System Step Response Using Convolution

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ECE 351-51

Lab Report 4

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1 Introduction

The objective of this lab is to become familiar with using convolution to compute a system's step response. To accomplish this, three provided transfer functions will be used in finding their respective step responses via the convolution function created in a previous lab. Additionally, the step responses will be calculated manually and plotted for comparison.

2 Methodology

I began by importing my convolution, step, and ramp functions created in previous labs. Using the appropriate functions, I created plots of the transfer functions provided. The equations for these transfer functions may be seen below and their plots may be seen in Figure 1 of the results section.

$$h_1(t) = e^{2t}u(1-t)$$

$$h_2(t) = u(t-2) - u(t-6)$$

$$h_3(t) = cos(\omega_0 t)u(t)$$

Next, I plotted the step response of each of the provided transfer functions. Plots of the three step responses may be seen in Figure 2 of the Results section. I also created plots based on hand calculated step responses to the transfer functions. The calculations for each of the step responses may be seen in the Calculations section and a plot of the results for each may be seen in Figure 3 of the Results section.

3 Calculations

$$
\begin{aligned}
h_1(t) * u(t) &= e^{2t}u(1-t) * u(t) \\
&= \int_{-\infty}^{\infty} e^{2\tau}u(1-\tau)u(t-\tau)d\tau \\
&= \int_{-\infty}^{t} u(1-\tau)d\tau + \int_{-\infty}^{t} e^{2\tau}u(\tau-1)d\tau \\
&= \int_{\infty}^{t} e^{2\tau}d\tau + \int_{-\infty}^{1} e^{2\tau}d\tau \\
&= \frac{1}{2}e^{2t}u(1-t) + \frac{1}{2}e^{2}u(t-1)
\end{aligned}
$$

$$
\begin{aligned}
h_2(t) * u(t) &= [u(t-2) - u(t-6)] * u(t) \\
&= \int_{-\infty}^{\infty} [u(\tau-2) - u(\tau-6)]u(t-\tau)d\tau \\
&= \int_{-\infty}^{t} u(\tau-2) - u(\tau-6)d\tau \\
&= r(t-2) - r(t-6)
\end{aligned}
$$

$$
\begin{aligned}
h_3(t) * u(t) &= [cos(\omega_0 t)u(t)] * u(t) \\
&= \int_{-\infty}^{\infty} [cos(\omega_0 \tau)u(\tau)]u(t-\tau)d\tau \\
&= \int_{-\infty}^{t} cos(\omega_0 \tau)u(\tau)d\tau \\
&= \int_{0}^{t} cos(\omega_0 \tau)d\tau \\
&= \frac{1}{\omega_0} sin(\omega_0 t)u(t) \\
&= \frac{1}{2\pi(.25)} sin(2\pi(.25)t)u(t)
\end{aligned}
$$

