm$$

$$\frac{M_{gmc} * \varepsilon_{sf}}{M_{sun}} = a \int_{0.1}^{20} m^{-1.35} dm$$

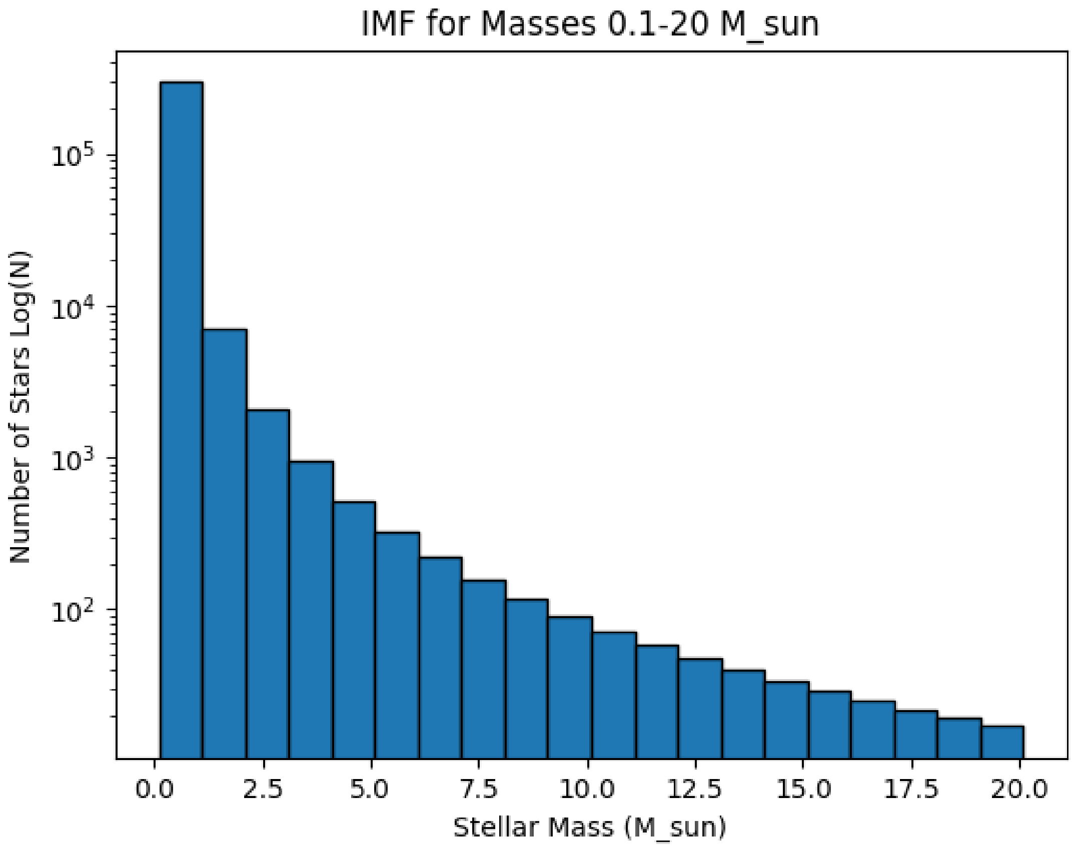
$$10^5 = a \int_{0.1}^{20} m^{-1.35} dm$$

$$a = \frac{10^5}{5.395}$$

$$\boxed{a = 1.85 * 10^4}$$

b) To determine the number distribution of the stars and place them in 1M_sun bins I used numpy arrays, the quad function from scipy.integrate, and the bar plot capabilities of matplotlib.pyplot. I made my code to iterate through the bins at 1 solar mass intervals and calculated the number using the IMF function including the proportionality constant calculated in part a.

```
bins = np.arange(0.1,21,1)
# print(bins)

a = 1.85e4      #constant calculated from part a

def IMF(m):     #IMF function
    return(a * m**-2.35)


num_stars = []


#Integrate over each bin
for i in range(len(bins)-1):
    num_stars.append(quad(IMF,bins[i], bins[i+1])[0])
print(num_stars)
num_stars = np.array(num_stars)

#make the bins I populate center themselves over the median value between their limits
cent = (bins[:-1] + bins[1:]) / 2

#ploting the histogram
plt.bar(cent,(num_stars),width=1,edgecolor = 'k')
plt.yscale('log')
plt.xlabel('Stellar Mass (M_sun)')
plt.ylabel('Number of Stars Log(N)')
plt.title('IMF for Masses 0.1-20 M_sun')
plt.savefig(fname = 'IMF')
```



For this plot I made the y-axis on a log plot as the number of stars in the 0.1-1.1 solar mass bin is so much greater than the number in the 19.1-20.1 solar mass bin.

