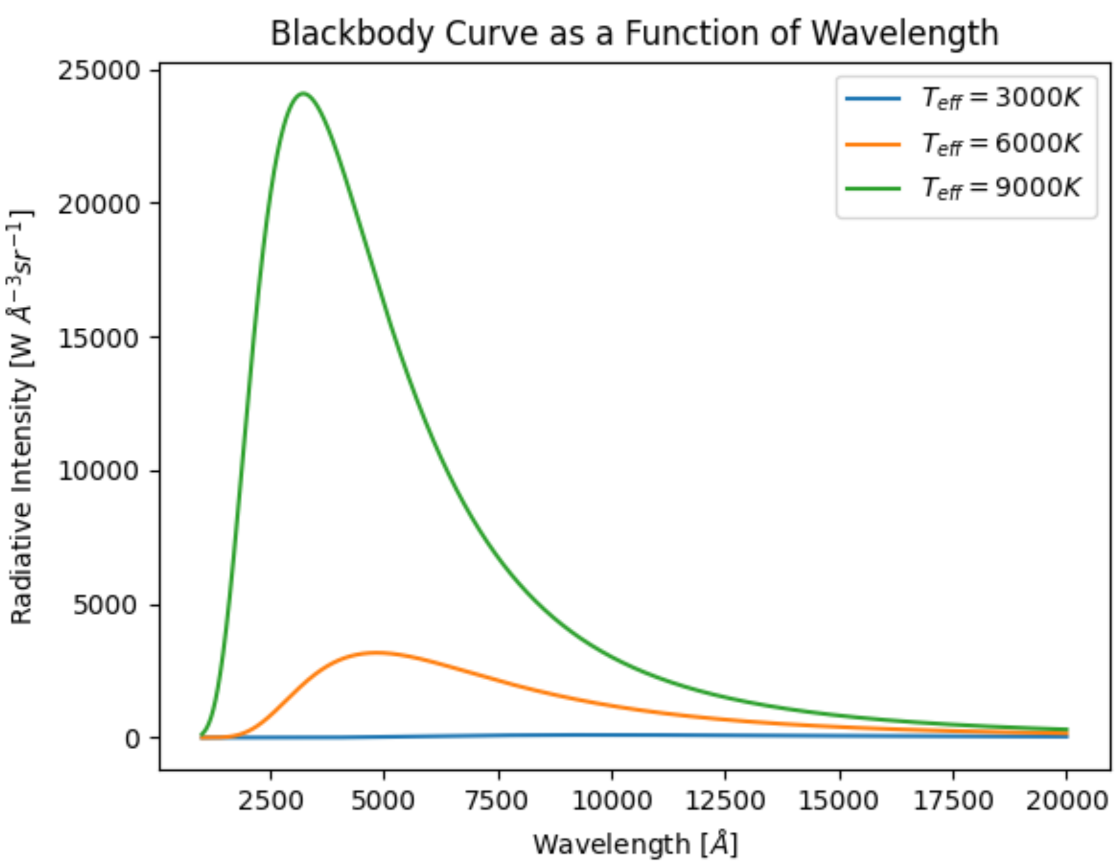
Stellar Astrophysics Homework 2 Solutions

Andrew Tom

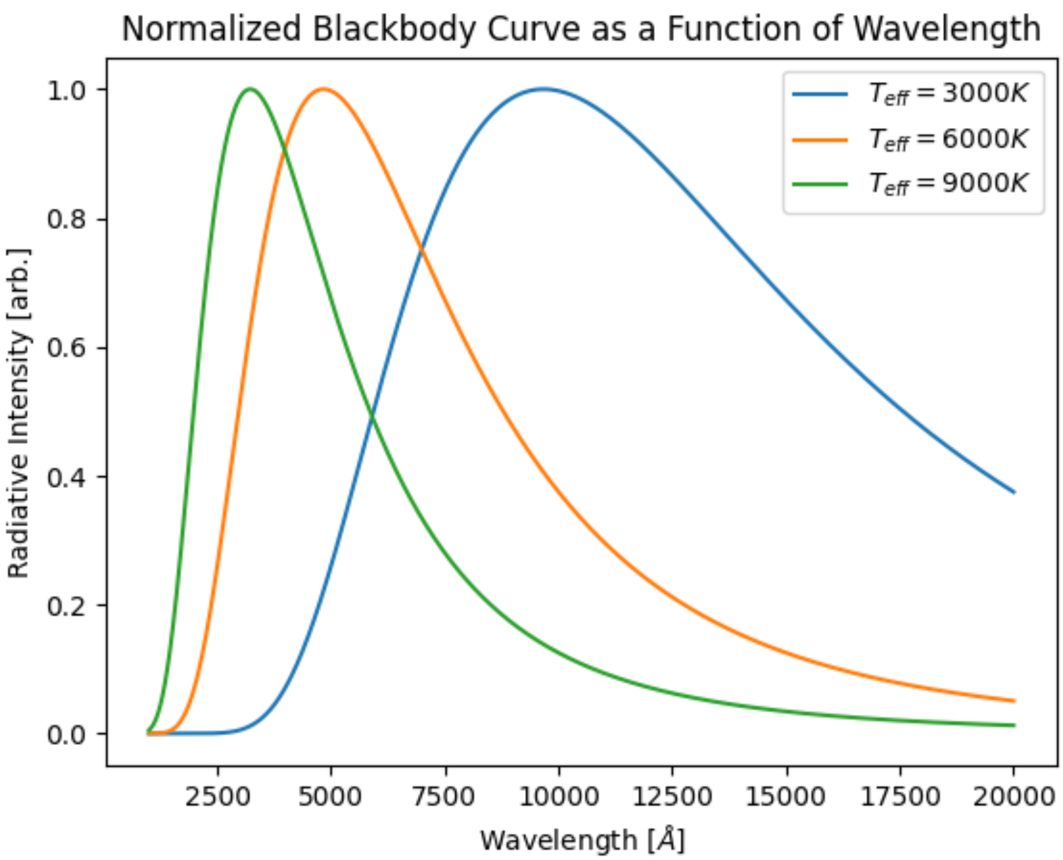
Question 1:

