Stellar Astrophysics Homework 3 Solutions

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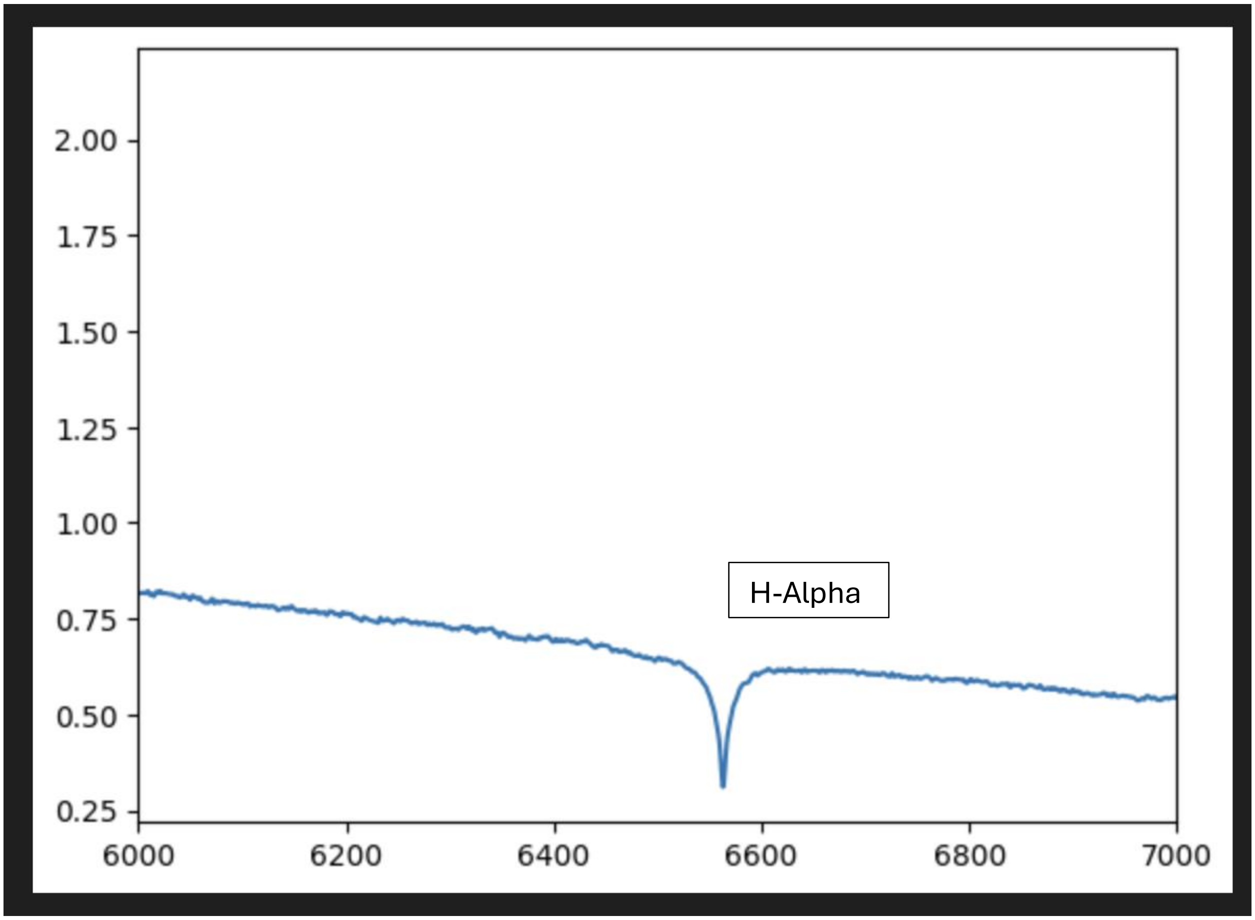
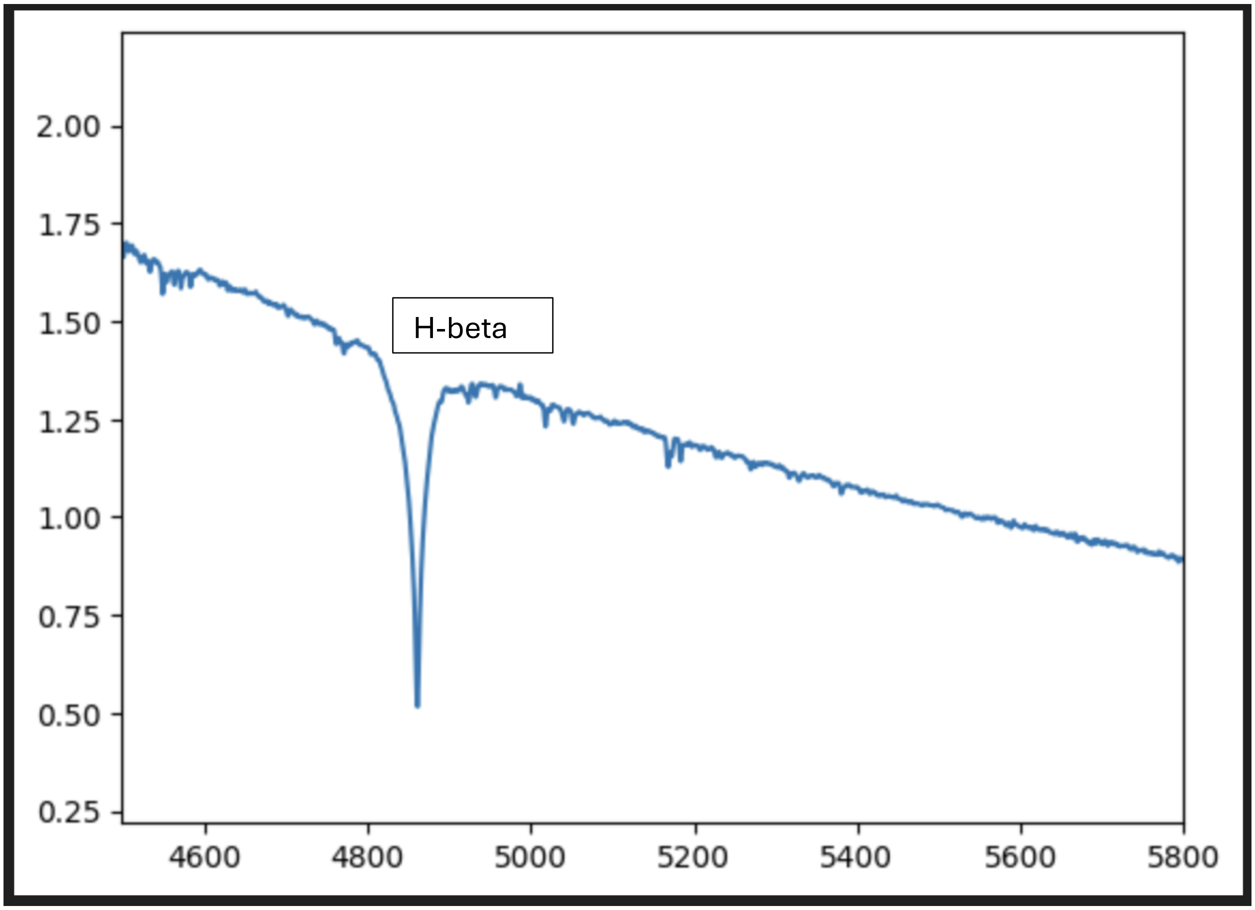
Question 1:

1.a) To normalize and plot the spectrum I first identified the locations of the H-alpha and H-beta absorption lines. I plotted the full spectra and played with the limits of the x-axis to include as much of the continuum as possible without including another absorption feature.





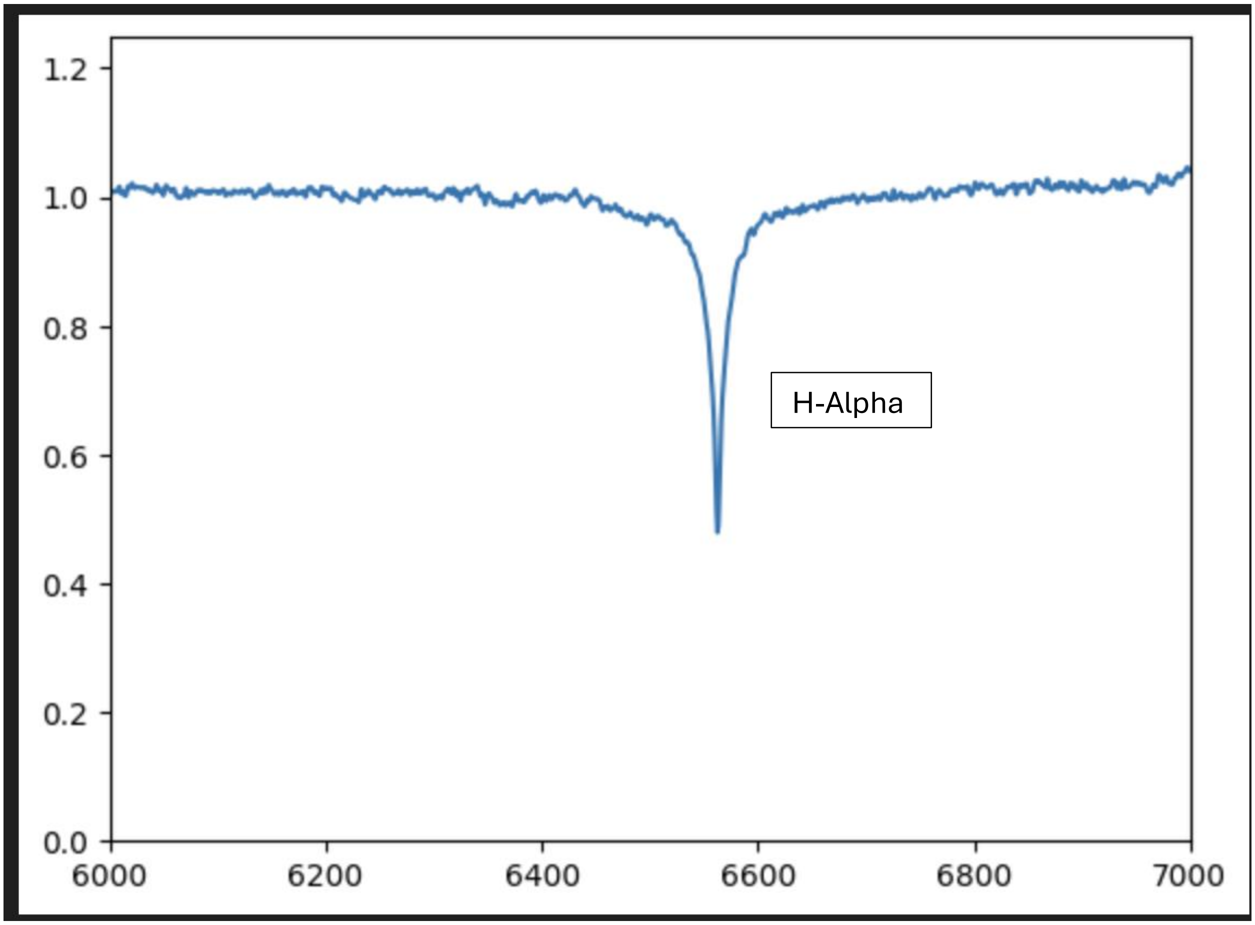
To normalize the spectrum I created masks that ignore all values except the continuum between 6000 and 7000 Angstrom (excluding the absorption feature for H-alpha) and between 4500 and 5800 (excluding the absorption feature for H-beta).

