Fast Fourier Transform

Cameron Williams

ECE 351-51

Lab Report 9

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1 Introduction

The objective of this lab was to become familiar with the use of fast Fourier transforms using Python.

2 Methodology

I started this lab by creating a user-defined fft() function in my Python script (submitted with this report). I used the example code for the fast fourier transform provided in the lab handout as a guide. I used the functions $cos(2\pi t)$, $5sin(2\pi t)$, and $2cos(2\pi 2t) + sin^2((2\pi 6t) + 3)$ to creat the plots for Task 1, Task 2, and Task 3 respectively. For each of these, I used the sampling frequency fs=100 and scaled the axes on the respective plots appropriately. The plots may be seen in the results section. As part of Task 4, I created a second user-defined function called fft_clean(). This function works almost exactly the same as fft, but sets the phase shift of any element with a corresponding magnitude less than 1e-10 to zero. This cleans up the appearance of many of the plots. I then recreated the plots the previous plots with fft_clean(). These plots may also bay seen in the Results section. Finally, for Task 5, I used my code from Lab 8 to recreate the square wave approximation with N=15 and ran the result through the fft_clean() function. A plot of the result may be seen in the Results section.

Results

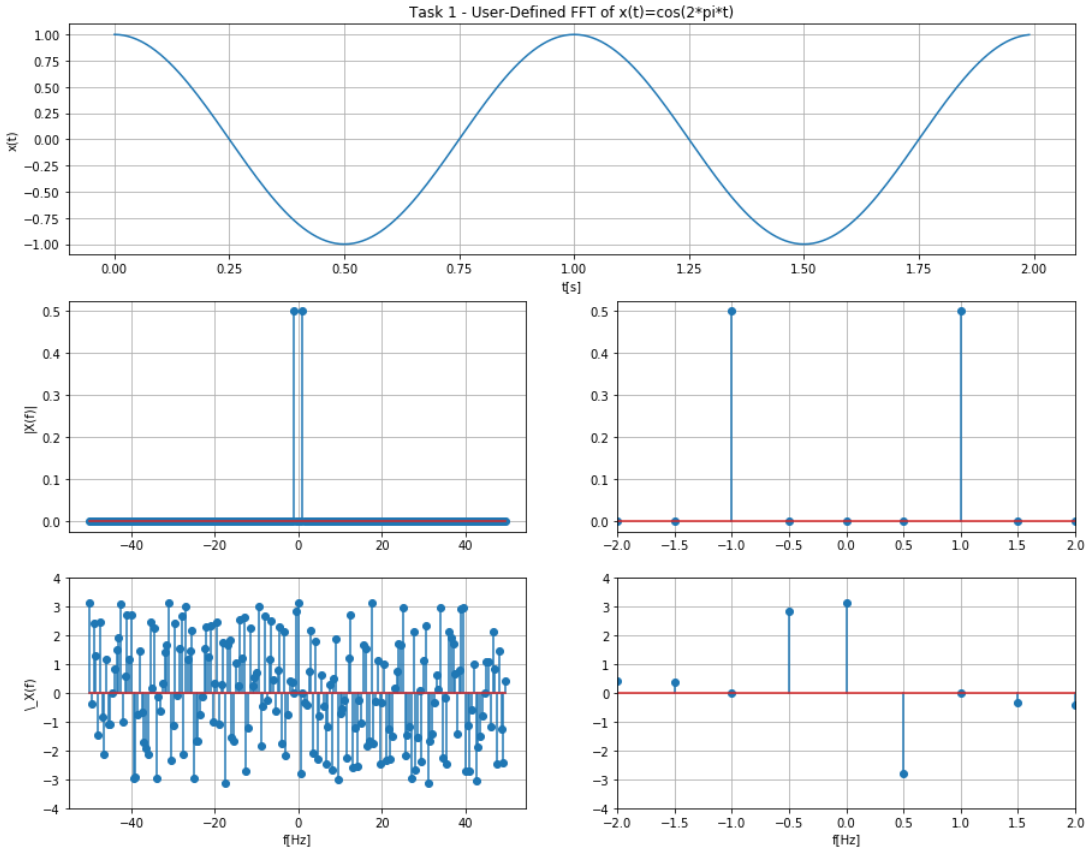

Figure 1: Plots for Task 1.