I also calculated the average mass of the stars made using:

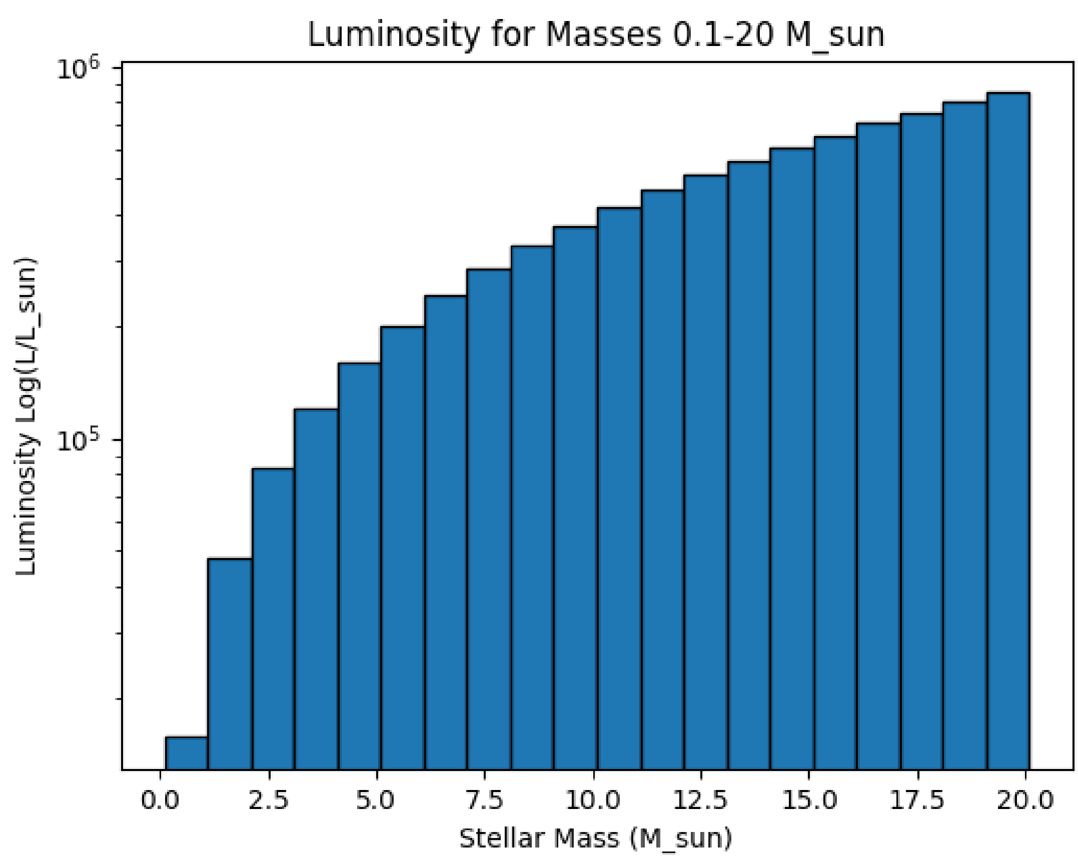
$$Avg\ M = \frac{Mass\ made\ into\ stars}{Total\ Number\ of\ Stars} = \frac{10^5}{300000} \sim 0.32 M_{sun}$$

Which based on this plot is obvious.

c) To find the luminosity for each bin I used the same coding method but changed the integrating function to:

$$L = \int L dN = a \int_{m\ low}^{m\ high} (1.5 * m^{3.5})\, m^{-2.35} dm$$

```
def luminosity(m):
    return((1.5 * m**3.5)*IMF(m))

num_stars = []
for i in range(len(bins)-1):
    num_stars.append(quad(luminosity,bins[i], bins[i+1])[0])
print(num_stars)
num_stars = np.array(num_stars)

cent = (bins[:-1] + bins[1:]) / 2

plt.bar(cent,(num_stars),width=1,edgecolor = 'k')
plt.yscale('log')
plt.xlabel('Stellar Mass (M_sun)')
plt.ylabel('Luminosity Log(L/L_sun)')
plt.title('Luminosity for Masses 0.1-20 M_sun')
plt.savefig(fname = 'Luminosity')
```



I also used a log scale on the y-axis to better show the relationship.
Without using log, this plot would have a practically linear relationship between the mass and luminosity. This is due to our model using a simple luminosity function for all mass bins. In reality, the slope of the luminosity function changes based on the

mass bin. For masses $0.1<M<10$, the luminosity function goes as m^3 for $M>10$, it goes as m^4.