1.a) To create plots of Intensity vs. Wavelength I used the Planck Function (with respect to wavelength). I created a python function that takes an np.linspace and Temperature as an input in order to plot the function. I plotted the equation for T_{eff} = 3000K, 6000K, 9000K with a range of wavelengths between 1000 Angstrom and 20000 Angstrom.

```python
def planck(wl,T):    #Planck Function (wavelength ver.)
    return(((2 * h * c**2) / (wl**5)) / (np.exp((h * c) / (wl * k * T)) - 1))
```



This plot is what we would expect for these temperatures, but because of the scale it would be easier to see if it were normalized:



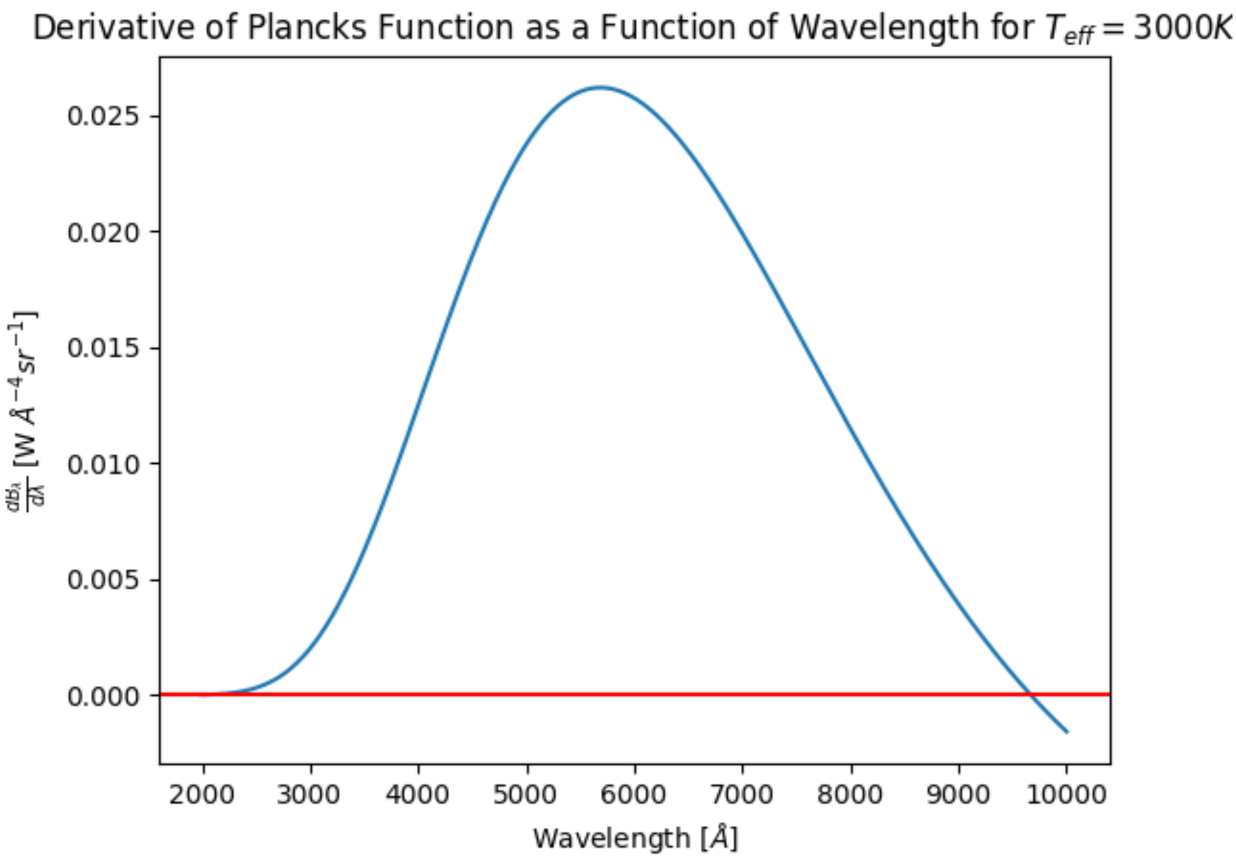
1.b) To differentiate the function, I coded a function that takes the Planck function as an input and uses the limit definition of the derivative to return a value for the differentiated curve at a given wavelength. In this function I set h = 10e^-6 as we cannot really take the limit as h goes to zero, so I just chose a small value of h to get a very close approximation.

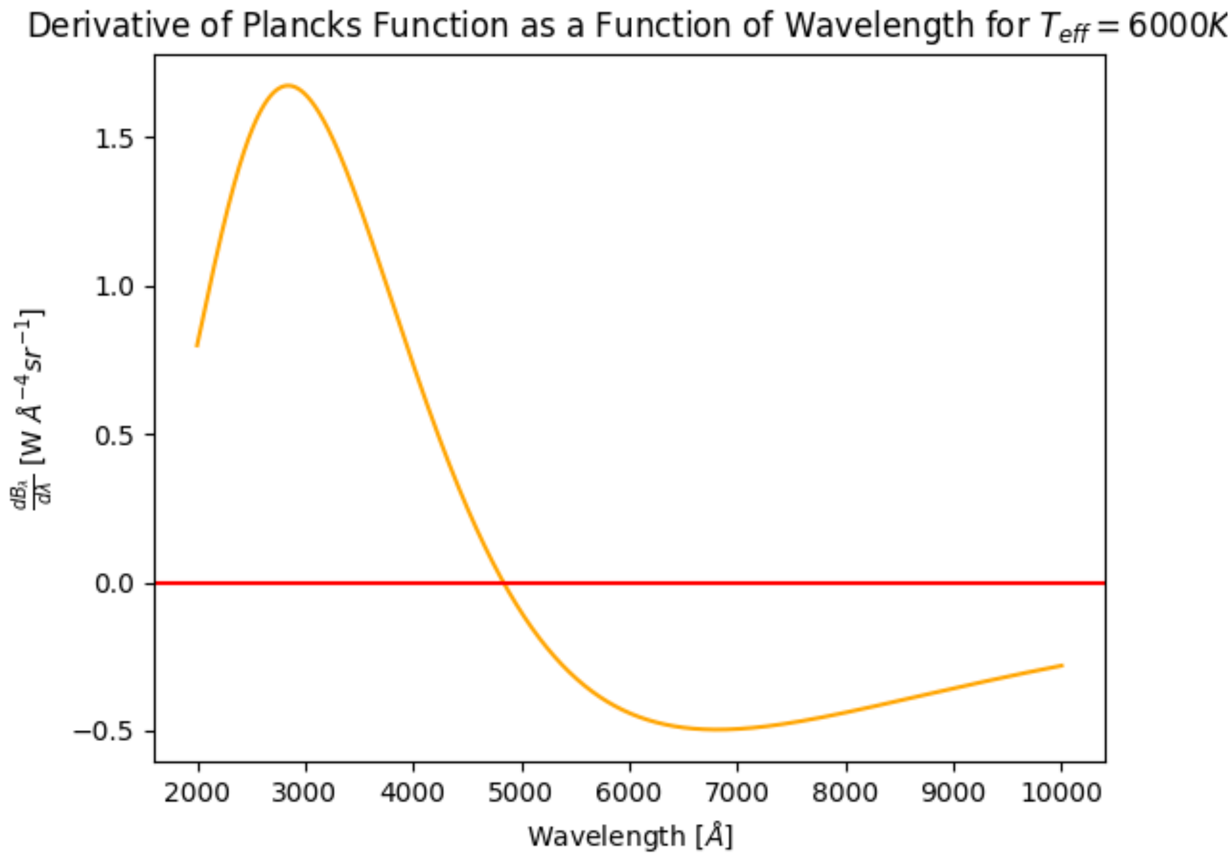
```
#Derivative of Planck's Function using the limit definition of a derivative. We are not really taking a limit here but we can set h to be really s
#a good approximation to the value of the derivative.

def derivative(wl,T,h=10e-6):
    return((planck(wl + h , T) - planck(wl,T)) / h)
```

