Step and Impulse Response of a RLC Band Pass Filter

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ECE 351-51

Lab Report 5

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1 Introduction

The objective of this lab is to use Laplace transforms to find the time-domain step and impulse responses of an RLC bandpass filter (pictured below).

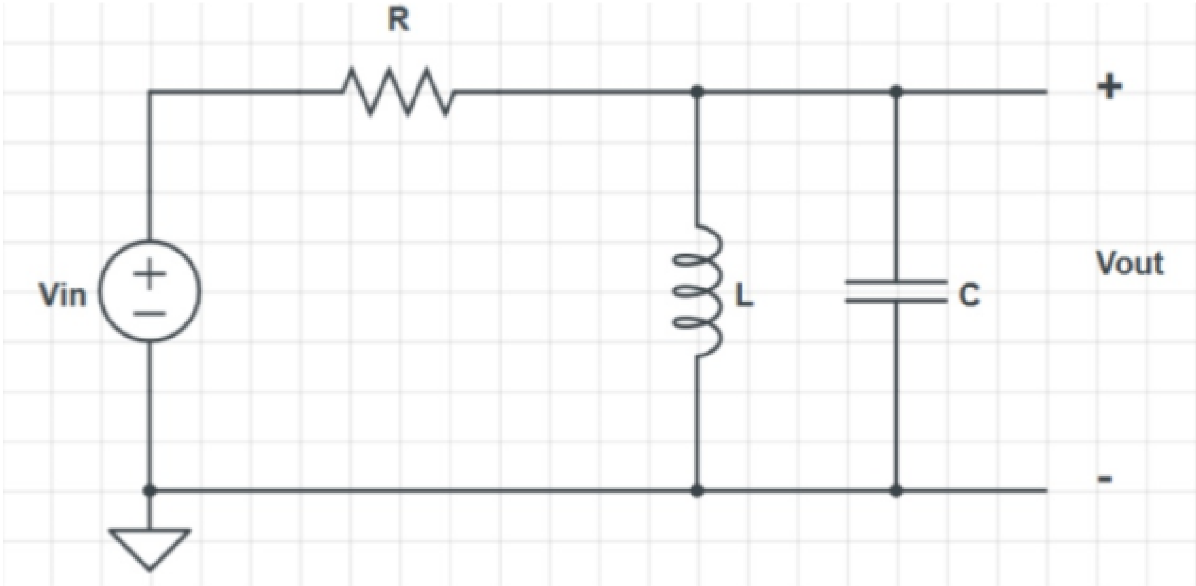

Figure 1: R = 1kΩ, L = 27 mH, C = 100 nF

2 Methodology

Before attending the lab session, I completed the requisite prelab assignment. However, I did not complete the calculation correctly so the corrected calculation may be seen in the first series of calculations of the Calculations section. Using the transfer function calculated, I plotted the step response by creating a function called sine_method() that solves for the transfer function of the RLC circuit symbolically. The function takes inputs for R, L, and C. I plotted the function by passing the defined R, L, and C values provided in the prelab to the sine_method function. For comparison, I also plotted the impulse response using the scipy.signal.impulse() function. This function takes two arrays of coefficients as input – one for the polynomial in the numerator and one in the denominator as well as an array for the times to calculate at. The plots of the manually calculated impulse response and the scipy.signal.impulse() may be seen in Figure 1 of the Results section.

Next, I found the step response of the RLC circuit by using the scipy.signal.step() function. Similar to the impulse function from the same library, it takes two arrays of polynomial coefficients and an array for times to perform the calculation at as inputs. A plot of the step response may be seen in Figure 2 of the Results section.

Finally, I used the Final Value Theorem on the transfer function that I calculated. The calculation may be seen in the Calculations section titled "Final Value Theroem calculation". The Final Value Theorem resulted in a value of 0. This agrees with the plots since the result converges to 0 as time goes on.

3 Calculations

Transfer function calculations:

$$
\begin{aligned}
H(s) &= \frac{V_{out}}{V_{in}} \\
&= \frac{L||C}{R + L||C} \\
L||C &= \frac{1}{\frac{1}{sL} + \frac{1}{\frac{1}{sC}}} \\
&= \frac{1}{\frac{1}{sL} + sC} \\
&= \frac{(\frac{1}{C}) * s}{s^2 + \frac{1}{LC}} \\
H(s) &= \frac{\frac{(\frac{1}{C})*s}{s^2+\frac{1}{LC}}}{R + \frac{(\frac{1}{C})*s}{s^2+\frac{1}{LC}}} \\
&= \frac{\frac{1}{RC} * s}{s^2 + \frac{1}{RC} * s + \frac{1}{LC}} \\
p &= \frac{\frac{1}{RC} + \sqrt{\frac{1}{RC}^2 - 4(\frac{1}{LC})}}{2} \\
p &= \alpha + j\omega \\
g &= \frac{1}{RC} * s\Big|_{s=p} \\
&= \left(\frac{-1}{2}\right)\left(\frac{1}{RC}\right)^2 + \left[\left(\frac{1}{2RC}\right)\sqrt{\left(\frac{1}{RC}\right)^2 - 4\left(\frac{1}{\sqrt{LC}}\right)^2}\right]^2 \\
g\angle &= \arctan \frac{\sqrt{\frac{1}{2RC}\sqrt{\frac{1}{2RC}}^2 - 4\left(\frac{1}{\sqrt{LC}}\right)^2}}{-\frac{1}{2}\left(\frac{1}{RC}\right)^2} \\
y_s(t) &= \frac{|g|}{\omega} e^{\alpha t} sin(\omega t + \angle g) u(t)
\end{aligned}
$$

Final Value Theorem calculation:

$$
\begin{aligned}
\lim_{t\to\infty}(f(t)) \to \lim_{s\to\infty}(s * F(s)) &= \lim_{s\to\infty}\left(\frac{\frac{1}{RC} * s^2}{s^2 + \frac{1}{RC} * s + \frac{1}{LC}}\right) \\
&= \left(\frac{0}{0 + 0 + \frac{1}{LC}}\right) \\
&= \frac{0}{\frac{1}{LC}} \\
&= 0
\end{aligned}
$$

4 Results

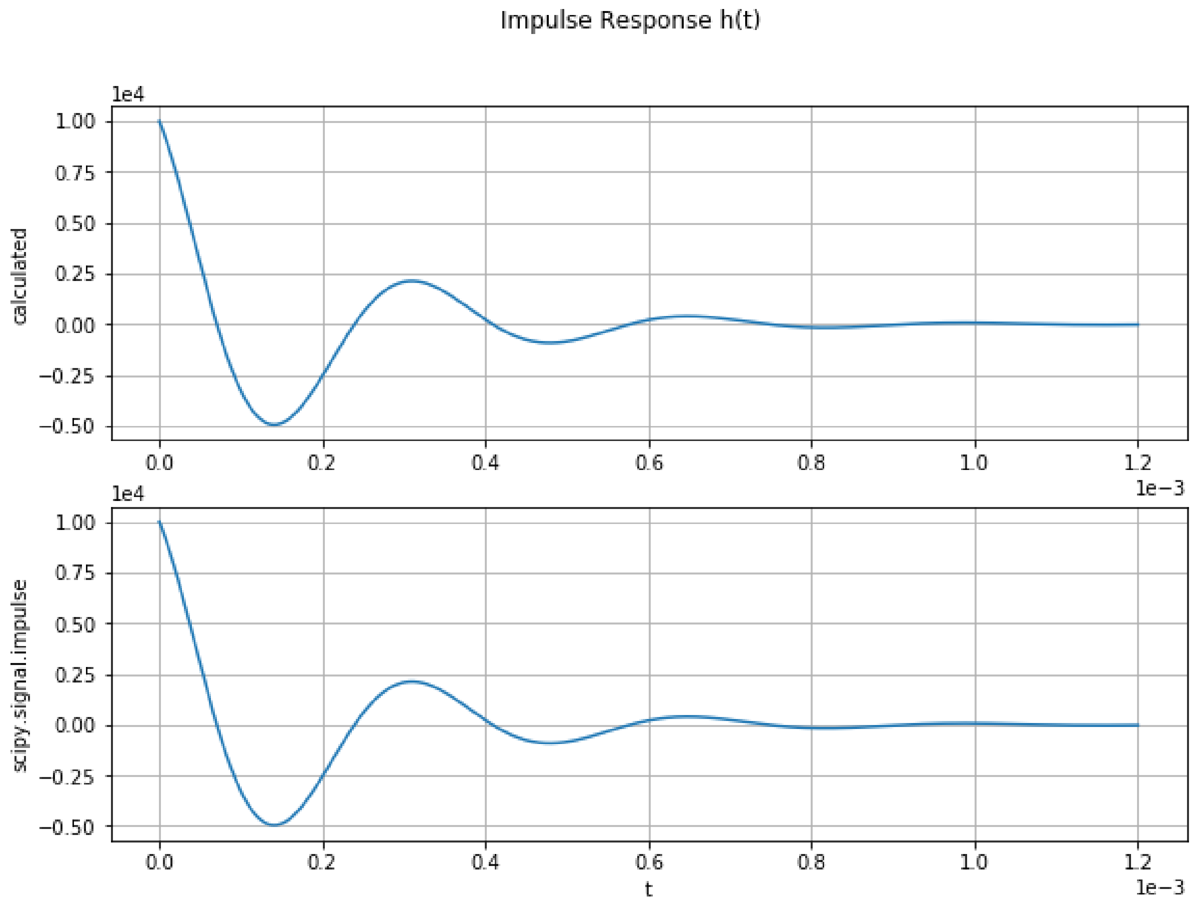

Figure 1: Impulse responses of the RLC circuit: hand-calculated then plotted (top) and found using scipy.signal.impulse() function then plotted (bottom).