4 Results

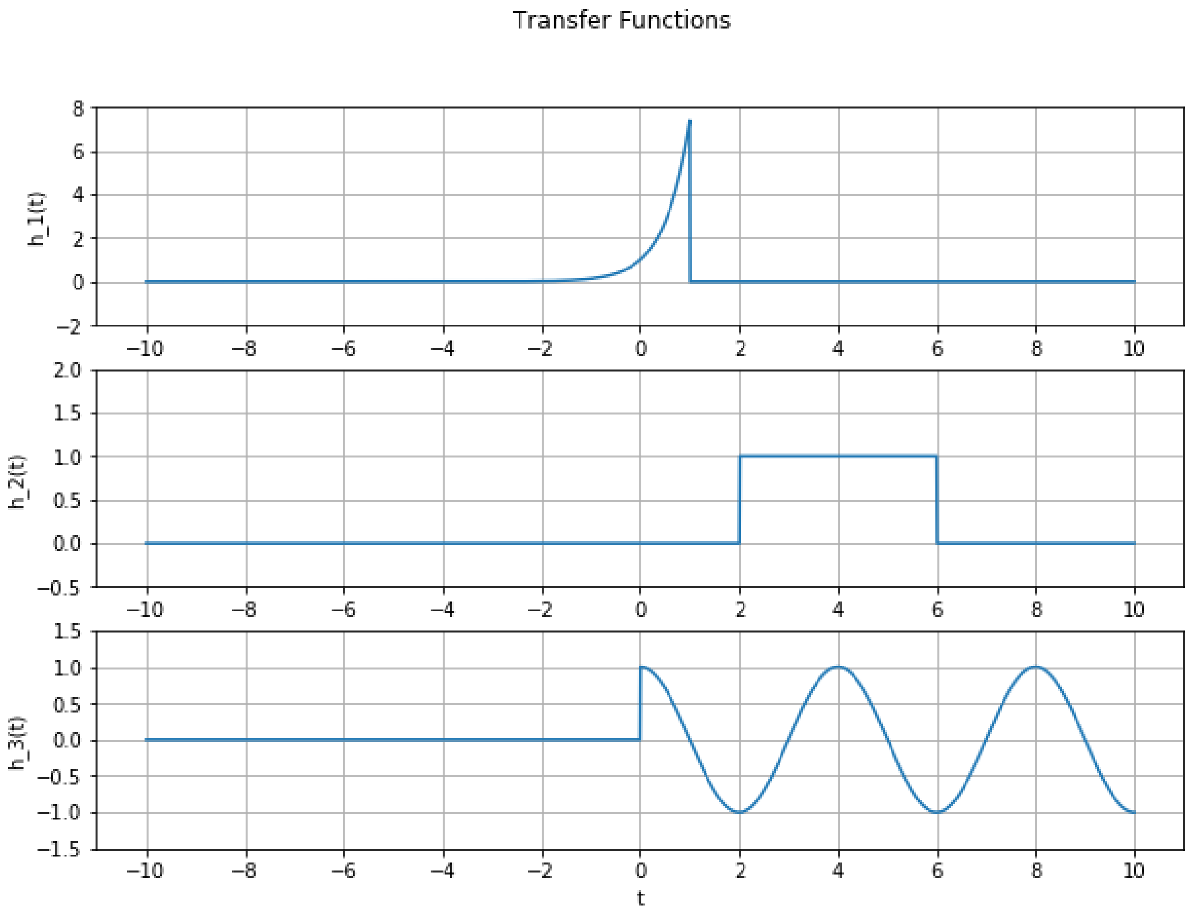

Figure 1: Plots of transfer functions provided.

