S2250 – Statistics

Important Series & summations

Geometric Series:

$$\sum_{i=k}^{i=n} p^i = p^k\left(\frac{1-p^{n-k+1}}{1-p}\right) = 1^{st}\ term\left(\frac{1-p^{number\ of\ terms}}{1-p}\right)$$

Common cases:

$\sum_{i=0}^{i=n} p^i = \left(\frac{1-p^{n+1}}{1-p}\right)$	$\sum_{i=1}^{i=n} p^i = p\left(\frac{1-p^{n}}{1-p}\right)$	$\sum_{i=k}^{i=\infty} p^i = p^k\left(\frac{1}{1-p}\right)$ $(p<1)$

Arithmetic Series:

$$\sum_{i=k}^{i=n} i = \left(\frac{(n-k+1)(n+k)}{2}\right) = \left(\frac{(number\ of\ terms)(last\ term + 1^{st} terms)}{2}\right)$$

Common cases:

$\sum_{i=1}^{i=n} i = \left(\frac{(n)(n+1)}{2}\right)$	$\sum_{i=0}^{i=n} i = \left(\frac{(n+1)(n)}{2}\right)$	$\sum_{i=0}^{i=n-1} i = \left(\frac{(n)(n-1)}{2}\right)$

Telescoping Series:

$$\sum_{i=k}^{i=n} \frac{1}{i(i+1)} = \sum_{i=k}^{i=n} \frac{1}{i} - \frac{1}{i+1} = \frac{1}{k} - \frac{1}{n+1}$$

$1^{st}\ term\ (i=k)$: $\frac{1}{k} - \boxed{\frac{1}{k+1}}$

$2^{nd}\ term\ (i=k+1)$: $\boxed{\frac{1}{k+1}} - \boxed{\frac{1}{k+2}}$

$3^{rd}\ term\ (i=k+2)$: $\boxed{\frac{1}{k+2}} - \boxed{\frac{1}{k+3}}$

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$last\ term\ (i=n)$: $\boxed{\frac{1}{n}} - \frac{1}{n+1}$

It's easy to notice that the terms are cancelling each other.

Examples:

$$\sum_{i=1}^{i=10} \frac{1}{i(i+1)} = \frac{1}{1} - \frac{1}{10+1} = 1 - \frac{1}{11}$$

$$\sum_{i=1}^{\infty} \frac{1}{i(i+1)} = \lim_{n\to\infty}\left(\frac{1}{1} - \frac{1}{n+1}\right) = 1$$

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