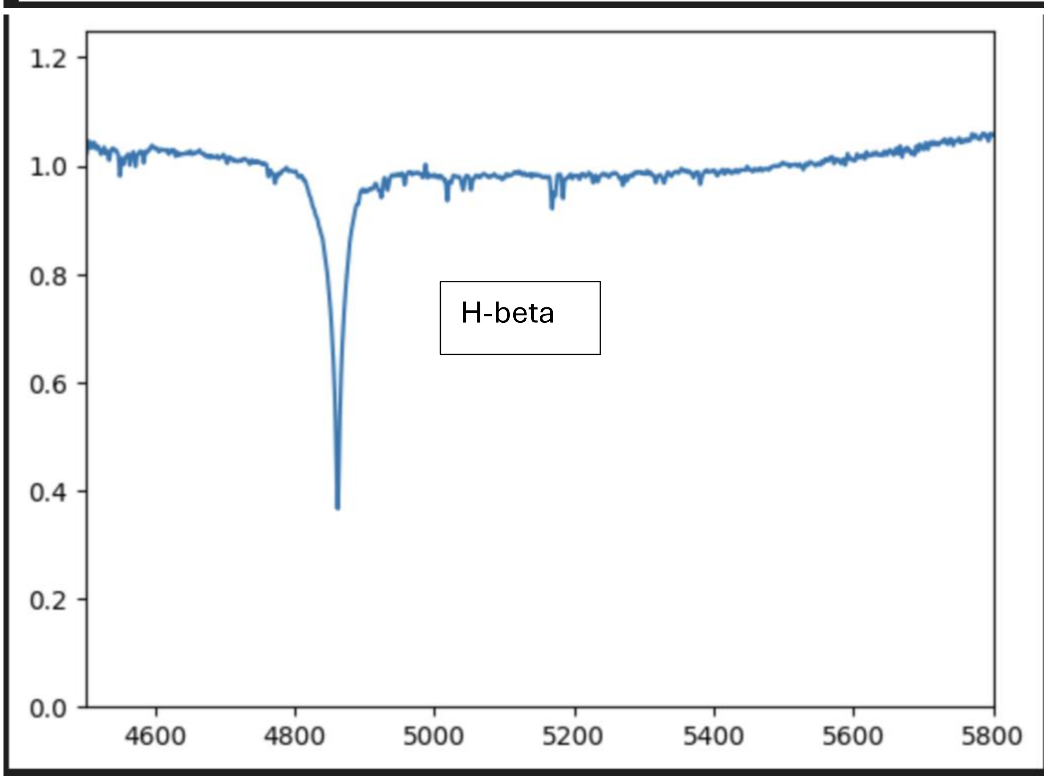
```python
Ha_Mask = ((star1['Wavelength'] > 6540) & (star1['Wavelength'] < 6580))                #Mask only including the absorption feature Ha
Ha_Mask = ~Ha_Mask                                                                      #Inverting the mask to include all but the absorption Ha
mask2_a = ((star1['Wavelength'][Ha_Mask] > 6000) & (star1['Wavelength'][Ha_Mask] < 7000))  #Masking all spectra except for the continuum just around the absorption Ha
# print(star1['Wavelength'][Ha_Mask][mask2_a])


Hb_Mask = ((star1['Wavelength'] > 4800) & (star1['Wavelength'] < 4900))                #Same thing here but for Hbeta
Hb_Mask = ~Hb_Mask
mask2_b = ((star1['Wavelength'][Hb_Mask] > 4500) & (star1['Wavelength'][Hb_Mask] < 5800))
# print(star1['Wavelength'][Hb_Mask][mask2_b])
```
✓ 0.0s Python

I then used np.polyfit to make a linear fit of this data.

Then plotting the spectra again but dividing by the polyfit for each flux value gives us the normalized spectra plots for these 2 absorption features.


1.2
1.0
0.8
0.6
0.4
0.2
0.0
6000
6200
6400
6600
6800
7000
H-Alpha


1.2
1.0
0.8
0.6
0.4
0.2
0.0
4600
4800
5000
5200
5400
5600
5800
H-beta

1.b) The next step is to find the equivalent widths of these two peaks. Applying the formula:

$$W = \int 1 - \frac{F_\lambda}{F_{\lambda,Continuum}} \frac{d\lambda}{\lambda_0}$$