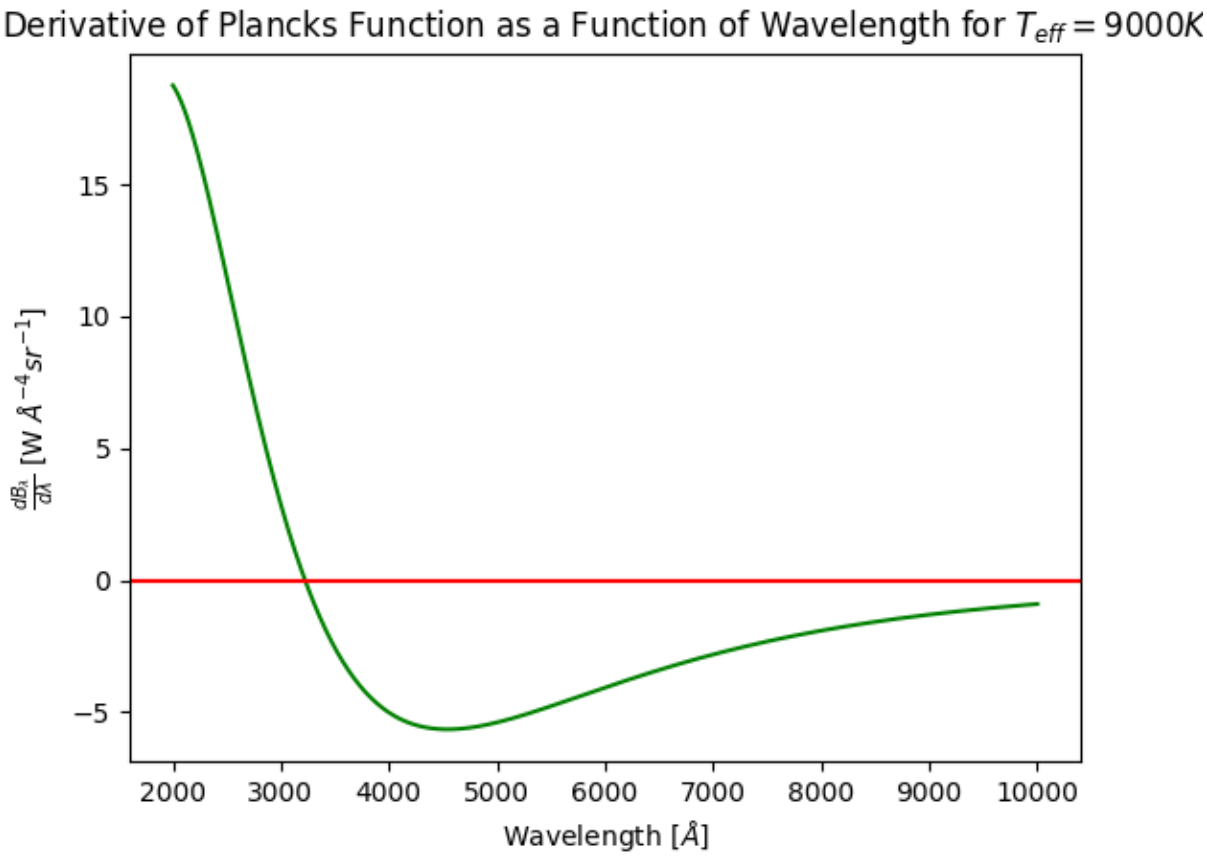
✓ 0.0s Python

I then plotted this new function for each of the temperatures from part a.





Derivative of Plancks Function as a Function of Wavelength for $T_{eff} = 6000K$
1.5
1.0
0.5
0.0
−0.5
$\frac{dB_\lambda}{d\lambda}$ [W $\AA^{-4} sr^{-1}$]
2000
3000
4000
5000
6000
7000
8000
9000
10000
Wavelength [$\AA$]



1.c) The red line on each graph represents when the derivative is equal to zero. This usually denotes where a maximum or minimum occurs on the non-differentiated plot. In our case we only will have maximums, so this point denotes where the peak wavelength is for each temperature.

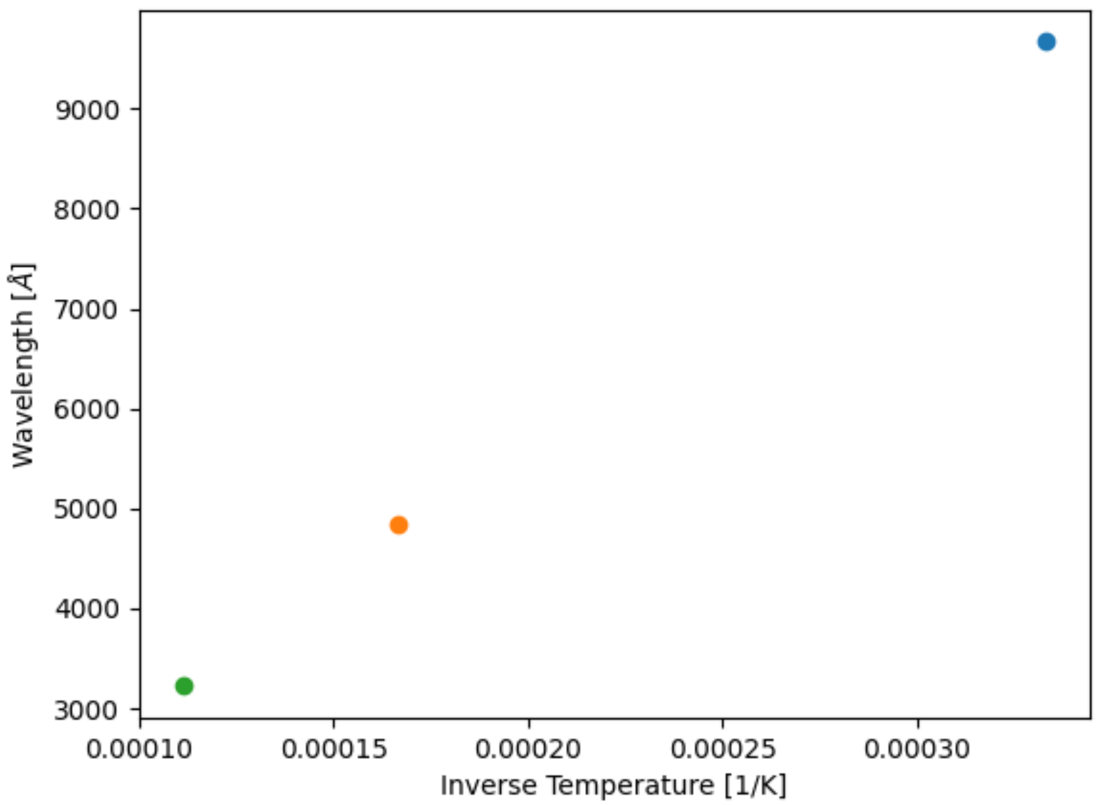
I estimated these values by masking the data around where the derivative function = 0 and found the following approximate peak wavelengths:

3000K: 9670 Angstrom

6000K: 4835 Angstrom

9000K: 3223 Angstrom

1.d) The last part of this question is basically to confirm Wein's Law by making a scatter plot with the peak wavelengths from part c and the inverse temperature.



Next I used scipy.optimize and a first degree polynomial function to create a best fit line to find the slope created by fitting these points.

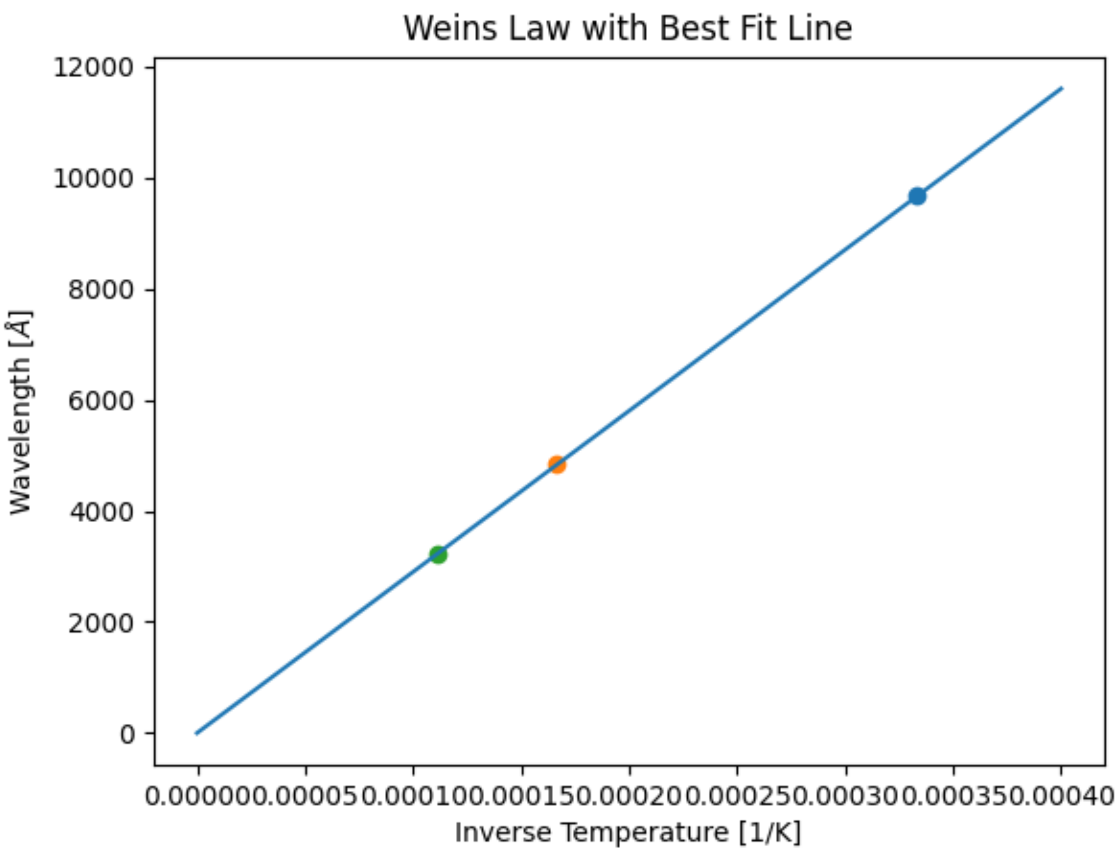
```python
#To fit these points with a line we chose a linear fit in the form of ax+b. Wein's law follows this form, so we will compare the optimal fit param
#the constant coefficent for Wein's law to see if our data matches.

def FitFunc(x,a,b):
    return(a * x + b)

popt, pcov = curve_fit(FitFunc, InvT, MaxWl)    #Using the curve fit function to find optimal parameters for the fit.
print('The slope of the fitted line is:',popt[0],'with an intercept of',popt[1])
```

✓ 0.0s Python

```
The slope of the fitted line is: 29011153.768677242 with an intercept of -0.34613467436184875
```

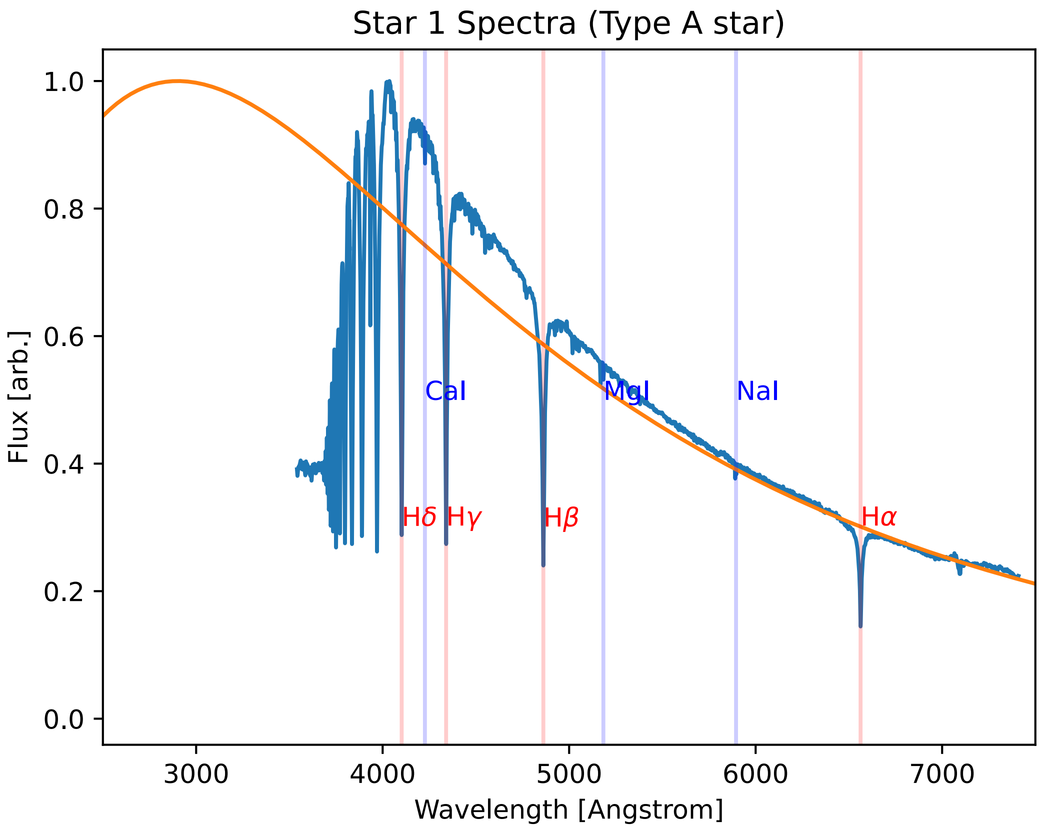


The slope of this fitted line is approximately 29011153 Angstrom*K which when converted to meters Kelvin is $2.90 \times 10^{-3}\ mK$ which is only 0.34% off the known value of Wein's displacement constant which shows the accuracy of our measurements.

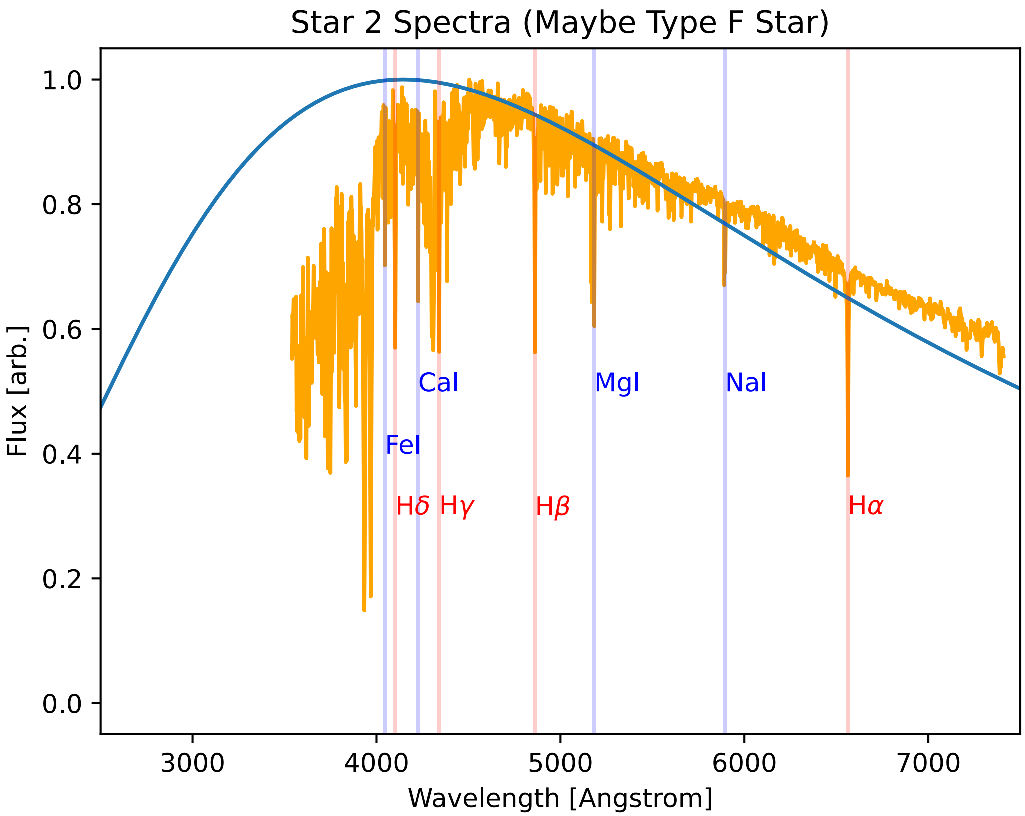
Question 2.

For each part of this question it asked to add something new to the plot of the stellar spectra for each star. To make it easier to read my solutions I will just go through each plot individually. I will also mention here that I slightly modified the text files for each star to include a name above the 2 columns. The purpose of doing this was to be able to call the list of values from the file by name to make the code more readable. So rather than the wavelengths just being star1[0] I can call them using star1['Wavelengths']. I included my modified text files with my homework submission.