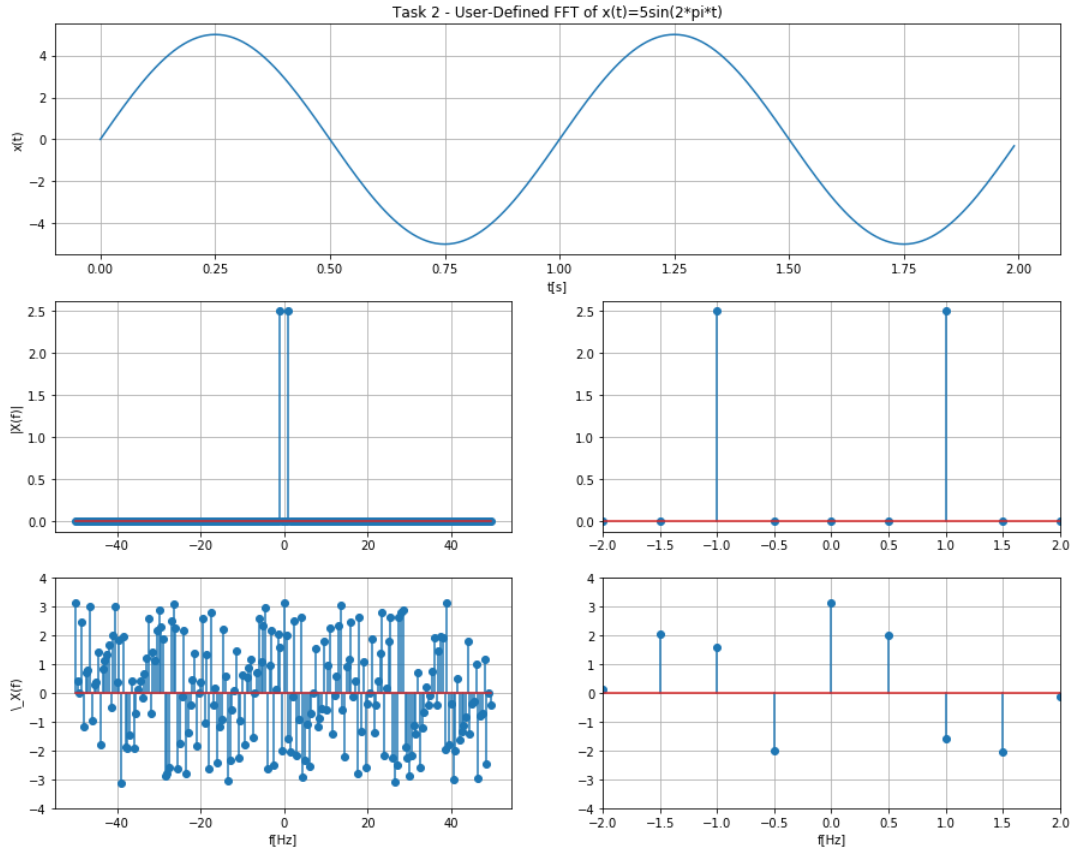

Figure 2: Plots for Task 2.