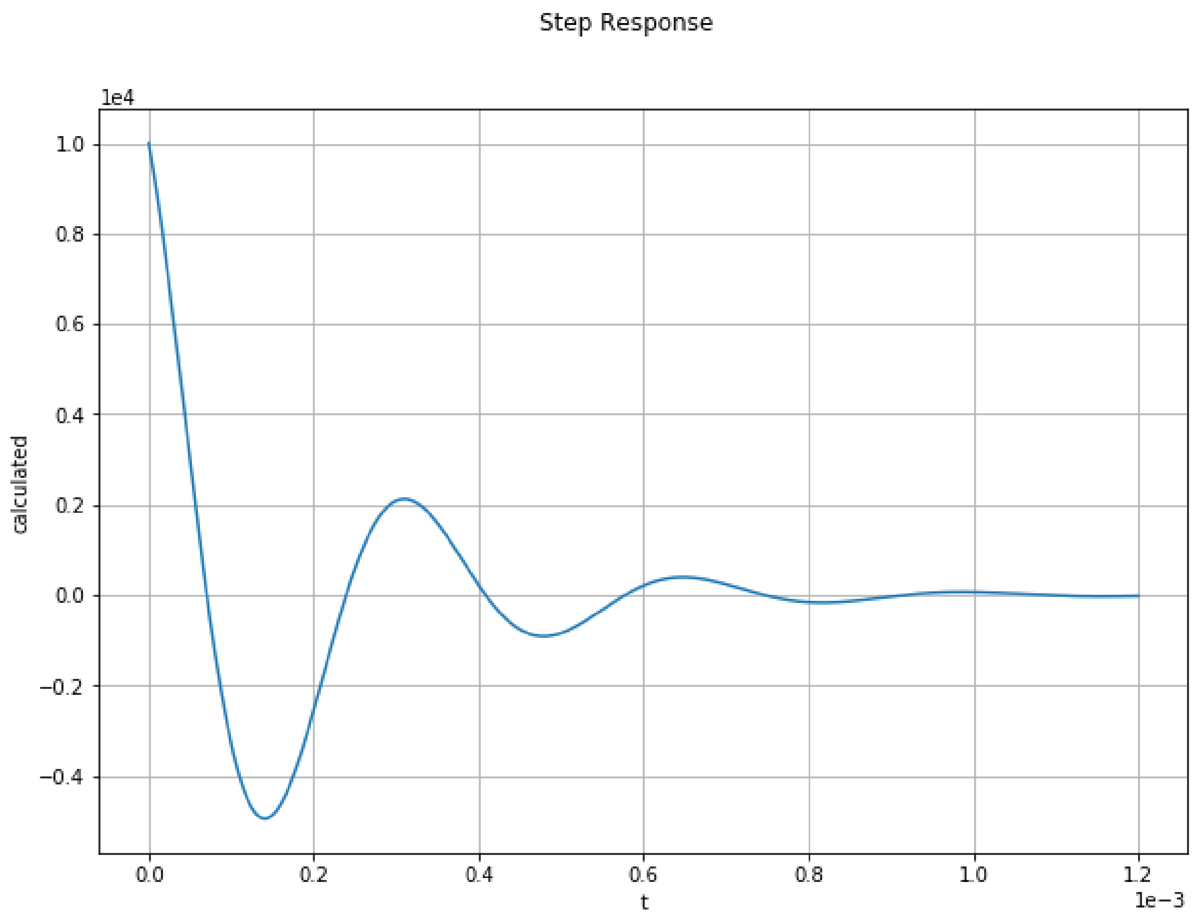

Figure 2: Plot of the step response of the RLC circuit found using the scipy.signal.step() function.

Questions

1. Explain the result of the FInal Value Theorem from Part 2 Task 2 in terms of the physical circuit components.

There are no active components, only passive components (resistor, inductor, and capacitor), so it makes sense that an applied impulse would eventually converge to zero as time went on.

1. Leave any feedback on the clarity of lab tasks, expectations, and deliverables.

The expectations and deliverables for this lab were expressed clearly.