Analytical ray tracing

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CHAPTER 1

Introduction

CHAPTER 2

Analytical ray tracing in two dimensions

Abstract

This chapter is a prologue to the next chapter, explores analytical ray tracing in two dimensions in both isometric and perspective projection.

2.1 Construction of the light rays

Let the camera be situated at point $C(c_x, c_y)$. The light rays drawn from this camera is represented by every line that passes through C. A general line equation is

$$L : mx + b = y \tag{2.1}$$

Let θ represent the pitch of the camera, which directly represents the slope of the line that passes through C; therefore,

$$L : \tan(\theta)x + b = y. \tag{2.2}$$

We put on a constraint that L must pass through C to find b in terms of θ, c_x, and c_y.

$$\tan(\theta)c_x + b = c_y \tag{2.3}$$

$$b = c_y - c_x\tan(\theta); \tag{2.4}$$

thus,

$$\tan(\theta)x + c_y - c_x\tan(\theta) = y \tag{2.5}$$

$$(x - c_x)\tan(\theta) = (y - c_y). \tag{2.6}$$

Given an arbitrary two-dimensional function $f(x) = y$ and θ, we have to find the intersection between $f(x)$ and L, which can be easily done:

$$(x - c_x)\tan(\theta) = f(x) - c_y \tag{2.7}$$

$$f(x) - x\tan(\theta) + (c_x\tan(\theta) - c_y) = 0. \tag{2.8}$$

For most functions, this equation might be even unsolvable. But it is analytically for polynomial $f(x)$ that has a degree less than five; most quintics are unsolvable. Or, we can use the Newton-Raphson's method to find the root.

2.2 Analytical demonstration

CHAPTER 3

Processes

3.1 The line of sight of the camera