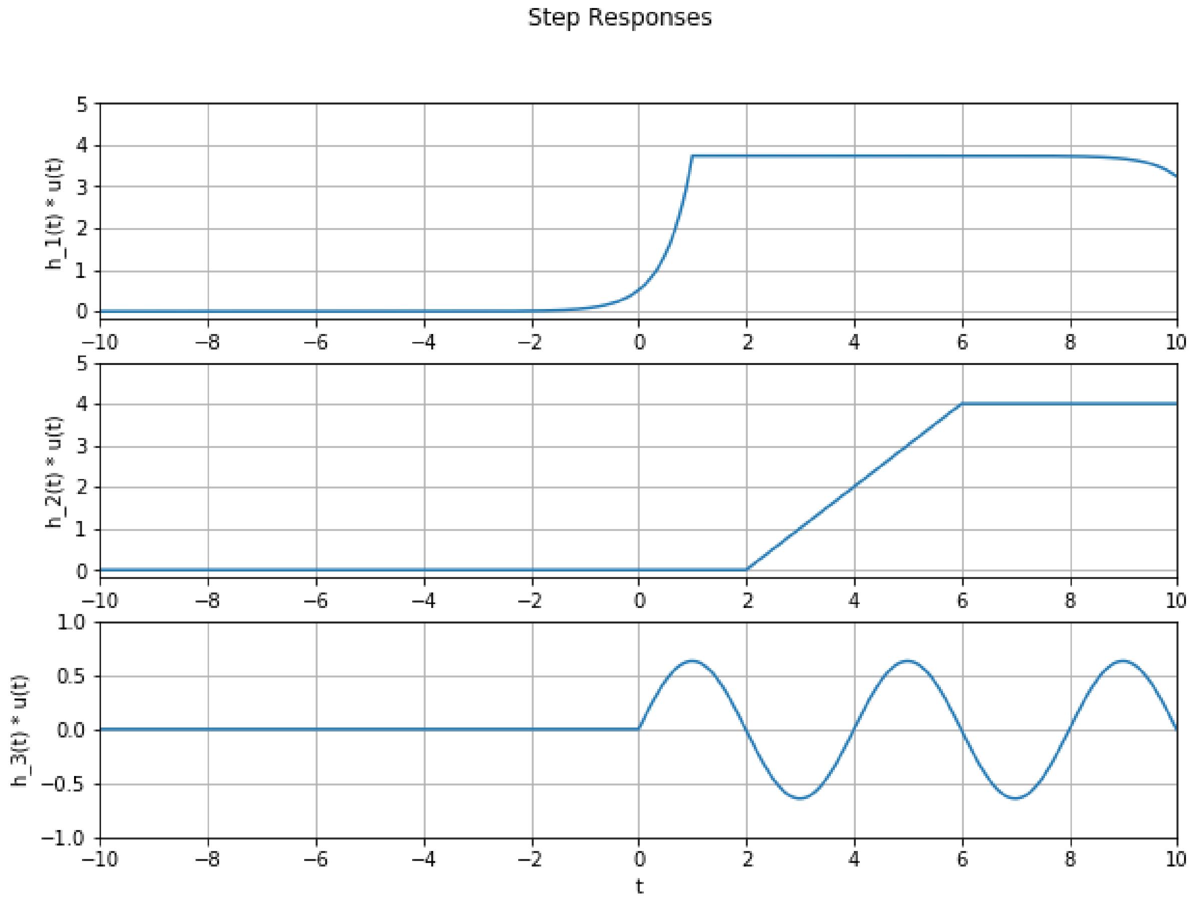

Figure 2: Plots of step responses via my convolution function.

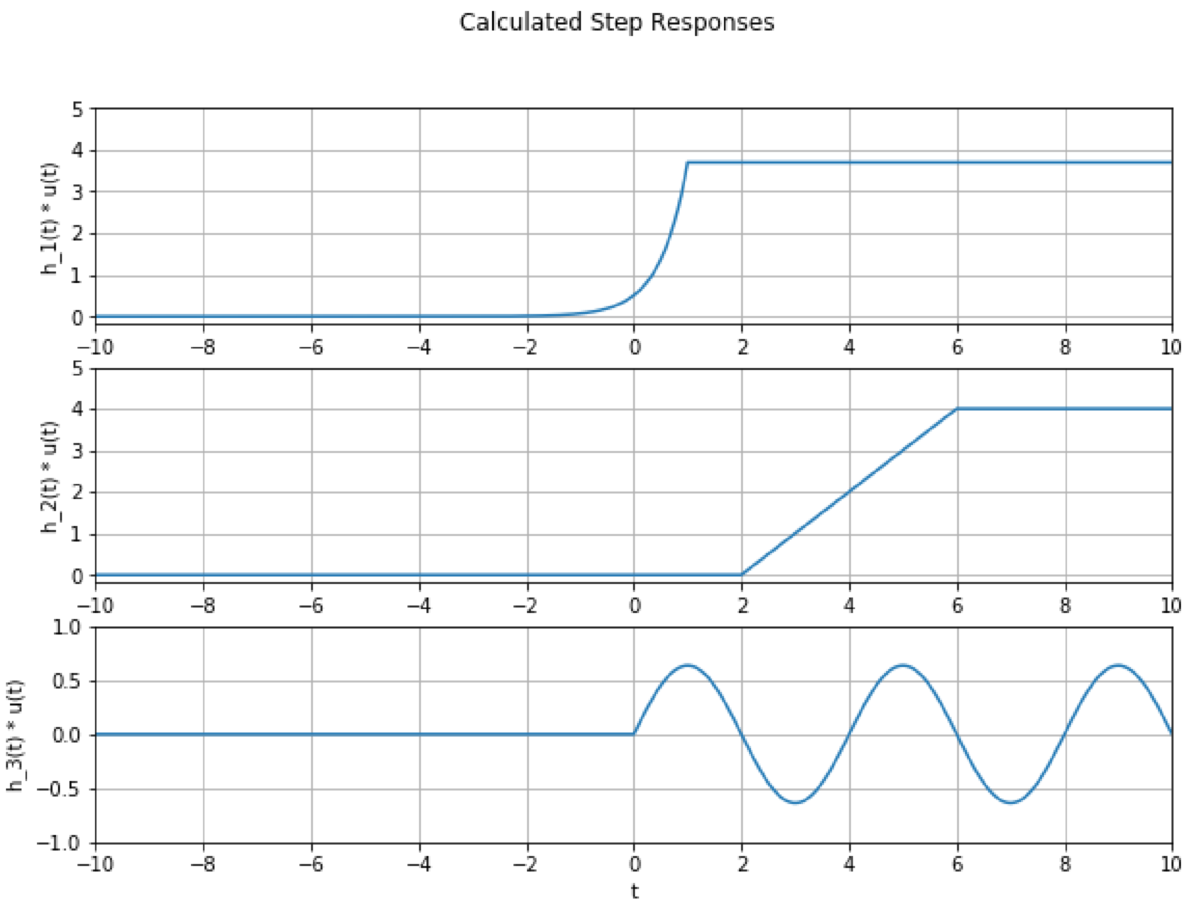

Figure 3: Plots of calculated step responses.

5 Conclusion

Creating the Python script for this lab was fairly straightforward. However, calculating the convolutions manually gave me some difficulty. I could use more practice with this.

Questions

1. Leave any feedback on the clarity of lab tasks, expectations, and deliverables.

The lab expectations were communicated clearly.