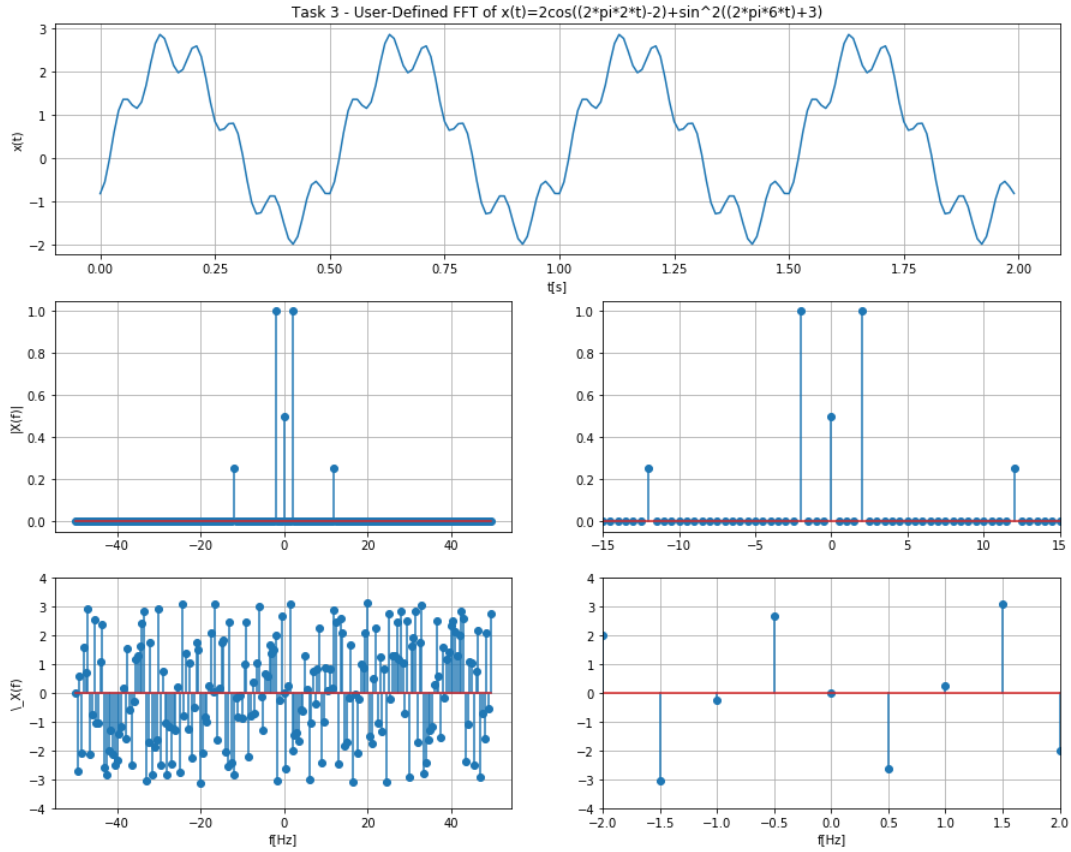

Figure 3: Plots for Task 3.

