

Fourier Series Approximation of a Square Wave

Cameron Williams

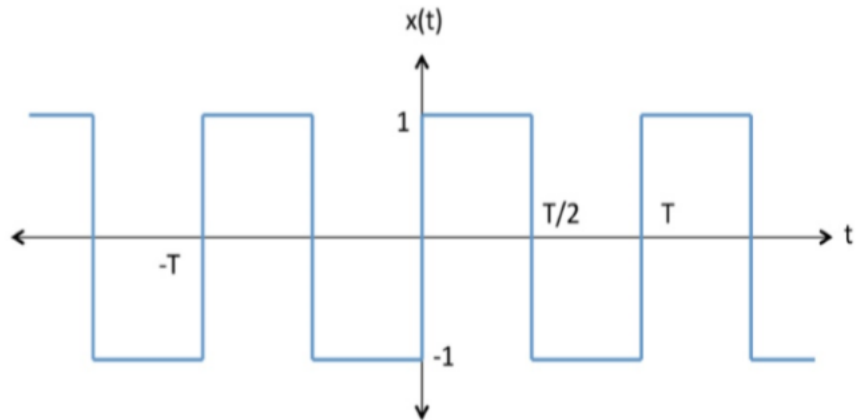
ECE 351-51

Lab Report 8

24 March 2020

1 Introduction

The objective of this lab was to use Fourier series to approximate periodic time-domain signals. For the purposes of this lab, the square wave function pictured below was used.



2 Methodology

The simplified a_k and b_k values for the Fourier series approximation may be seen in the Equations section. I implemented them as functions in my Python script and used them to print out the first two values of a_k , a_0 and a_1 . I also printed out the values of b_1 , b_2 , and b_3 . These values may be seen in the Appendix. Next, I implemented a summation of the Fourier series in my Python script and plotted it for values $N = 1$, $N = 3$, $N = 15$, $N = 50$, $N = 150$, and $N = 1500$. These plots may be seen in the Results section.

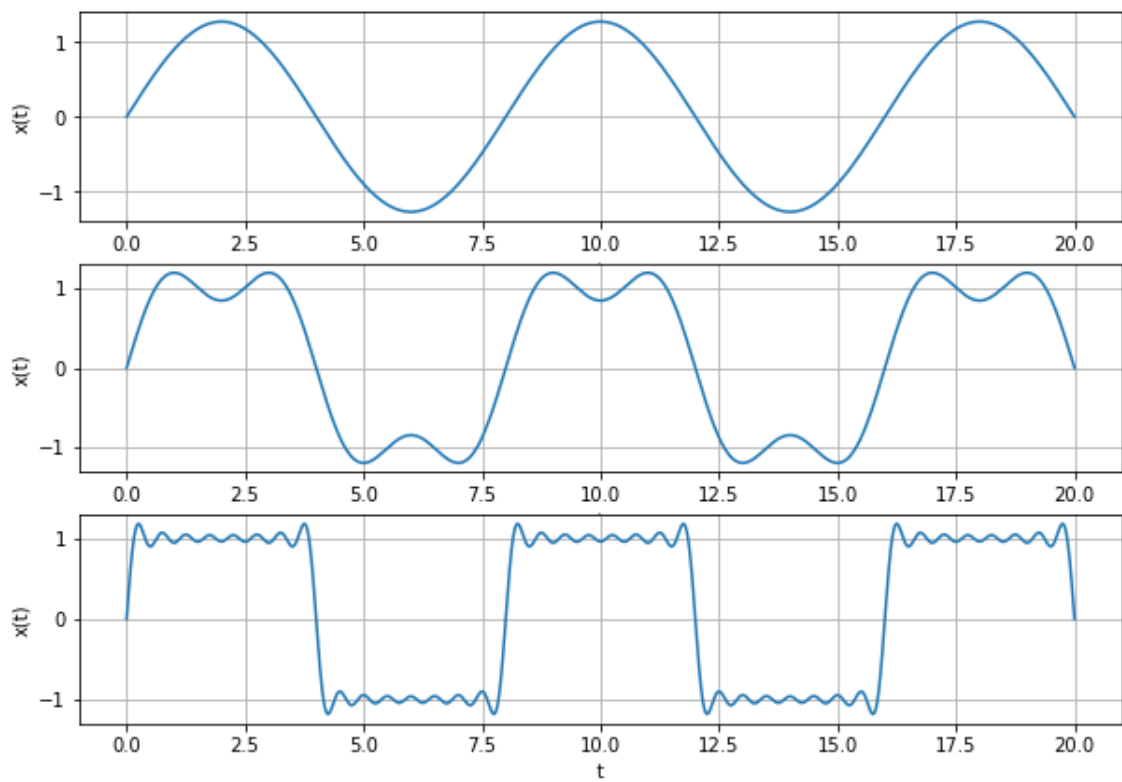
3 Equations

$$a_k = 0$$

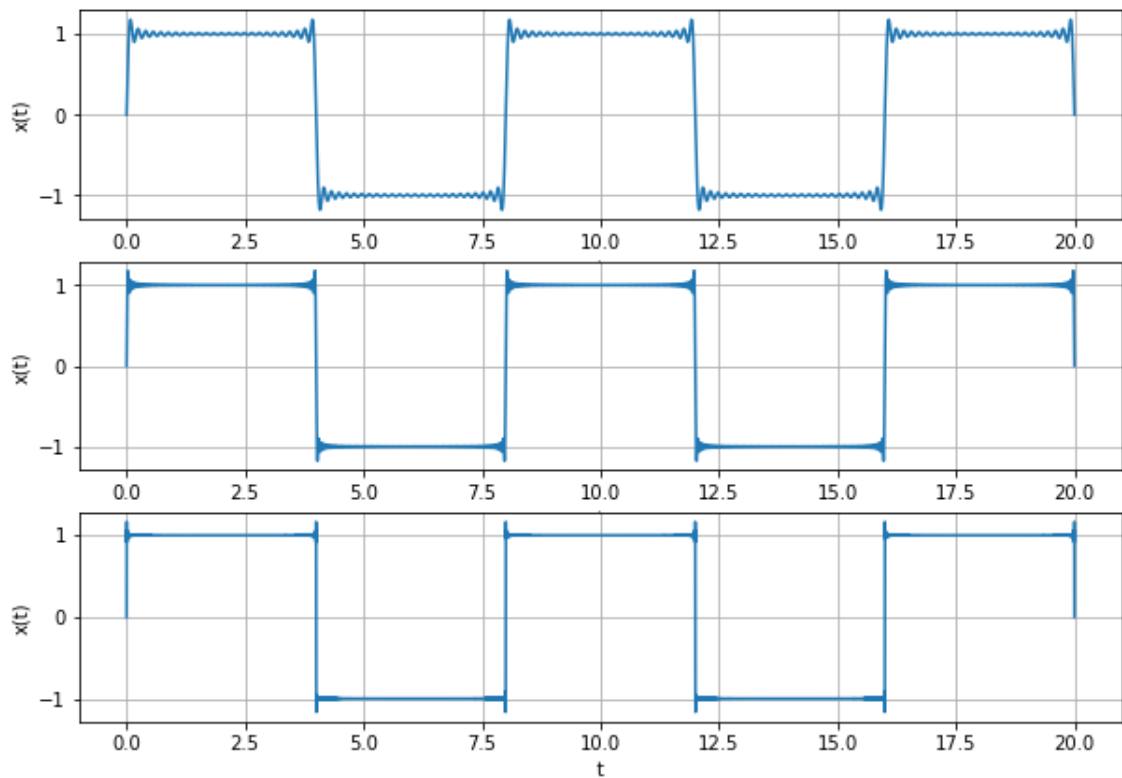
$$b_k = \frac{2}{k\pi}[1 - \cos(k\pi)]$$

Results

Fourier Series Approximations of $x(t)$ ($N=1$, $N=3$, $N=15$)



Fourier Series Approximations of $x(t)$ ($N=50$, $N=150$, $N=1500$)



Questions

1. Is $x(t)$ an even or an odd function? Explain why.

The function is odd since it is not mirrored across the y-axis ($X_n = -X_{-n}$).

2. Based on your results from Task 1, what do you expect the values of a_2, a_3, \dots, a_n to be? Why?

I expect all values of a_k to be 0 because that's what the equation for a_k simplifies to.

3. How does the approximation of the square wave change as the value of N increases?
In what way does the Fourier series struggle to approximate the square wave?

The approximation gets closer and closer to the square wave as N increases. The Fourier series struggles to approximate the square wave at the straight vertical edges.

4. What is occurring mathematically in the Fourier series summation as the value of N increases?

As the value of N increases, each new component gets smaller and smaller, but pushes the total $x(t)$ ever closer to resembling the waveform being approximated.

5. Leave any feedback on the clarity/usefulness of the purpose, deliverables, and expectations for this lab.

The purpose, deliverables, and expectations for this lab were communicated clearly.

Appendix

Python output of requested a_k and b_k values:

$a_0 = 0, a_1 = 0$

$b_1 = 1.2732395447351628, b_2 = 0.0, b_3 = 0.4244131815783876$