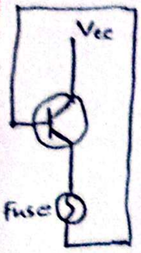
12/01/2025 – D.S.D.



PROM cell

PROM: The manufacturer does not program this ROM during manufacturing. Rather user's programs this ROM as per their needs. Once programmed this ROM cannot be erased. It cannot be re-programmed. So, if there is a mistake while writing the program in the PROM, then that PROM has to be thrown away. That's why PROM is called OTP (one time program). Therefore it can be said that ROM is not programmed at the time of preparation but is programmed by the user as per their wish is called PROM. This PROM can be programmed only once.

EPROM: EPROM is a type of ROM that the user can program at will and reprogram at will by erasing any stored data. Once the EPROM is programmed, the information is stored in it indefinitely unless erased. EPROM is programmed by applying a specific voltage (10–25 volt) for specified time to a specific input pin of the EPROM IC through a special programming circuit.

EPROM has some disadvantages:

* To erase and reprogram an EPROM, it must be moved from the circuit
* Erasing the EPROM erases the entire chip; as a result the entire EPROM has to be reprogrammed.
* Erasing and reprogramming the EPROM takes at least 20mins.

4/01/25 / **EEPROM:** The EPROM which can be erased by electrical signal is called EEPROM. It is made by mosfet like EPROM. However, some modifications are made to the MOSFET for EEPROM. EEPROM MOSFET has a floating gate like EPROM. But very thin oxide layer is formed on the MOSFET drain. By creating a thin oxide layer on the drain, it can be removed by electrical signal.
Applying a high voltage (21 volt) between the gate and drain of the MOSFET causes the floating gate to accumulate charge. then even if the power supply is turned off the charge remains. Applying some voltage in reverse to the MOSFET destroys the charge stored in the gate. Because the MOSFET requires very little current to store and remove, the EEPROM can be programmed and erased while still in the circuit.

Advantages of EEPROM:

* Its main advantage is that data can be erased and reprogrammed at any location or cell.
* EEPROM can be erased and reprogrammed at high speed.
* EEPROM doesn't need to be removed from circuit while programming

ROM APPLICATION

IC No	Organization	O/P	Access time	Max power dissipation	Power Supply	No of Pin	Techno	Type
1702A	256 × 8	Ts	1000	885	5–9	24	MOS	EP ROM
2308	1024 × 8	Ts	450	840	5, 12, –5	24	MOS	ROM
2316E	2048 × 8	Ts	450	620	5	24	MOS	ROM
2704	512 × 8	Ts	450	800	5, 12, –5	24	MOS	EP ROM
2708	512 × 8	Ts	450	800	5, 12, –5	24	MOS	EP ROM

01861031939

2018 | 3(c) |

Table for calculation

Daily sales	Num of Shops (f)	Mid value (x)	fx	f log x	f/x
20 - 30	4	25	100	5.59	0.16
30 - 40	7	35	245	10.80	0.2
40 - 50	16	45	720	26.45	0.36
50 - 60	12	55	660	20.88	0.218
60 - 70	6	65	390	10.87	0.092
70 - 80	5	75	375	9.37	0.067
	$N = 50$		$\Sigma fx = 2490$	$\Sigma f \log x = 83.96$	$\Sigma f/x = 1.0927$

∴ Arithmetic mean,

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{2490}{50}$$

$$= 49.8$$

∴ Geometric mean $= \text{Antilog}\left(\frac{\Sigma f \log x}{N}\right)$

$$= \text{Antilog}\left(\frac{83.96}{50}\right)$$

$$= 47.77$$

∴ Harmonic mean $= \frac{N}{\Sigma \frac{f}{x}}$

$$= \frac{50}{1.0927}$$

$$= 45.758$$

12/01/2025 — L.A. [Eigen value & Eigen vector]

① Suppose A be a square matrix
Then a non-zero vector V is form a relation with A

i.e. $AV = \lambda V$

for some scalar λ

The scalar λ is called an eigen value of the matrix A and v is called eigen vector of A corresponding to λ.

② (i) Characteristic matrix/Eigen matrix: Suppose A be a square matrix and v be a vector such that $Av = \lambda v$ for some scalar λ. Then,

$$Av - \lambda v = 0$$
$$\Rightarrow [A - \lambda I] v = 0$$

Here $[A-\lambda I]$ is called characteristic matrix.

(ii) Characteristic equation/Eigen equation: Suppose A be a square matrix and v be a vector such that $Av = \lambda v$ for some scalar λ, then,

$$Av - \lambda v = 0$$
$$\Rightarrow [A - \lambda I] v = 0$$
$$\Rightarrow |A - \lambda I| |v| = 0$$

Since $|v| \neq 0$ then $|A - \lambda I| = 0$

Here $|A - \lambda I| = 0$ is called characteristic equation.

Find the eigen value of the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

Soln: Given that, $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

The characteristic equation

$\Rightarrow |A - \lambda I| = 0$

$\Rightarrow \left| \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \left| \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)(4-\lambda) - 3 = 0$

$\Rightarrow 8 - 4\lambda - 2\lambda + \lambda^2 - 3 = 0$

$\Rightarrow \lambda^2 - 6\lambda + 5 = 0$

$\Rightarrow \lambda^2 - 5\lambda - \lambda + 5 = 0$

$\Rightarrow \lambda(\lambda - 5) - 1(\lambda - 5) = 0$

$\Rightarrow (\lambda - 5)(\lambda - 1) = 0$

$\therefore \lambda = (5, 1)$

Given that, $A = \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix}$

The

$\Rightarrow |A - \lambda I| = 0$

$\Rightarrow \left| \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \left| \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} 3-\lambda & 3 \\ 1 & 5-\lambda \end{vmatrix} = 0$

$\Rightarrow (3-\lambda)(5-\lambda) - 3 = 0$

$\Rightarrow 15 - 3\lambda - 5\lambda + \lambda^2 - 3 = 0$

$\Rightarrow \lambda^2 - 8\lambda - 12 = 0$

$\Rightarrow \lambda^2 - 6\lambda - 2\lambda - 12 = 0$

$\Rightarrow \lambda(\lambda - 6) - 2(\lambda - 6) = 0$

$\Rightarrow (\lambda - 2)(\lambda - 6) = 0$

$\therefore \lambda = (2, 6)$

Given that, $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix}$

The chara

$$\Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & -2-\lambda & 0 \\ 0 & -5 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\{(-2-\lambda)(2-\lambda) - 0\} = 0$$

$$\Rightarrow (1-\lambda)(-2-\lambda)(2-\lambda) = 0$$

$$\therefore \lambda = 1, -2, 2$$

13/01/2025 - DM

Boolean algebra:

complement

sum

product

Find the value of $1.0 + (\overline{0+1})$

Soln $1.0 + (\overline{0+1})$

$= 0 + \overline{1}$

$= 0 + 0$

$= 0$

Prove that $x(x+y) = x$ using the identities of Boolean Algebra.