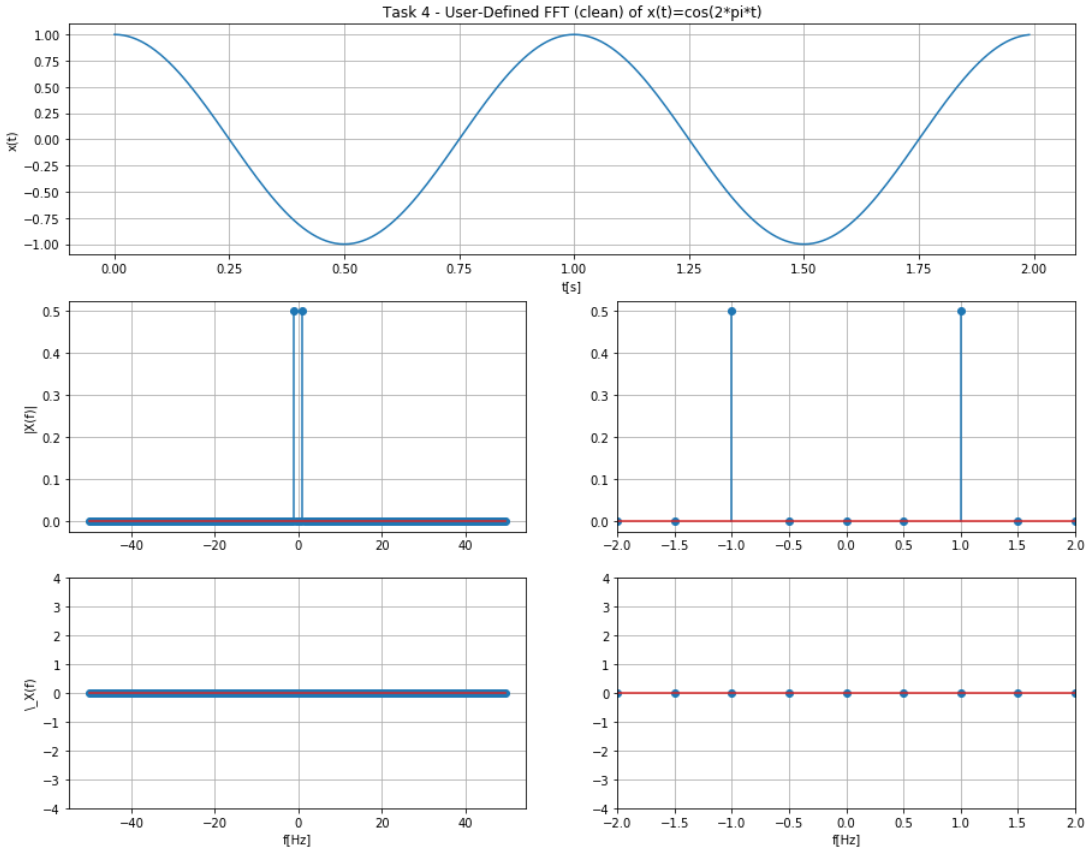

Figure 4: Task 1 Plots redone for Task 4.