It is important to note that:

$$\frac{F_\lambda}{F_{\lambda,Continuum}}$$

Is just the values of flux that we read on the plots we just made as they have been normalized to the continuum, and λ_0 is the wavelength that each absorption takes place at: 6562.8 Å and 4861.3 Å. So by using the Simpson function from scipy.integrate, it is fairly straightforward to find the equivalent widths for each line.

```python
F_lambda_a = star1['Flux'][~Ha_Mask] / (Ha_fit[0] * star1['Wavelength'][~Ha_Mask] + Ha_fit[1]) #Flux values during the absorption Ha

Lambda_a = star1['Wavelength'][~Ha_Mask]                                                    #Wavlength values during the absorption

# plt.plot(Lambda_a,F_lambda_a)

Wa = 1/6562.3 * simpson(1-F_lambda_a, Lambda_a)                                             #Using simpsons rule to evaluate the integral to get the equivalent widtr

print(Wa)
```

✓ 0.0s Python

```
0.0014030522833585986
```

```python
F_lambda_b = star1['Flux'][~Hb_Mask] / (Hb_fit[0] * star1['Wavelength'][~Hb_Mask] + Hb_fit[1])
Lambda_b = star1['Wavelength'][~Hb_Mask]
Wb = 1/4861.3 * simpson(1-F_lambda_b,Lambda_b)
plt.plot(Lambda_b,F_lambda_b)
print(Wb)
```

✓ 0.0s Python

```
0.0032806760944173313
```

$$\boxed{\begin{aligned} W_\alpha &= 0.0014\ \text{Å} \\ W_\beta &= 0.0033\ \text{Å} \end{aligned}}$$

Which based on 9.5 in the textbook we would expect the widths to be on the order of 0.01nm which these are!

1.c) The final step for question 1 is to calculate the column density for each line using the "optically thin regime" of the curve of growth:

$$N_a = 1.130 \times 10^{12}\ \text{cm}^{-2} \frac{W}{f\left(\lambda_o/\text{Å}\right)}$$

We are given the values of *f* in the problem, those being: $f_{H\alpha} = 0.6407$ and $f_{H\beta} = 0.1193$, and we are given λ_0. These values and the calculated widths give us the values:

```
fa = 0.6407
fb = 0.1193
Na_alpha = 1.130e12 * Wa / (fa * 6562.8)
Na_beta = 1.130e12 * Wb / (fb * 4861.3)
print(Na_alpha)
print(Na_beta)

print(np.log10(Na_alpha)/np.log10(Na_beta)) #!!!!!! You must take the log for the ratio, The curve of growth is a log plot
✓ 0.0s

377058.2130166779
6392179.041052413
0.8193793805197289
```

$$\boxed{\begin{aligned} N_{H\alpha} &= 3.7\ x\ 10^5\ \frac{atoms}{cm^2} \\ N_{H\beta} &= 6.3\ x\ 10^6\ \frac{atoms}{cm^2} \\ \frac{N_{H\alpha}}{N_{H\beta}} &= 0.059 \end{aligned}}$$

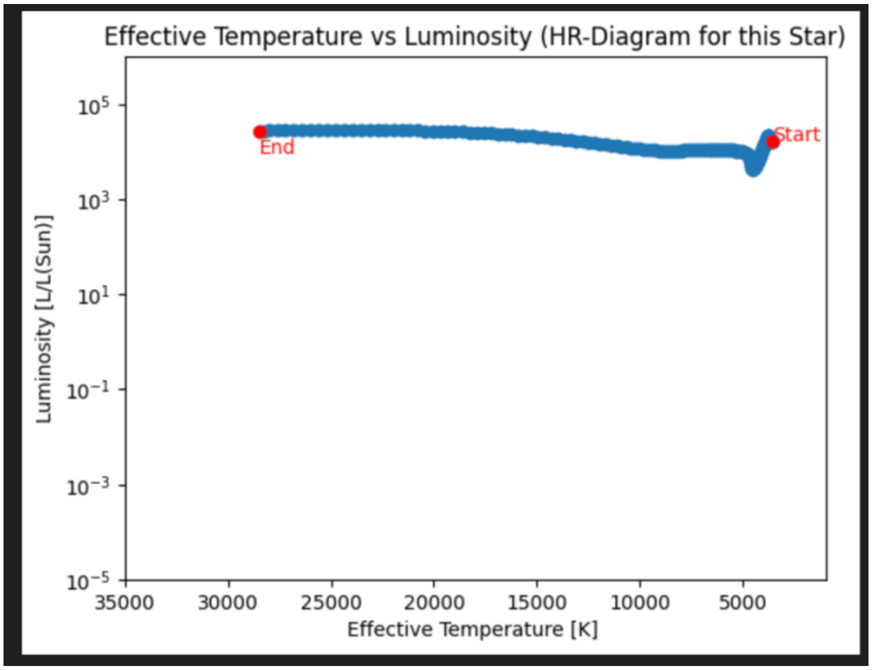
To me this ratio did not make much sense. I am not entirely sure why this ratio is not believable, but when reading the section on the curve of growth, it almost always referred to N using log. By taking the log of both column densities and then taking the ratio, I get:

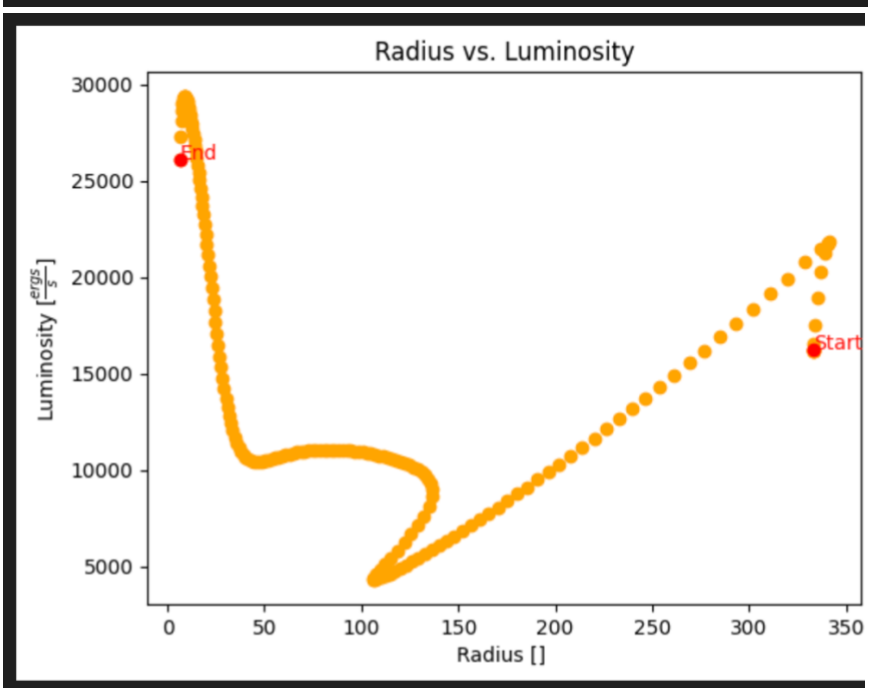
$$\boxed{\frac{Log(N_{H\alpha})}{Log(N_{H\beta})} = 0.82}$$

This value seems more believable as a ratio. Since we are using the optically thin regime formula, I would assume that our spectra lines are in the same regime. This means that the equivalent widths are linearly proportional to the column density. I think that these values would represent the upper limits as this star is an A type. The hydrogen lines are strongest on these types of stars so this would show the maximum column densities possible with hydrogen lines.

Question 2:

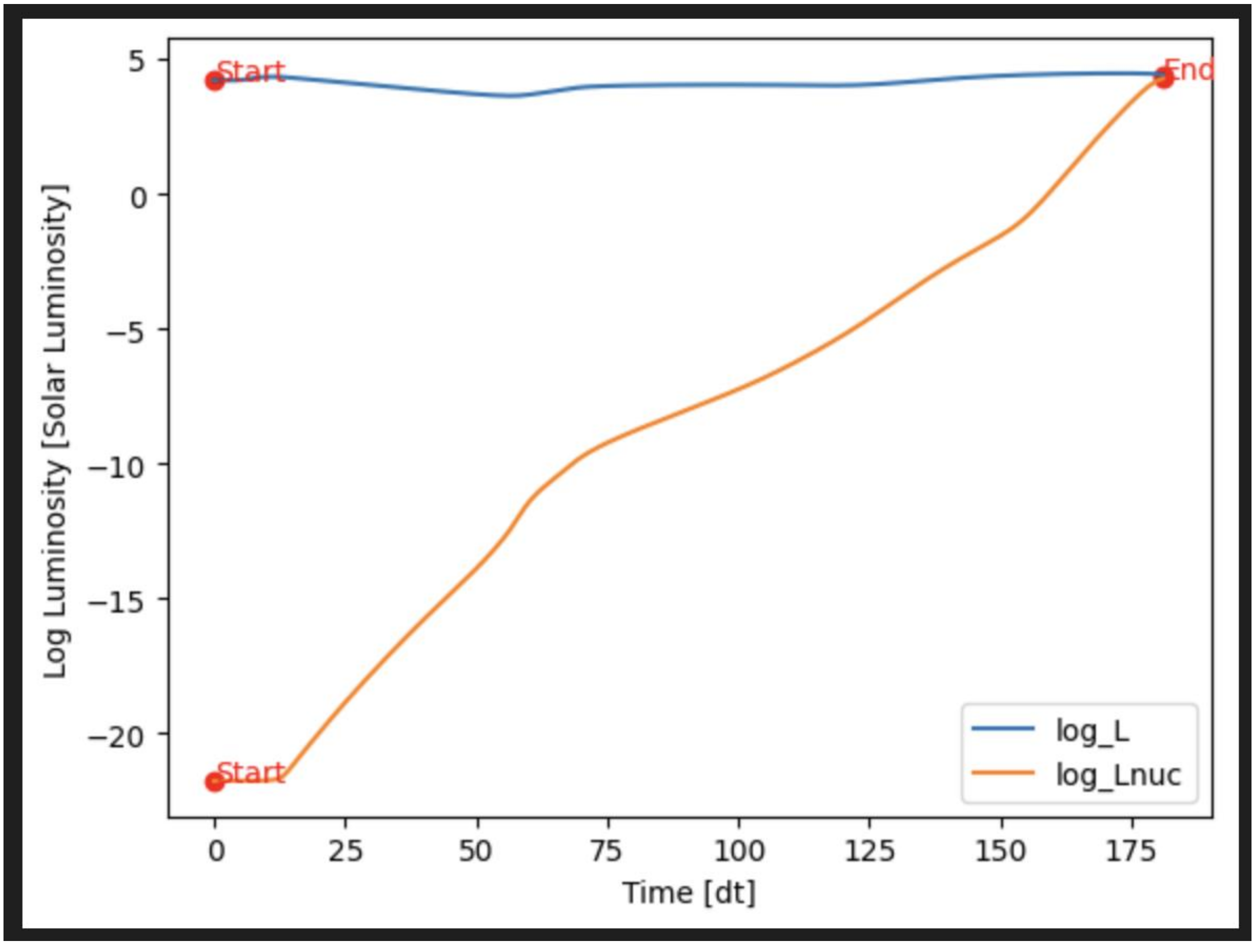
2.a)





The HR-diagram I made was scaled to have x and y limits that would be expected from a normal HR-diagram. When plotting the data from this star, I removed the log from the data by taking 10 to the power of every value. This lets the plotted data match the location where it would be on the HR-diagram. As seen in the plot this star started in the red-giant branch and began traveling across the HR diagram towards becoming a white dwarf. The radius of the star starts at almost 350 times the radius of the sun and ends at around the radius of the sun. While traveling the luminosity stays mostly constant and the Temperature and radius are the only things that change. R is proportional to the inverse of T^2.

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$



2.b)Based on the above plot, we see that the surface luminosity remains relativly constant while the nuclear luminosity increases linearly with time. This is the process of a star dying. The star starts as a red giant and as it starts to run out of fuel, its core shrinks which stars increaing the luminosity. The core begins fusing helium into heavier elements which increases the temperature of the core which also increases the luminosity.