Proof The following table proves that $x(x+y) = x$

x	y	$x+y$	$x(x+y)$
1	1	1	1
1	0	1	1
0	1	1	0
0	0	0	0

Find the sum of product expression for the function $F(x,y,z) = (x+y)\overline{z}$

Soln: The following table shows the sum of product expression for the function

x	y	z	$x+y$	$\overline{z}$	$(x+y)\overline{z}$
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	1	0	0
0	1	1	1	0	0
1	0	0	1	1	1
0	1	0	1	1	1
0	0	1	0	0	0
0	0	0	0	1	0

$xy + \overline{z}$

x	y	z	$\overline{z}$	xy	$xy+\overline{z}$
0	0	0	1	0	1
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	1	1

CamScanner

$1+2+3+\dots+\infty = ? \; -\frac{1}{12}$

Logic Gate: Digital system are said to be constructed by using logic ga[tes]. These gates are AND, OR, NOT, NAND, NOR, X-OR, X-NOR.

AND Gate: The AND gate is an electronic circuit that gives

$0.9999\dots\infty = 1$

0.999

9.99

$S = 0.9999\dots\infty$

$10S = 9.99999\dots\infty$

$10S - S = 9.9999\dots\infty - 0.999\dots\infty$

$9S = 9$

$S = 1$

$S =$

$1 \quad 0$

0

$S_1 = 1-1+1-1+1\dots\infty$

$S_2 =$

$S = \frac{1}{2}$

6

~~$1 = 1$~~ $n^2 =$

~~$1+3 = 4$~~

~~$1+3+5 = 9$~~

D.S.D. 14/01/25 Arm

x64 - 64
x86 - 32

GTX / RTX   Xe   DLSS SLRRT core

GPT 3.5   CUDA core

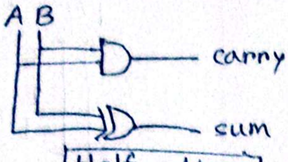
Arithmetic circuit

Half adder: The adder ~~circuit~~ circuit is used to add two bits is called half adder circuit. By adding two bits this circuit prepares the SUM and number of hand CARRY. For two bits the following four conditions are available for addition

- 0+0=0
- 1+0=1
- 0+1=1
- 1+1=0 (carry 1)

The output of the half adder is SUM and CARRY, and its boolean expression is as follows:

$$SUM = A \oplus B$$
$$CARRY = AB$$

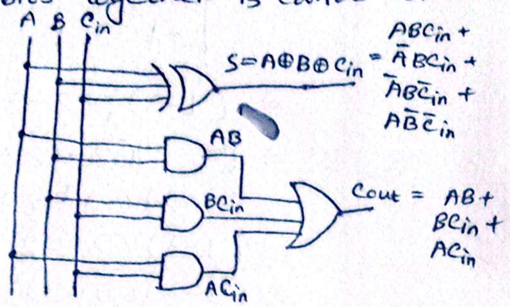


Half adder

Full adder: An adder circuit that can add three binary bits together is called full adder circuit.

Here, X-OR gate is used to add the three bits and after AND gate operation from the three input. The final carry (C_{out}) is obtained from the output of OR gate.

I/P			O/P	
A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



14/01/25

3. $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$

we have, characteristic equation

$|A - \lambda I| = 0$

$\Rightarrow \left| \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 2 \\ 1 & 2-\lambda & -1 \\ -1 & 1 & 4-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)\{(2-\lambda)(4-\lambda)+1\} - 2\{1(4-\lambda)-1\} + 2\{1+1(2-\lambda)\} = 0$

$\Rightarrow (1-\lambda)(8-4\lambda-2\lambda+\lambda^2+1) - 2(4-\lambda-1) + 2(3-\lambda) = 0$

$\Rightarrow (1-\lambda)(\lambda^2-6\lambda+9) - 2(3-\lambda) + 2(3-\lambda) = 0$

$\Rightarrow (1-\lambda)(\lambda^2 - 2\cdot\lambda\cdot 3 + 3^2) = 0$

$\Rightarrow (1-\lambda)(\lambda-3)^2 = 0$

$\therefore \lambda = 1, 3, 3$

Let the eigen vector, $v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

~~when λ=1 then~~

$[A - \lambda I]v = 0$

$\Rightarrow \begin{bmatrix} 1-\lambda & 2 & 2 \\ 1 & 2-\lambda & -1 \\ -1 & 1 & 4-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ — (i)

when $\lambda = 1$ then

$\begin{bmatrix} 0 & 2 & 2 \\ 1 & 1 & -1 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore 2y + 2z = 0$ — (ii)

$x + y - z = 0$ — (iii)

$-x + y - 3z = 0$ — (iv)

from (ii) $\Rightarrow y + z = 0$

$\Rightarrow z = -y$

$\Rightarrow \frac{z}{-1} = \frac{y}{1}$ — (v)

from (iii) $\Rightarrow x + y - (-y) = 0$

$\Rightarrow x + 2y = 0$

$\Rightarrow x = -2y$

$\Rightarrow \frac{x}{-2} = \frac{y}{1}$ — (vi)

from (v) and (vi)

$\frac{x}{-2} = \frac{y}{1} = \frac{z}{-1}$

$\therefore$ eigen vector, $v_1 = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$

Again, when $\lambda = 3$ then

$\begin{bmatrix} -2 & 2 & 2 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore \left. \begin{array}{l} -2x + 2y + 2z = 0 \\ x - y - z = 0 \\ -x + y + z = 0 \end{array} \right\} \Rightarrow x - y - z = 0$

let $z = 1$ & $y = 2$

then $x = 3$

$\therefore$ eigen vector, $v_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Again, ~~[illegible]~~ Let $z = 2$, $y = 1$,

then $x = 3$

$\therefore$ eigen vector, $v_3 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

2,3,6

4(i) $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$

We have,

$|A - \lambda I| = 0$

$\Rightarrow \left| \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ 1 & 1 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow (3-\lambda)\{(4-\lambda)(3-\lambda) - 2\} - 1\{2(3-\lambda) - 2\} + 1\{2 - 1(4-\lambda)\} = 0$

$\Rightarrow (3-\lambda)\{12 - 4\lambda - 3\lambda + \lambda^2 - 2\} - \{6 - 2\lambda - 2\} + \{2 - 4 + \lambda\} = 0$

$\Rightarrow (3-\lambda)\{\lambda^2 - 7\lambda + 10\} - (-2\lambda + 4) + (\lambda - 2) = 0$

$\Rightarrow (3-\lambda)\{\lambda^2 - 5\lambda - 2\lambda + 10\} + 2\lambda - 4 + \lambda - 2 = 0$

$\Rightarrow (3-\lambda)\{\lambda(\lambda-5) - 2(\lambda-5)\} + 3\lambda - 6 = 0$

$\Rightarrow (3-\lambda)\underline{(\lambda-2)}(\lambda-5) + 3\underline{(\lambda-2)} = 0$

$\cancel{\Rightarrow (\lambda-2)\{(3-\lambda)(\lambda-5)+3\} = 0}$

$\cancel{\Rightarrow (\lambda-2)\{3\lambda -}$

4(ii) $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

we have,

$$|A - \lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 2 \\ 3 & 1-\lambda & 0 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\{(1-\lambda)^2\} - 2\{3(1-\lambda)\} + 2\{3-(1-\lambda)\} = 0$$

$$\cancel{\Rightarrow (1-\lambda)^3 \quad - 2(3-3\lambda) + 2(3-\lambda).} = 0$$

$$\cancel{\Rightarrow (1-\lambda)^3 = 0}$$

$$\Rightarrow (1-\lambda)^3 - 2(3-3\lambda) + 2(3-1+\lambda) = 0$$

$$\Rightarrow (1-\lambda)^3 - \cancel{2\{3(1-\lambda)\}} - 6 + 6\lambda + 4 - 2\lambda = 0$$

$$= (1-\lambda)^3 + 4\lambda - 2 = 0$$

Let, x variable having n number of positive observations with mean $\bar{x}$ and standard deviation σ.

we have, $\bar{x} = \frac{\sum x_i}{n}$

$\Rightarrow \sum x_i = n\bar{x}$

also, $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

$\Rightarrow \sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$

$\Rightarrow \sigma = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2}$

we know, $(\sum x_i)^2 = \sum x_i^2 + 2\sum\sum x_{ij}$

$\Rightarrow (\sum x_i)^2 > \sum x_i^2$

$\Rightarrow n^2\bar{x}^2 > \sum x_i^2$ $\quad [\because \sum\sum x_{ij} > 0]$

$\Rightarrow n\bar{x}^2 > \frac{\sum x_i^2}{n}$

$\Rightarrow n\bar{x}^2 - \bar{x}^2 > \frac{\sum x_i^2}{n} - \bar{x}^2$

$\Rightarrow \bar{x}^2(n-1) > \frac{\sum x_i^2}{n} - \bar{x}^2$

$\Rightarrow \sqrt{\bar{x}^2(n-1)} > \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2}$

$\Rightarrow \bar{x}\sqrt{n-1} > \sigma$

$\Rightarrow \sqrt{n-1} > \frac{\sigma}{\bar{x}}$

$\Rightarrow 100\sqrt{n-1} > \frac{\sigma}{\bar{x}} \times 100$

$\Rightarrow 100\sqrt{n-1} > C.V$

[Proved]

2020
5(b)

Soln: let, variable x having n values $x_1, x_2, \dots x_n$ with their mean $\bar{x}$ and any arbitrary value 'a' where $a \neq \bar{x}$ then the

rth raw moments, $\mu_r' = \frac{\Sigma(x_i - a)^r}{n}$; $r = 1, 2, 3, \dots$

1st " " , $\mu_1' = \frac{\Sigma(x_i - a)}{n} = \bar{x} - a$

2nd " " , $\mu_2' = \frac{\Sigma(x_i - a)^2}{n}$

3rd " " , $\mu_3' = \frac{\Sigma(x_i - a)^3}{n}$

4th " " , $\mu_4' = \frac{\Sigma(x_i - a)^4}{n}$

rth central moments, $\mu_r = \frac{\Sigma(x_i - \bar{x})^r}{n}$; $r = 1, 2, 3, \dots$

1st " " , $\mu_1 = \frac{\Sigma(x_i - \bar{x})}{n} = 0$

2nd " " , $\mu_2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$

$$= \frac{\Sigma(x_i - a + a - \bar{x})^2}{n}$$

$$= \frac{\Sigma\{(x_i - a) - (\bar{x} - a)\}^2}{n}$$

$$= \frac{\Sigma\{(x_i - a) - \mu_1'\}^2}{n}$$

$$= \frac{\Sigma(x_i - a)^2}{n} - 2\frac{\Sigma(x_i - a)}{n}\mu_1' + \frac{n\mu_1'^2}{n}$$

$$= \mu_2' - 2\mu_1'\mu_1' + (\mu_1')^2$$

$$= \mu_2' - 2(\mu_1')^2 + (\mu_1')^2$$

$$= \mu_2' - (\mu_1')^2$$

3rd " " , $\mu_3 = \frac{\Sigma(x_i - \bar{x})^3}{n}$

$$= \frac{\Sigma\{(x_i - a) - (\bar{x} - a)\}^3}{n}$$

$$= \frac{\Sigma\{(x_i - a) - \mu_1'\}^3}{n}$$

$$= \frac{\Sigma\{(x_i - a)^3 - 3(x_i - a)^2\mu_1' + 3(x_i - a)(\mu_1')^2 - (\mu_1')^3\}}{n}$$

$$= \frac{\Sigma(x_i - a)^3}{n} - 3\frac{\Sigma(x_i - a)^2}{n}\mu_1' + 3\frac{\Sigma(x_i - a)}{n}(\mu_1')^2 - \frac{n(\mu_1')^3}{n}$$

$$= \mu_3' - 3\mu_2'\mu_1' + 3\mu_1'(\mu_1')^2 - (\mu_1')^3$$

$$= \mu_3' - 3\mu_2'\mu_1' + 3(\mu_1')^3 - (\mu_1')^3$$

$$= \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

4th central moments, $\mu_4 = \frac{\sum (x_i - \bar{x})^4}{n}$

$$= \frac{\sum\{(x_i - a) - (\bar{x} - a)\}^4}{n}$$

$$= \frac{\sum\{(x_i - a) - \mu_1'\}^4}{n}$$

$$= \frac{\sum\{(x_i-a)^4 - 4(x_i-a)^3\mu_1' + 6(x_i-a)^2(\mu_1')^2 - 4(x_i-a)(\mu_1')^3 + (\mu_1')^4}{n}$$

$$= \frac{\sum(x_i-a)^4}{n} - 4\frac{\sum(x_i-a)^3}{n}\mu_1' + 6\frac{\sum(x_i-a)^2}{n}(\mu_1')^2 - 4\frac{\sum(x_i-a)^3}{n}(\mu_1')^3 + \frac{n(\mu_1')^4}{n}$$

$$= \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 4\mu_1'(\mu_1')^3 + (\mu_1')^4$$

$$= \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 4(\mu_1')^4 + (\mu_1')^4$$

$$= \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1)^4$$

\# $\overline{(A+B+\bar{C})}\cdot B\bar{C}$

$=(\bar{A}\cdot\bar{B}\cdot\bar{\bar{C}})\cdot B\bar{C}$ $[\because \overline{X+Y}=\bar{X}\cdot\bar{Y}]$

$=(\bar{A}\cdot\bar{B}\cdot C)\cdot B\cdot\bar{C}$ $[\because \bar{\bar{X}}=X]$

$=0$ $[\because X\cdot\bar{X}=0]$

\# $\bar{X}\cdot\overline{\bar{Y}(Z+\bar{X})}$ $[\because \overline{X\cdot Y}=\bar{X}+\bar{Y}]$

$=\bar{X}(\bar{\bar{Y}}+\overline{(Z+\bar{X})})$ $[\because \bar{\bar{X}}=X]$

$=\bar{X}(Y+(\bar{Z}\cdot\bar{\bar{X}}))$

$=\bar{X}(Y+(\bar{Z}X))$ $[\because X(Y+Z)=XY+XZ]$

$=\bar{X}Y+\bar{X}\bar{Z}X$

$=\bar{X}Y$ $[\because \bar{X}\cdot X=0$ and $X\cdot 0=0]$

\# $ABC+\bar{A}BC+A\bar{B}C+\bar{A}\bar{B}C$

$=BC(A+\bar{A})+\bar{B}C(A+\bar{A})$

$=BC+\bar{B}C$

$=C(B+\bar{B})$

$=C$

\# $\overline{(A+B+\bar{C})}+BC$

$=(\bar{A}\cdot\bar{B}\cdot\bar{\bar{C}})+BC$

$=(\bar{A}\cdot\bar{B}\cdot C)+BC$

$=C(\bar{A}\bar{B}+B)$

$=C(\bar{A}+B)(\bar{B}+B)$

$=C(\bar{A}+B)$

\# $\overline{(\bar{A}+C)}+\overline{(B+\bar{D})}$

$=(\bar{\bar{A}}\cdot\bar{C})+(\bar{B}\cdot\bar{\bar{D}})$

$=A\cdot\bar{C}+\bar{B}D$

\# $A\bar{B}C+ABC+\bar{A}C$

$\cancel{=AC(B+\bar{B})+C(AB+\bar{A})}$

$\cancel{=AC+C((A+\bar{A})(B+\bar{A}))}$

$\cancel{=AC+C(B+\bar{A})}$

$=AC(\bar{B}+B)+\bar{A}C$

$=AC+\bar{A}C$

$=C(A+\bar{A})$

$=C$

29/01/25 - D.M.

<u>k-map</u> (4 variable)

	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	0	1	3	2
$\bar A B$	4	5	7	6
AB	12	13	15	14
$A\bar B$	8	9	11	10

$F = ABCD + AB\bar C\bar D + A\bar B\bar C\bar D + AB\bar C D + ABC\bar D$

AB \ CD	$\bar C\bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A\bar B$ 00				
$\bar A B$ 01				
AB 11	1	1	1	1
$A\bar B$ 10	1			

→ AB

↓ $A\bar C\bar D$

∴ simplification : $A\bar C\bar D + AB$

$F = \bar A\bar B\bar C\bar D + \bar A\bar B\bar C D + \bar A\bar B C\bar D + \bar A B C\bar D + A\bar B\bar C\bar D$
$+ A\bar B\bar C D + A\bar B C\bar D$

or, $F(A,B,C,D) = \Sigma(0,1,2,6,8,9,10)$

	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	1	1		1
$\bar A B$				1
AB				
$A\bar B$	1	1		1

→ $\bar A C\bar D$

~~$\bar A\bar B\ \bar A\bar B\ \bar C\bar D\ \bar C D$~~

$A\bar B(\bar C\bar D + \bar C D) + \bar A\bar B(\bar C\bar D + \bar C D)$

$\Rightarrow A\bar B\bar C + \bar A\bar B\bar C$

$\rightarrow \bar B\bar C$

$F(A,B,C,D) = \Sigma(0,4,5,7,8,12,13,15)$

	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	1			
$\bar A B$	1	1	1	
AB	1	1	1	
$A\bar B$	1			

→ $\bar C\bar D + BD$

DSD. CT - 09/02/25

→ concepts of number system & code
→ logic gate
→ boolean algebra
→ combinational circuit design
→ universal gate
→ algebric simplification
→ k-map

S&P CT

~~[illegible]~~

04/02/25 - S&P

** Difference between standard deviation/variance and coefficient of variation.

Possible questions from regression

- Definition of regression
- Regression ~~coefficient~~ analysis
- write down the properties of regression coefficient
- Difference between correlation/correlation analysis and regression/regression analysis.

Topic suggestion for final exam

* Frequency distribution table
* Graph
* Mean, median, mode, geometric mean, harmonic mean
* Dispersion, combined mean, combined standard deviation, standard deviation, c.v.
* Correlation and rank correlation
* χ^2 test.

04/02/25 - L.A.

Cayley Hamilton Theorem

Statement: Every square matrix satisfy of its own characteristic equation

Ex: $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

Characteristics equation

$$|A-\lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 \\ 4 & 6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(6-\lambda) - 8 = 0$$

$$\Rightarrow 6 - 6\lambda - \lambda + \lambda^2 - 8 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda - 2 = 0 \quad \text{—— (i)}$$

By the statement of caley hamilton theorem matrix A satisfy equation

$$A^2 - 7A - 2I = 0$$

Now, $A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 1+8 & 2+12 \\ 4+24 & 8+36 \end{bmatrix} = \begin{bmatrix} 9 & 14 \\ 28 & 44 \end{bmatrix}$

L.H.S. $= A^2 - 7A - 2I$

$$= \begin{bmatrix} 9 & 14 \\ 28 & 44 \end{bmatrix} - 7\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 14 \\ 28 & 44 \end{bmatrix} - \begin{bmatrix} 7 & 14 \\ 28 & 42 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-7-2 & 14-14-0 \\ 28-28-0 & 44-42-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence, Cayley Hamilton theorem is verified

CamScanner

05/02/25 – D.M.

$F(A,B,C,D) = \Sigma(2,4,6,7,10,12,14,15)$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				1
$\bar{A}B$	1		1	1
AB	1		1	1
$A\bar{B}$				1

$Y = C\bar{D} + BC + B\bar{D}$

$F(A,B,C,D) = \Sigma(1,2,3,5,6,7,9,10,11,13,14,15)$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		1	1	1
$\bar{A}B$		1	1	1
AB		1	1	1
$A\bar{B}$		1	1	1

$Y = C + D$

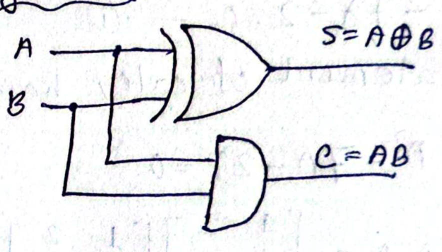
Construct Half adder circuit with truth table.

⇒ Half adder: Half adder is a circuit that can add two bits without considering a carry from previous addition, the inputs of half adder circuit are A and B and the outputs are the sum bit(S) and carry bit (C).

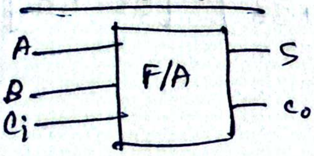
Truth table:

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Logic ckt:



⇒ Full adder:



A	B	C_i	S	C_o
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$S = \bar{A}\bar{B}C_i + \bar{A}B\bar{C_i} + A\bar{B}\bar{C_i} + ABC_i$

$= \bar{A}(\bar{B}C_i + B\bar{C_i}) + A(\bar{B}\bar{C_i} + BC_i)$

$= \bar{A}(B \oplus C_i) + A(\overline{B \oplus C_i})$

$= \bar{A}P + A\bar{P}$

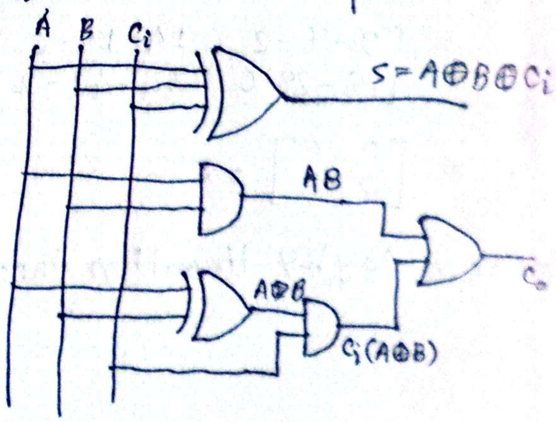
$= A \oplus P$ | Let, $B \oplus C_i = P$

$= A \oplus B \oplus C_i$

$C_o = \bar{A}BC_i + A\bar{B}C_i + \ldots$

$= C_i(\bar{A}B + A\bar{B}) + AB(\ldots$

$= C_i(A \oplus B) + AB$



06/02/25 - D.S.D

Prove that, $A(B+C) = AB+AC$

A	B	C	B+C	A(B+C)	AB	AC	AB+AC
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Since the truth values of $A(B+C)$ and $AB+AC$ are same we can say they are equivalent.

$\therefore A(B+C) = AB+AC$

[Proved]

06/02/25 – S&P

2019 5(b) **Proof:** Let, x and y are two variables with n pairs of observati $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ with their respective means. $\bar{x}$ and $\bar{y}$. Then the coefficient of correlation, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \Sigma(y-\bar{y})^2}}$.

Let, $u = \frac{x-a}{c}$ and $v = \frac{y-b}{d}$ where a, b = origin, c, d = scale

$\Rightarrow x - a = cu$ $\qquad \Rightarrow y - b = dv$

$\Rightarrow x = a + cu$ $\qquad \Rightarrow y = b + dv$

$\Rightarrow \bar{x} = a + c\bar{u}$ $\qquad \Rightarrow \bar{y} = b + d\bar{v}$,

Now, $r_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \Sigma(y-\bar{y})^2}}$

$$\Rightarrow r_{xy} = \frac{\Sigma(a+cu-a-c\bar{u})(b+dv-b-d\bar{v})}{\sqrt{\Sigma(a+cu-a-c\bar{u})^2 \Sigma(b+dv-b-d\bar{v})^2}}$$

$$\Rightarrow r_{xy} = \frac{cd\,\Sigma(u-\bar{u})(v-\bar{v})}{cd\sqrt{\Sigma(u-\bar{u})^2 \Sigma(v-\bar{v})^2}}$$

$$\Rightarrow r_{xy} = \frac{\Sigma(u-\bar{u})(v-\bar{v})}{\sqrt{\Sigma(u-\bar{u})^2 \Sigma(v-\bar{v})^2}}$$

$$\Rightarrow r_{xy} = r_{uv}$$

Hence it is proved that, coefficient of correlation is independent of origin and scale.

2019 5(d) Given information,

$r = 0.75$

$\sigma x = 3$

$cov(xy) = 8.2$

variance of y, $\sigma^2_y = ?$

We know, $r = \frac{cov(xy)}{\sigma x \, \sigma y}$

$$\Rightarrow 0.75 = \frac{8.2}{3\,\sigma y}$$

$$\Rightarrow \sigma y = \frac{8.2}{3 \times 0.75} = 3.64$$

$$\Rightarrow \sigma^2 y = 13.28$$

09/02/25 - S&P

2020 6(d) Let, the regression equation of y on x is

$$y = a + bx \quad \text{—} (i)$$

Table for calculation

x	y	xy	x^2	y^2
30	28	840	900	
45	40	1800	2025	
32	27	864	1024	
28	24	672	784	
27	26	702	729	
24	18	432	576	
$\Sigma x = 186$	$\Sigma y = 163$	$\Sigma xy = 5310$	$\Sigma x^2 = 6038$	

Computing a and b,

$$b = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$= \frac{5310 - \frac{186 \times 163}{6}}{6038 - \frac{186^2}{6}}$$

$$\approx 0.94$$

$$a = \bar{y} - b\bar{x}$$

$$= 27.167 - 0.94 \times 31$$

$$= -1.973$$

from eq (i) we get,

$$y = -1.973 + 0.94x$$

12/02/25 - S&P

2020
5(d)

Table for calculation

X	Y	Rank of x R_1	Rank of Y R_2	Rank difference $d = R_1 - R_2$	d^2
50	11	9.5	10	-0.5	0.25
50	13	9.5	8	1.5	2.25
55	14	8	7	-1	1
60	16	5.5	5	0.5	0.25
65	12	2	9	-7	49
65	19	2	2	0	0
65	20	2	1	1	1
60	15	5.5	6	-0.5	0.25
60	17	5.5	4	1.5	2.25
60	18	5.5	3	2.5	6.25

$\Sigma d^2 = 62.5$

$\therefore$ the rank correlation coefficient,

$$\rho = 1 - \frac{6(\Sigma d^2 + T_x)}{n(n^2-1)}$$

$$= 1 - \frac{6(62.5 + 7.5)}{10(10^2-1)}$$

$$= 0.57$$

65 repeated 3 times

$\therefore T_{x_1} = \frac{m^2(m^2-1)}{12} = \frac{3(3^2-1)}{12} = 2$

60 repeated 4 times

$\therefore T_{x_2} = \frac{m^2(m^2-1)}{12} = \frac{4(4^2-1)}{12} = 5$

50 repeated 2 times

$\therefore T_{x_3} = \frac{m^2(m^2-1)}{12} = \frac{2(2^2-1)}{12} = 0$

$\therefore T_x = T_{x_1} + T_{x_2} + T_{x_3}$

$= 2 + 5 + 0.5$

$= 7.5$

11/02/25 - L.A.

☑ verify caley-hamilton theorem $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Soln: we have, characteristics equation,

$$|A - \lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 \\ 2 & -1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(-1-\lambda) - 4 = 0$$

$$\Rightarrow -1 + \lambda - \lambda + \lambda^2 - 4 = 0$$

$$\Rightarrow \lambda^2 - 5 = 0$$

By the statement of calex-Hamilton theorem, every square matrix satisfy of its own characteristics equation,

$$A^2 - 5I = 0 \quad \text{———} (i)$$

Now, $A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

from (i) $\Rightarrow$ L.H.S $= A^2 - 5I$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence cayley-Hamilton theorem is verified.

⑦ We have, characteristic equation

$$|A-\lambda I|=0$$

$$\Rightarrow \left| \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ 1 & 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)\{(4-\lambda)(3-\lambda)-2\} - \{(6-2\lambda)-2\} + \{2-(4-\lambda)\} = 0$$

$$\Rightarrow (3-\lambda)\{12-4\lambda-3\lambda+\lambda^2-2\} - 6+2\lambda+2+2-4+\lambda = 0$$

$$\Rightarrow (3-\lambda)\{10-7\lambda+\lambda^2\} - 6+3\lambda = 0$$

$$\Rightarrow 30-21\lambda+3\lambda^2-10\lambda+7\lambda^2-\lambda^3-6+3\lambda=0$$

$$\Rightarrow 24-28\lambda+10\lambda^2-\lambda^3=0$$

$$\Rightarrow \lambda^3-10\lambda^2+28\lambda-24=0 \quad \text{——— (i)}$$

$$A^2 = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 8 & 8 \\ 16 & 20 & 16 \\ 8 & 8 & 12 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 12 & 8 & 8 \\ 16 & 20 & 16 \\ 8 & 8 & 12 \end{bmatrix}\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 60 & 52 & 52 \\ 104 & 112 & 104 \\ 52 & 52 & 60 \end{bmatrix}$$

from (i) $\Rightarrow$ L.H.S $= A^3-10A^2+28A-24I$

$$= \begin{bmatrix} 60 & 52 & 52 \\ 104 & 112 & 104 \\ 52 & 52 & 60 \end{bmatrix} - 10\begin{bmatrix} 12 & 8 & 8 \\ 16 & 20 & 16 \\ 8 & 8 & 12 \end{bmatrix} + 28\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 60 & 52 & 52 \\ 104 & 112 & 104 \\ 52 & 52 & 60 \end{bmatrix} - \begin{bmatrix} 120 & 80 & 80 \\ 160 & 200 & 160 \\ 80 & 80 & 120 \end{bmatrix} + \begin{bmatrix} 84 & 28 & 28 \\ 56 & 112 & 56 \\ 28 & 28 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

∴ Caley hamilton theorem is proved.

13/02/25 - D.S.D

Difference between combinational & sequential logic ckt.

Combinational	Sequential
(i) Output of any instance of time depends only upon the input variables	(i) Output is generated dependent upon the present input variables and also on the basic of past history of these inputs.
(ii) Memory unit is not required. It does not allocate any memory to elements	(ii) Memory unit is required. It allocates memory to the elements.
(iii) Faster	(iii) Slower
(iv) Easy to design	(iv) Difficult
(v) Parallel adder	(v) Serial adder
(vi) Ex: Half & Full adder, MUX, DEMUX	(vi) Ex: flip-flop, shift register

Race around problem in flip-flop

In a J-K flip-flop when input J=1, K=1 we find a trouble when J=1 and K=1 this output are toggle, then any position this output are race condition, this problem is known as race condition.

13/02/25 - L.A.

(6) Given that,
$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

we have,

characteristic equation,

$$|A - \lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 0 & -1 \\ 5 & 1-\lambda & 0 \\ 0 & 1 & 3-\lambda \end{vmatrix} = 0$$

$$= (2-\lambda)\{(1-\lambda)(3-\lambda)\} - 0 - 1(5-0) = 0$$
$$= (2-\lambda-2\lambda+\lambda^2)(3-\lambda) - 5 = 0$$
$$= (\lambda^2 - 3\lambda + 2)(3-\lambda) - 5 = 0$$
$$= 3\lambda^2 - 9\lambda + 6 - \lambda^3 + 3\lambda^2 - 2\lambda - 5 = 0$$
$$= -\lambda^3 + 6\lambda^2 - 11\lambda + 1 = 0$$
$$= \lambda^3 - 6\lambda^2 + 11\lambda - 1 = 0$$

By the statement of caley-hamilton theorem every square matrix satisfy of its own characteristic equation:

$$\therefore A^3 - 6A^2 + 11A - I = 0$$
$$\Rightarrow A^3A^{-1} - 6A^2A^{-1} + 11AA^{-1} + IA^{-1} = 0$$
$$\Rightarrow A^2(AA^{-1}) - 6A(AA^{-1}) + 11AA^{-1} = A^{-1}$$
$$\Rightarrow A^2I - 6AI + 11I = A^{-1}$$
$$\therefore A^{-1} = A^2 - 6A + 11I \text{ --- (i)}$$

Now,
$$A^2 = A \cdot A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix} - 6\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix} - \begin{bmatrix} 12 & 0 & -6 \\ 30 & 6 & 0 \\ 0 & 6 & 18 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

16/02/25 - S&P

2021 6(c)

Table for calculation:

x	y	xy	x^2	y^2
10	8	80	100	64
12	7	84	144	49
21	17	357	441	289
4	2	8	16	4
7	6	42	49	36
$\Sigma x=54$	$\Sigma y=40$	$\Sigma xy=571$	$\Sigma x^2=750$	$\Sigma y^2=442$

$$\therefore r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left\{\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right\}\left\{\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right\}}}$$

$$= \frac{571 - \frac{54\times 40}{5}}{\sqrt{\left(750-\frac{54^2}{5}\right)\left(442-\frac{40^2}{5}\right)}}$$

$$= 0.974$$

2021 4(d) Given that,

Factory A: $\overline{x}_1 = 460$, $\sigma_1 = 50$, $n_1 = 100$

Factory B: $\overline{x}_2 = 490$, $\sigma_2 = 40$, $n_2 = 80$

(i) For factory A

$$\overline{x}_1 = \frac{\Sigma x_1}{n_1}$$

$$\Rightarrow 460 = \frac{\Sigma x_1}{100}$$

$$\therefore \Sigma x_1 = 460\times 100 = 46000$$

For factory B

$$\overline{x}_2 = \frac{\Sigma x_2}{n_2}$$

$$\Rightarrow 490 = \frac{\Sigma x_2}{80}$$

$$\Rightarrow \Sigma x_2 = 490\times 80 = 39200$$

$\therefore$ Factory A page larger amount for weekly wages.

(ii) For factory A

$$C.V_1 = \frac{\sigma_1}{\overline{x}_1}\times 100 = \frac{50}{460}\times 100 = 10.87$$

$$C.V_2 = \frac{\sigma_2}{\overline{x}_2}\times 100 = \frac{40}{490}\times 100 = 8.16$$

$\therefore$ Factory A shows larger variability in the distribution of wages.

2020 4(d) Factory A: $\overline{x}_1 = 1560$, $\sigma_1 = 90$, $n_1 = 200$

Factory B: $\overline{x}_2 = 1580$, $\sigma_2 = 70$, $n_2 = 160$

$\therefore$ Combined mean, $\overline{x}_c = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1+n_2} = \frac{200\times 1560 + 160\times 1580}{200+160}$

$= 1568.89 \approx 1569$

$$\therefore \text{combined s.d., } \sigma = \sqrt{\frac{n_1(\sigma_1^2+d_1^2) + n_2(\sigma_2^2+d_2^2)}{n_1+n_2}}$$

$$= \sqrt{\frac{200(90^2+81)+160(70^2+121)}{200+160}}$$

$$= 82.32$$

$d_1^2 = (\overline{x}_1 - \overline{x}_c)^2 = (1560-1569)^2 = 81$

$d_2^2 = (\overline{x}_2 - \overline{x}_c)^2 = (1580-1569)^2 = 121$

CamScanner

16/02/25 - L.A.

8) Given that, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 3 & 8 \\ 3 & 6 & 6 \\ 8 & 6 & 11 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 14 & 3 & 8 \\ 3 & 6 & 6 \\ 8 & 6 & 11 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 44 & 33 & 53 \\ 33 & 6 & 21 \\ 53 & 21 & 41 \end{bmatrix}$$

$$L.H.S = A^3 - A^2 - 15A - 15I$$

$$= \begin{bmatrix} 44 & 33 & 53 \\ 33 & 6 & 21 \\ 53 & 21 & 41 \end{bmatrix} - \begin{bmatrix} 14 & 3 & 8 \\ 3 & 6 & 6 \\ 8 & 6 & 11 \end{bmatrix} - 15\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix} - 15\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 44-14-15-15 & 33-3-30-0 & 53-8-45-0 \\ 33-3-30-0 & 6-6+15-15 & 21-6-15-0 \\ 53-8-45-0 & 21-6-15-0 & 41-11-15-15 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

[Proved]

17/02/25 - D.M.

Tree

Tree: A tree is a connected undirected graph with no simple circuits.

Rooted: A rooted tree is a tree in which one vertex has been designated as the root and every edge is directed away from the root.

Full m-ary tree: The tree is called a full m-ary tree if every internal vertex has exactly m childry.

Binary tree: A full m-ary tree is called binary tree if an m-ary tree has m=2 children.

Q: What is the value of prefix expression: + - * 2 3 5 / ↑ 2 3 4 ?

Solⁿ:

+ - * 2 3 5 / ↑ 2 3 4

+ - * 2 3 5 / ↑ 2 3 4

2*3=6

+ - 6 5 / ↑ 2 3 4

6-5=1

+ 1 / ↑ 2 3 4

2↑3=8

+ 1 / 8 4

8/4=2

+ 1 2

1+2=3

Q: Postfix

7 2 3 * - 4 ↑ 9 3 / +

7 2 3 * - 4 ↑ 9 3 / +

2*3=6

7 6 - 4 ↑ 9 3 / +

7-6=1

1 4 ↑ 9 3 / +

1↑4=1

1 9 3 / +

9/3=3

1 3 +

1+3=4

Prefix: ~~+ - * 4 6 8 / ↑ 3 2 3~~

~~3↑2=9~~

~~+ - * 4 6 8 / 9 3~~

~~9/3=3~~

~~+ - * 4 6 8 3~~

~~+ - 24 8 3~~

+ - * 2 4 5 / ↑ 2 2 4

2↑2=4

+ - * 2 4 5 / 4 4

4/4=1

+ - * 2 4 5 1

2*4=8

+ - 8 5 1

8-5=3

+ 3 1

3+1=4

18/02/25 - S&P.

2019 4(c) First four raw moments about the value $a=5$ is

$\mu_1' = 2$, $\mu_2' = 20$, $\mu_3' = 40$ and $\mu_4' = 50$

1st central moments, $\mu_1 = 0$

2nd central moments, $\mu_2 = \mu_2' - (\mu_1')^2$
$= 20 - 2^2$
$= 16$

3rd central moments, $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$
$= 40 - 3\times20\times2 + 2\times2^3$
$= -64$

4th central moments, $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1)^2 - 3(\mu_1')^4$
$= 50 - 4\times40\times2 + 6\times20\times2^2 - 3\times2^4$
$= 162$

$\therefore \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-64}{\sqrt{16^3}} = -1$ $\qquad$ $\therefore \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{162}{16^2} = 0.63$

$\therefore$ distribution is negatively skewed $\qquad$ $\therefore$ distribution is platykurtic

2021 5(d) ~~The worker's age are given be~~

Table for calculation

x	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$
25	-4.2	17.64	-74.088	311.1696
27	-2.2	4.84	-10.648	23.4256
30	0.8	0.64	0.512	0.4096
31	1.8	3.24	5.832	10.4976
33	3.8	14.44	54.872	208.5136
$\Sigma x = 146$	$\Sigma(x-\bar{x}) = 0$	$\Sigma(x-\bar{x})^2 = 40.8$	$\Sigma(x-\bar{x})^3 = -23.52$	$\Sigma(x-\bar{x})^4 = 554.016$

$\therefore \bar{x} = \frac{\Sigma x}{n}$
$= \frac{146}{5}$
$= 29.2$

$\therefore$ First four central moments

$\mu_1 = \frac{\Sigma(x-\bar{x})}{n} = \frac{0}{5} = 0$

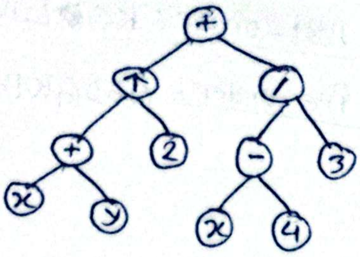
$\mu_2 = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{40.8}{5} = 8.16$

$\mu_3 = \frac{\Sigma(x-\bar{x})^3}{n} = \frac{-23.52}{5} = -4.704$

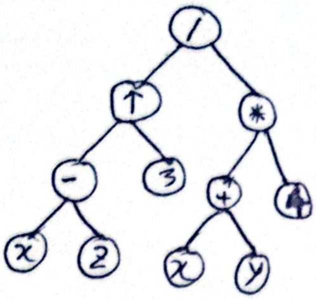
$\mu_4 = \frac{\Sigma(x-\bar{x})^3}{n} = \frac{554.016}{5} = 110.8032$

19/02/25 - D.M.