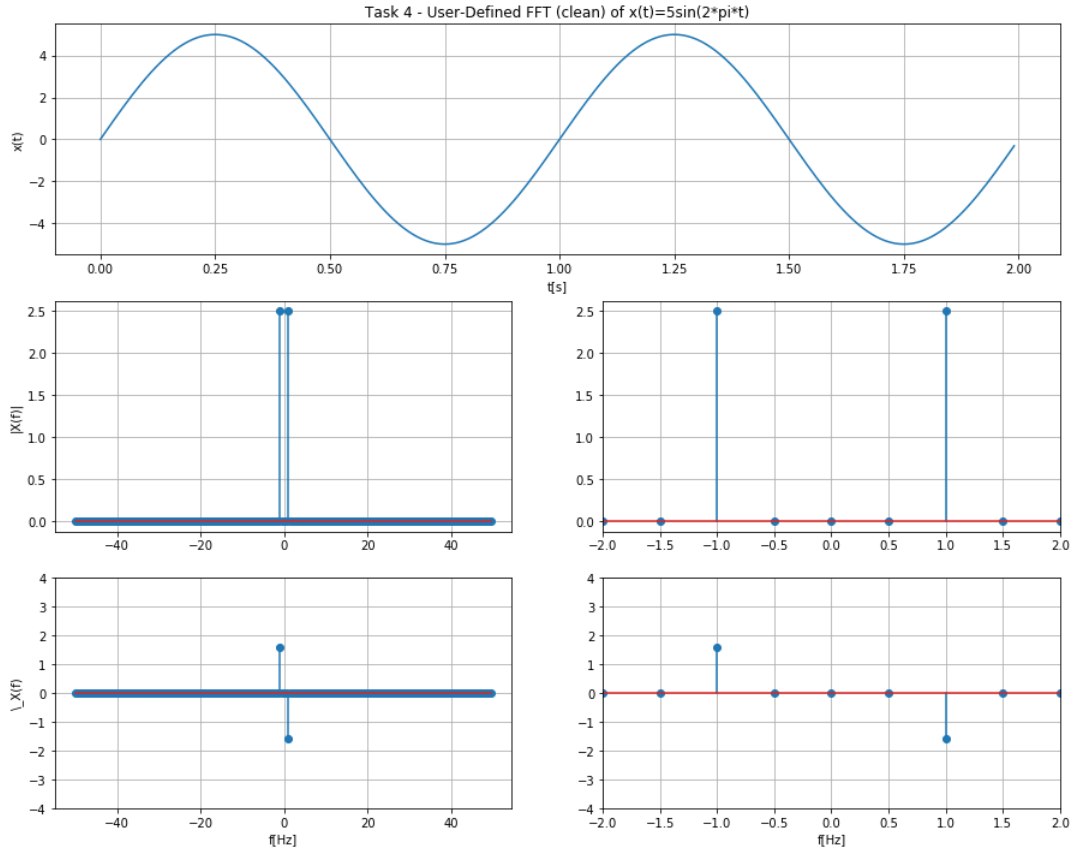

Figure 5: Task 2 Plots redone for Task 4

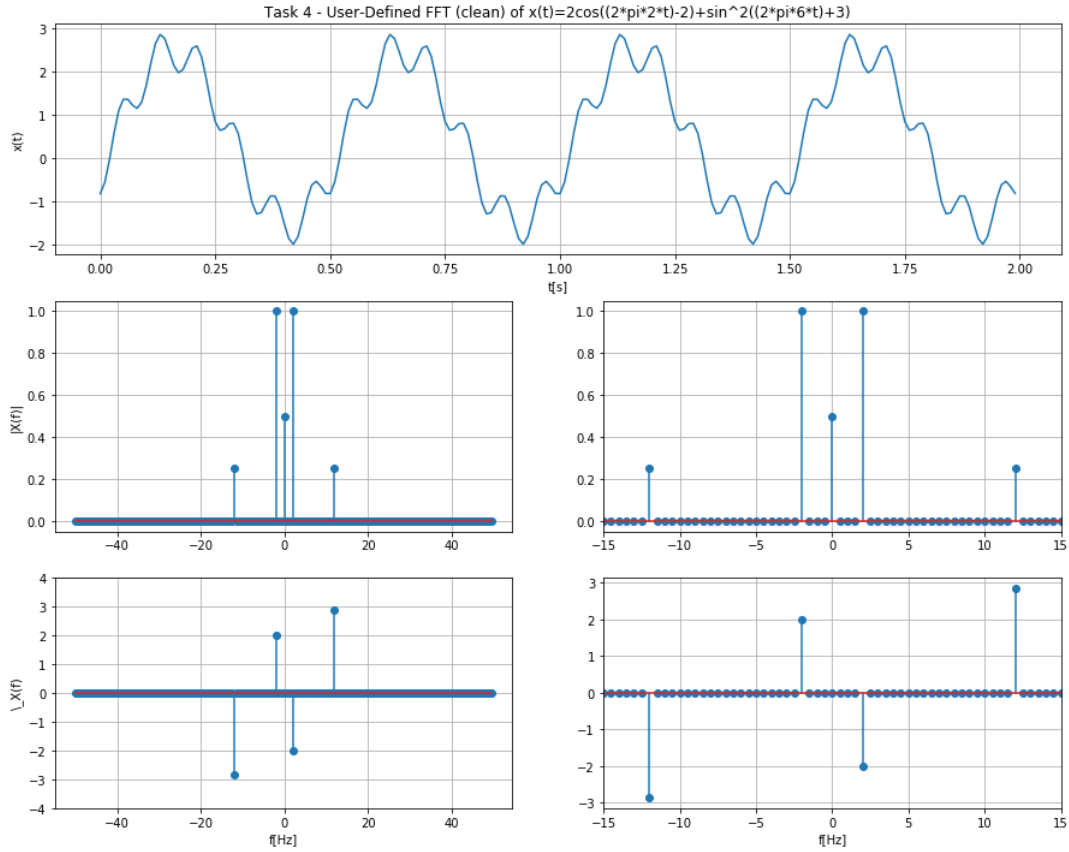

Figure 6: Task 3 Plots redone for Task 4.