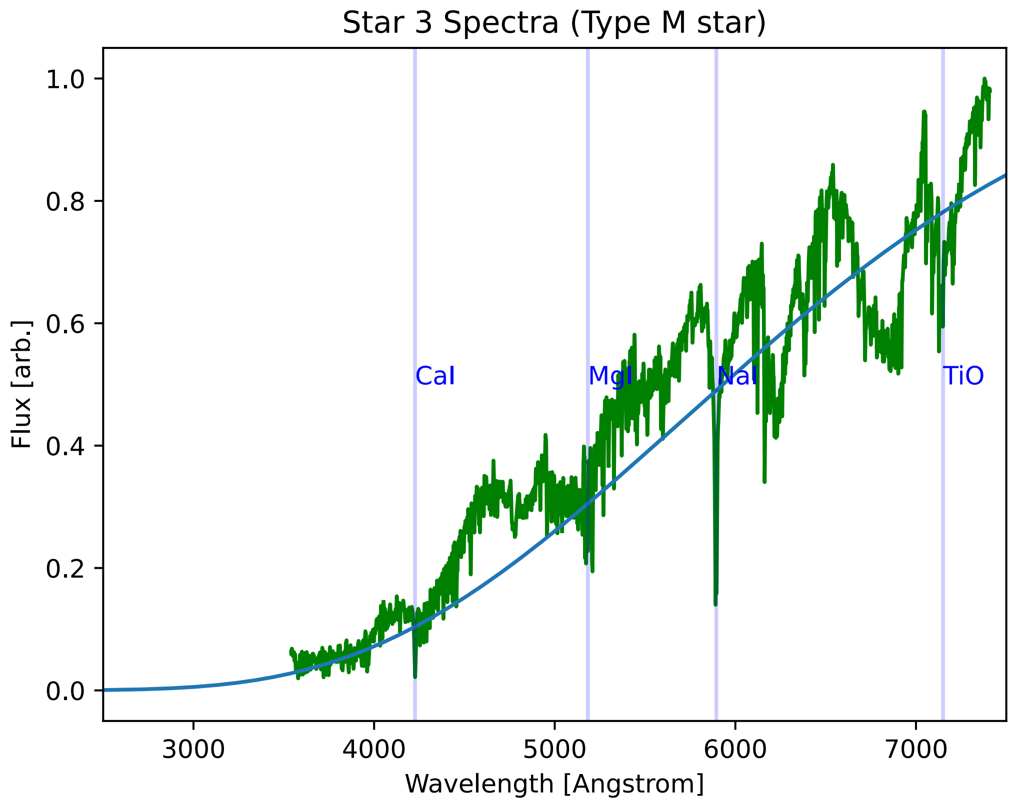
Star 1: As seen in the plotted spectra this star has very strong hydrogen lines (Balmer series) and some very weak almost unnoticable metal lines between 4000 and 7000 Angstrom. All lines are marked and labeled on the plot. For the hydrogen lines, H alpha is discribed by the absoption of energy by the electron making it jump from n=2 to n=3. This is the lowest transition in the balmer series. The next is H beta which is the jump from n=2 to n=4, then H gamma which is n=2 to n=5 and finally H delta which is n=2 to n=6. This spectra does have more hydrogen lines with absorption wavelengths less that 4000 Angstrom for further transitions but I was only asked to find absorbption features between 4000 and 7000 Angstrom. The metal lines seen in this star are neutral Calcium (CaI), Magnesium(MgI), and Sodium(NaI). Based on the strength of the hydrogen lines and the very very weak metal lines I believe this star is a type A star. Type A stars typically have a thermal continuity curve based on an effective temeperature of around 9000K. This line is over plotted on the spectrum in orange. To be able to compare the 2 plots I normalized each curve where the peak flux in arbitrary units is 1. As it can clearly be seen, this curve does not perfectly fit the spectra of the stars except for the Rayleigh-Jeans Tail where they begin to coincide well. I believe this is due to the fact that cool gas, like that found in the atmosphere of a star, is not well modeled by a blackbody distribution. A pure blackbody requires hot and dense gas, like the interior of a star. The light that leaves the interior could be modeled well by a blackbody, but this light interacts with the neutral hydrogen and other particles in the atmosphere of the sun and can be absorbed and reemmited. This leads to the higher energy photons (UV level energy) to be less abundant in our spectra as it is more easily absorbed by the hydrogen particles causing the bound electrons to be excited, and the spectrum drops rapidly at lower UV wavelengths as the star is not even hot enough to be able to emmit these wavelengths of light. This leads to the peak of the spectra to be shifted right to a higher peak wavelength compared to the 9000K Blackbody. This also would describe why we are not detecting the higher energy photons that we would expect from a true blackbody. The lower energy photons on the right of the plot are less likely to excite the hydrogen atoms in the atmosphere which makes it better match the thermal continuum.



Star 2: For star 2 I plotted the given spectra and identified the absorption features. Similar to the type A star I found strong hydrogen lines and metal lines. The difference here is that the metal lines are much stronger. This led me to determine that the star was most likely an F type star. Fitting the Planck distribution was similar to the first star. I normalized both functions and observed that the blackbody curve for an F type star does not perfectly match our observed spectra. This is due to atmospheric effects where the higher energy photons can interact with the metal and hydrogen in the atmosphere of the star and cause very low counts of high energy photons in our spectra. The right side of the plot fits better than the left as we would expect the lower energy photons to not interact with the neutral particles in the star's atmosphere. On this plot we are seeing higher flux amounts for higher wavelengths which I think could be caused by the remittance of photons that collide with the particles in the atmosphere.



Star 3: For star 3 I used the same plotting techniques as before. However, this star had basically nonexistent hydrogen lines and strong, but messy, metal lines. The most important feature here is the existence of TiO absorption. This made it obvious that we are looking at an M type star. That and M type star spectra always look very messy like this and peak more in the IR wavelength. Fitting a blackbody curve gives the line in blue. This curve fit this star's spectra the best out of the 3 stars, at least if we are looking at the average flux at each point. A few things would affect this curve fit. The atmospheres of M type stars are very cold compared to the interior, but it's also very thin. This atmosphere holds heavier metals and the hydrogen it has is neutral, but the star does not produce high enough energy photons to excite these particles and so they do not really show up in a spectrum. The photons leaving the star do not have much to interact with to absorb and be reemitted so the measured spectra would be closer to a thermal continuum and a star that interacts more with its atmosphere.



Star 3 Spectra (Type M star)
Flux [arb.]
Wavelength [Angstrom]
CaI
MgI
NaI
TiO
0.0
0.2
0.4
0.6
0.8
1.0
3000
4000
5000
6000
7000