Q) Represent the expression $((x+y)\uparrow 2) + ((x-4)/3)$ using binary tree



Q) $((x-z)\uparrow 3)/((x+y)*4)$

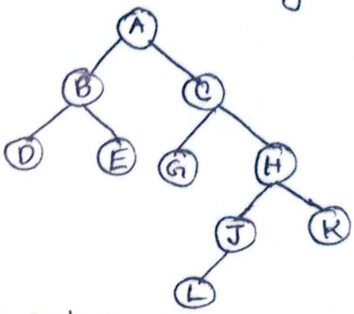


$\boxed{\text{24/02/25 - D.M}}$ # Traversing a binary tree

There are three standard way of traversing a binary tree. These are called Pre-order, In-Order, Post-order traversing. These are as follows.

Pre-order

i) Process the Root
ii) Traverse the left sub-tree of Root in pre-order
iii) Traverse the right sub-tree in pre-order



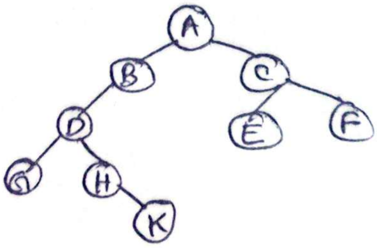
Pre-order: ABDECGHJLK
In-order: DBEAGCLJHK
Post-order: DEBGLJKHCA

In-order

i) Traverse the left sub-tree of Root in in-order
ii) Process the Root
iii) Traverse the right sub-tree in-in-order

Post-order

i) Traverse the left sub-tree of Root in post-order
ii) Traverse the right sub-tree of Root in post-order
iii) Process the Root



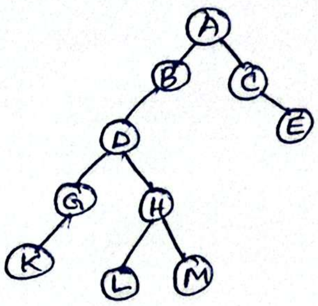
Pre-order: ABDGHKCEF
In-order: GDHKBAECF
Post-order: GKHDBEFCA

Draw Binary tree from traversal

In-order: KGDLHMBAEC

Post-order: KGDLMHDBECA

Pre-order: ABDGKHLMCE



25/02/25 - L.A.

State and Prove Caley-Hamilton theorem.

Statement: Every square matrix satisfies its own characteristic equation.

If the characteristic equation of the nth order matrix A is

$$f(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_{n-1}\lambda + a_n = 0$$

then Caley-Hamilton theorem states that

$$f(A) = A^n + a_1A^{n-1} + a_2A^{n-2} + \dots + a_{n-1}A + a_nI = 0$$

Where I is the nth order Unit matrix and 0 is the nth order zero matrix.

Proof: The characteristic equation for the matrix A is

$$|\lambda I - A| = 0$$ which can be written as

$$\Rightarrow \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_{n-1}\lambda + a_n = 0 \quad \text{—(i)}$$

where $a_1, a_2, \dots, a_n$ are scalars

Multiplying (i) by vector x (say)

$$\lambda^n x + a_1\lambda^{n-1}x + a_2\lambda^{n-2}x + \dots + a_{n-1}\lambda x + a_n x = 0 \quad \text{—(ii)}$$

Here x is the eigen vector corresponding to the eigen value λ.
then by the definition we can write

$$Ax = \lambda x$$

$$A^2x = A\cdot Ax = A\cdot\lambda x = \lambda Ax = \lambda\lambda x = \lambda^2 x$$

$$A^3x = A^2Ax = A^2\lambda x = \lambda A^2 x = \lambda\lambda^2 x = \lambda^3 x$$

$$\vdots$$

$$A^{n-1}x = \lambda^{n-1}x$$

$$A^n x = \lambda^n x$$

Adding above term

$$A^n x + a_1A^{n-1}x + \dots + a_{n-1}Ax + a_nIx = \lambda^n x + a_1\lambda^{n-1}x + \dots + a_{n-1}\lambda x + a_n\lambda$$

$$\Rightarrow A^n x + a_1A^{n-1}x + \dots + a_{n-1}Ax + a_nIx = 0 \quad \text{—(iii)}$$

This will be true for all eigen vectors. So equation (iii) holds for all n-order vectors and is true in general

$$A^n + a_1A^{n-1} + a_2A^{n-2} + \dots + a_{n-1}A + a_nI = 0$$

$\therefore$ A satisfies its own characteristic equation.

(Proved)

2/3/25 - S&P

2021 2(c) For two non-zero positive numbers x_1 and x_2 we have,

arithmetic mean, A.M. $= \frac{x_1+x_2}{2}$

geometric mean, G.M. $= \sqrt{x_1 x_2}$

harmonic mean, H.M. $= \dfrac{2}{\frac{1}{x_1}+\frac{1}{x_2}}$

$$\therefore A.M. \times H.M. = \frac{x_1+x_2}{\cancel{2}} \times \frac{\cancel{2}}{\frac{1}{x_1}+\frac{1}{x_2}}$$

$$= \frac{x_1+x_2}{\frac{1}{x_1}+\frac{1}{x_2}}$$

$$= \frac{x_1+x_2}{\frac{x_1+x_2}{x_1x_2}}$$

$$= \cancel{x_1+x_2} \cdot \frac{x_1x_2}{\cancel{x_1+x_2}}$$

$$= x_1x_2$$

$$= (\sqrt{x_1x_2})^2$$

$$= (G.M.)^2$$

$\therefore$ A.M. $\times$ H.M $= (G.M.)^2$ [Proved]

2018 3(b) For two non-zero positive numbers x_1 and x_2 we have,

A.M. $= \frac{x_1+x_2}{2}$

G.M. $= \sqrt{x_1 x_2}$

H.M. $= \dfrac{2}{\frac{1}{x_1}+\frac{1}{x_2}}$

we know, $(x_1-x_2)^2 \geq 0$

$$\Rightarrow (x_1+x_2)^2 - 4x_1x_2 \geq 0$$

$$\Rightarrow (x_1+x_2)^2 \geq 4x_1x_2$$

$$\Rightarrow x_1+x_2 \geq 2\sqrt{x_1x_2}$$

$$\Rightarrow \frac{x_1+x_2}{2} \geq \sqrt{x_1x_2}$$

$\Rightarrow$ A.M. $\geq$ G.M.

we also know, $\left(\frac{1}{x_1} - \frac{1}{x_2}\right)^2 \geq 0$

$$\Rightarrow \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 - 4\frac{1}{x_1}\frac{1}{x_2} \geq 0$$

$$\Rightarrow \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 \geq 4\frac{1}{x_1}\frac{1}{x_2}$$

$$\Rightarrow \left(\frac{1}{x_1} + \frac{1}{x_2}\right) \geq 2\frac{1}{\sqrt{x_1 x_2}}$$

$$\Rightarrow \sqrt{x_1 x_2} \geq \left(\frac{2}{\frac{1}{x_1} + \frac{1}{x_2}}\right)$$

$$\Rightarrow G.M. \geq H.M$$

$\therefore$ AM $\geq$ GM $\geq$ H.M [Proved]