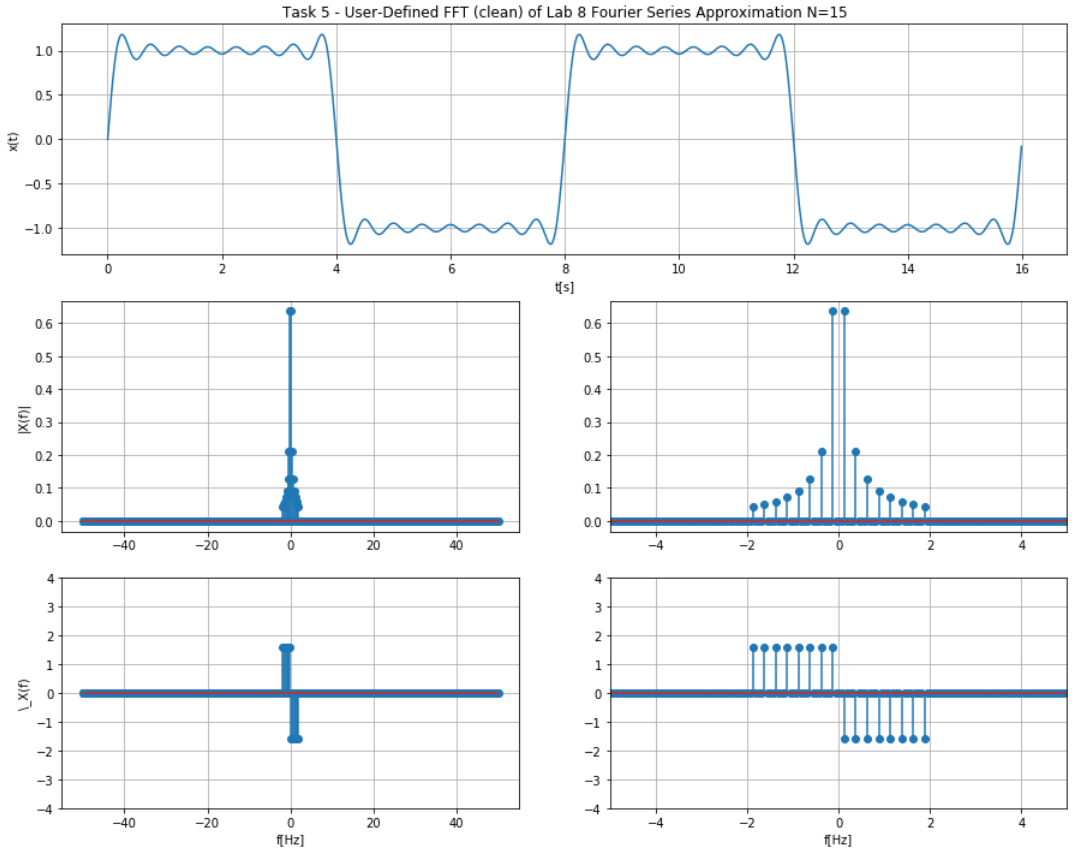

Figure 7: Task 5 plots.

Questions

1. What happens if fs is lower? If it is higher? fs in your report must span a few orders of magnitude.

With fs set lower (I tried 10) the resolution of the output plot is lower and there are far fewer phase shift delta functions in the bottom plots. With fs set higher (I tried 1000), the resolution is improved and there are a lot more phase shift delta functions in the bottom plots. Finally, I tried fs=10000. The larger fs is, the longer the run time and at fs=10000 it took over 10 minutes. The phase shift output plot was so crowded with delta functions that it looked solid blue.

2. What difference does eliminating the small phase magnitudes make?

In terms of the plots, it makes them much more readable. In terms of the Fourier transform results themselves, though it eliminates many of the terms contributing to the output, the change is negligible since the magnitude lower limit specified (1e-10) is so small.

3. Verify your results from Tasks 1 and 2 using the Fourier transforms of cosine and sine. Explain why your results are correct. You will need the transforms in terms of Hz, not rad/s. For example, the Fourier transform of cosine (in Hz) is:

$$\mathcal{F}\{cos(2\pi f_0 t)\} = \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$$

As described above, the Fourier transform of cosine in Hz results in two delta functions, one negative and one positive corresponding to the frequency of the function with a magnitude equal to half of the coefficient in front of the cosine function. The sine function results in similar delta functions appearing when transformed. The difference between the sine and cosine fourier transforms is the phase shifting. As seen in the fft_clean() result of cosine, there is no phase shift. This makes sense since cosine has no imaginary components. The sine function does on the other hand, resulting in two delta functions appearing in the phase shift (one negative, one positive).

4. Leave any feedback on the clarity/usefulness of the purpose, deliverables, and expectations for this lab.

The expectations for this lab were expressed clearly in the lab handout.