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Number Representation & Boolean Function

TD 1

Objectives

- ✯ Natural numbers representation
- ✯ Basic conversion
- ✯ Strings of characters representation
- ✯ Boolean Function

1 Numbers Representation

Exercise 1

Fill in the table which might help you for the following exercises.

base 10	base 2	base 16
0		
1		
2		
3		
4		
5		
6		
7		

base 10	base 2	base 16
8		
9		
10		
11		
12		
13		
14		
15		

Exercise 2

Give the representation in binary form on 8 bits of the following decimal numbers:

Decimal	Binary
0	
7	
8	
78	
125	
255	

Exercise 3

Give the hexadecimal representation on 8 bits of the following decimal numbers:

Decimal	Hexadecimal
0	
8	
16	
84	
170	
255	

Exercise 4

Fill the table where the first column contains only positives numbers on 8 bits.

Binary	Decimal	Hexadecimal
0000 0000		
0000 0001		
0000 0101		
0000 1000		
0000 1001		
0000 1010		
0000 1011		
0000 1111		
0001 0000		
0001 1100		
1010 1101		

Exercise 5

Fill the table with two's complement values on 8 bits.

$(x)_{10}$	$(x)_2$	$(-x)_2$	$(-x)_{10}$
0			
	0000 1001		
110			
		1110 1010	
			-42
		0101 0010	
			-1

Exercise 6

Convert the following signed numbers

base 2 (on 8 bits)	base 10	base 16(on 1 byte)
	10_d	
$0000\ 0010_b$		
		10_h
	57_d	
	127_d	
		17_h
		$5B_h$
$0010\ 1001_b$		
$1010\ 1010_b$		

Exercise 7

How many bits are at least needed to encode the following numbers:

127_d

32_d

-127_d

-32_d

How many numbers can be encoded on a binary word of length n ?
What are the interval of values that can be represented by an unsigned binary word of length n ?

Exercise 8

How many numbers can be encoded on a hexadecimal word of length n ?
What are the interval of values that can be represented by an unsigned hexadecimal word of length n ?

Exercise 9

Let suppose that the following words represent natural numbers. Which one is the biggest ?

exp1	exp2	winner
$(20)_8$	$(100)_2$	
$(0F0)_{16}$	$(710)_8$	
$(56)_8$	$(56)_{10}$	
$(BDE)_{16}$	$(3038)_{10}$	
$(EC)_{16}$	$(100000000)_2$	

2 Characters representation

Exercise 10

Encode the following letters:

Letter	code ASCII (base 16)
'C'	
'c'	
	$0C_{16}$
'#'	
	64_{16}
	$7A_{16}$

Exercise 11

For the following exercises we will consider that each character is encoded in ASCII

1. What are the hexadecimal representation of ”BIT” ?
2. Which word is represented by 5444203100_{16} ?
3. What sentence is encoded by the following binary word:
   01010110011010010111011001100101001000000110110000100111011010010110111001100110011010111100000000

3 Operations

Exercise 12

Let A, B and C 3 numbers in twos complement, compute the following operations:

①

A:		0110 0011
B:	+	0001 1001
C:	=	

②

A:		0110 0011
B:	+	0011 1001
C:	=	

③

A:		1010 1011
B:	+	1001 1001
C:	=	

④

A:		1110 0011
B:	+	0011 1001
C:	=	

- For which operation the result on 8 bits was correct?

- What does “overflow” means?

Exercise 13

Fill up the following table, where *carry out* is the output carry, *carry* the carry of the preceding rank and *Overflow* specify if the operation has produced a value that is outside of the range that can be represented on 8 bits. Deduce the operation which can compute the overflow.

carry out	carry	Overflow
0	0	
0	1	
1	0	
1	1	

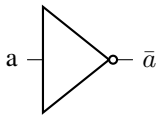
4 Boolean Expressions

Exercise 14 Logic Gates

Let a, b and c be 3 Boolean variables, fill in the following truth table:

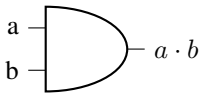
- **Not:**

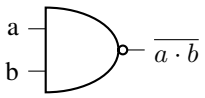
a	$\bar{a}$
0	
1	



- **And/Nand**

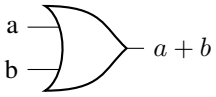
a	b	$a \cdot b$	$\overline{a \cdot b}$
0	0		
0	1		
1	0		
1	1		

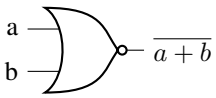




- **Or/Nor**

a	b	$a + b$	$\overline{a + b}$
0	0		
0	1		
1	0		
1	1		





Exercise 15

Give the truth table of the following Boolean equation:

$s = b + \overline{b}.a.c$

Exercise 16

Does the above expression could be simplified ? Justify your answer.

Exercise 17

- Explain how do we construct the Disjunctive Normal Form of a function out of its truth table.
- Give the Disjunctive Normal Form of the function f described by the following truth table. f has 3 inputs: a, b and c.

a	b	c	s
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Exercise 18

Give the Conjunctive Normal Form of the function f described by the above truth table

Exercise 19

Using truth table show that in Boolean logic we have:

$$a + (b.c) = (a + b).(a + c)$$

Using distribution on the expression on the right, explain how one could simplify it to get the expression on the left.

Exercise 20

Give the truth table of the function `XOR`.

Exercise 21

For the `XOR` truth table, give the conjunctive and disjunctive normal form of that function.
Show that both expressions are equivalent.

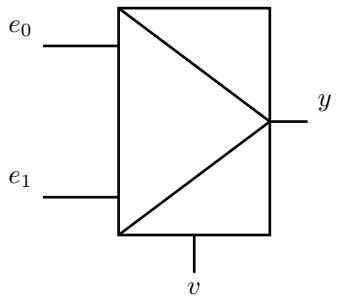
Exercise 22

Using De Morgan rules, find an expression for the function $\overline{(a \texttt{ XOR } b)}$.

Exercise 23 2 Entries Multiplexer

A multiplexer with two entries is an electronic circuit with two bits e_0 and e_1 and a third bit v as inputs. v is a selector which allow to chose the first entry e_0 when $v = 0$ and the second entry e_1 when $v = 1$.

Give the truth table of a two input multiplexer by expressing the output value y ($y = e_0$ or $y = e_1$) with respect of the value of v.



e_0	e_1	v	y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Give the disjunctive normal form of such a multiplexer.

Exercise 24 8 Entries Multiplexer

Now, we want to find the Boolean expression describing an 8 entries multiplexer. The value of the selector is encoded by a set of bits $i_0, i_1, \cdots$. We decide that i_0 is the least significant bit. That is, the value v of the selector is given by **v**= $\sum_k i_k.2^k$.

Let $f_i(v)$ with $i \in [0, 7]$ be Boolean functions which equals 1 when $v = i$ and 0 otherwise. So, for a given value $v \in [0, 7]$, the function $f_v(v)$ equals 1 and the $f_i(v)$ for $i \neq v$ equals 0.

Give the Boolean expression off the output of a 8 input multiplexer. It a function which equals e_v if and only if $f_i(v)$ = 1, for any $v \in [0, 7]$.

Exercise 25

Now, we want to give the expressions for the functions $f_i(v)$. The following table give the decimal value corresponding to the selector value:

i_2	i_1	i_0	$decimale\ value$
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Fill the following truth table with values of functions $f_i(v)$ for each entries:

i_2	i_1	i_0	$decimale\ value$	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7
0	0	0	0								
0	0	1	1								
0	1	0	2								
0	1	1	3								
1	0	0	4								
1	0	1	5								
1	1	0	6								
1	1	1	7								

Exercise 26

Give the Boolean expression of each function $f_i(v)$ which depends of v so of i_2, i_1 and i_0.

Exercise 27

Starting for the Boolean expression obtained at question 12 and Boolean expressions of function $f_i(v)$, give the Boolean expression for a 8 inputs multiplexer.

5 Annexes

5.1 ASCII table

ASCII Code Chart

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2		!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL