Digital System Design [Mahmud sir] 03/10/24

Introductory concept:

Number system: Number system is the method by which numbers are calculated or revealed.

Types of number system:
(i) decimal (0-9)
(ii) binary (0-1)
(iii) octal (0-7)
(iv) hexadecimal (0-9, A-F)

Number system conversion:

Decimal → Binary: $(25)_{10} = (?)_2$

$$
\begin{array}{r|l l}
2 & 25 & \\
2 & 12 & 1 \\
2 & 6 & 0 \\
2 & 3 & 0 \\
2 & 1 & 1 \\
 & 0 & 1
\end{array}
$$

$(11001)_2$

$(.8125) = (?)_2$

$$
\begin{array}{r|l}
 & .8125 \times 2 \\
1 & .625 \times 2 \\
1 & .25 \times 2 \\
0 & .5 \times 2 \\
1 & .0
\end{array}
$$

$(.1101)_2$

$(15.303)_{10}$

$$
\begin{array}{r|l l}
2 & 15 & \\
2 & 7 & 1 \\
2 & 3 & 1 \\
2 & 1 & 1 \\
 & 0 & 1
\end{array}
$$

$$
\begin{array}{r|l}
 & .303 \times 2 \\
0 & .606 \times 2 \\
1 & .212 \times 2 \\
0 & .424 \times 2 \\
0 & .848 \times 2 \\
1 & .696
\end{array}
$$

$(1111.01001\ldots)_2$

Statistics and Probability

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Population (গণনাযোগ্য বৈশিষ্ট্য)
- finite
- infinite

variable
পরিবর্তনশীল বৈশিষ্ট্য
- qualititive
- quantitive
  - discrete
  - continuous

constant
অপরিবর্তনশীল বৈশিষ্ট্য

Problem 8:

(i) Highest value ~~value~~ = 86
lowest value = 31
Range = 86 − 31 = 55

We know, the number of classes lie between 5 and 25

class interval: $\frac{R}{25} = \frac{55}{25} = 2.2$ and $\frac{R}{5} = \frac{55}{5} = 11$

let us consider, class interval, c = 10

Table for calculation

Class interval	Tally	Frequency
30-40	~~IIII~~	5
40-50	~~IIII~~ III	8
50-60	~~IIII~~ I	6
60-70	IIII	4
70-80	IIII	4
80-90	III	3
		N = 30

(ii) highest value = 86
lowest value = 31
Range = 86 − 31 = 55

stem and leaf display is given below

stem	leaf	frequency
3	4 6 1 2 3	5
4	6 2 4 6 0 2 6 0	8
5	4 6 0 6 7 7	6
6	6 0 3 4	4
7	6 7 6 0	4
8	6 0 1	3
		N=30

stem = শুরুর ডিজিটস
leaf = শেষের ডিজিট

34 → stem, leaf

324 → stem, leaf

highest value = 68

lowest value = 15

range = 68 − 15 = 53

stem and leaf display is given below

stem	leaf	frequency
1	5 7	2
2	0 7 4 6	4
3	2 3 6 8 3 8 1 0 6 9	10
4	7 2 5 5 9 1 3 8 2	9
5	5 1 2 7	4
6	8	1
		N = 30

26/10/24 - Statistics

Problem 20: ① Highest value = 210

2009 1(d)

lowest value = 68

Range, R = H.V − L.V = 210 − 68 = 142

We know, the number of classes lie between 5 and 25

class interval, $c = \frac{R}{1+3.322 \log N} = \frac{142}{1+3.222 \log 40}$ [N = 40]

$= 22.46$

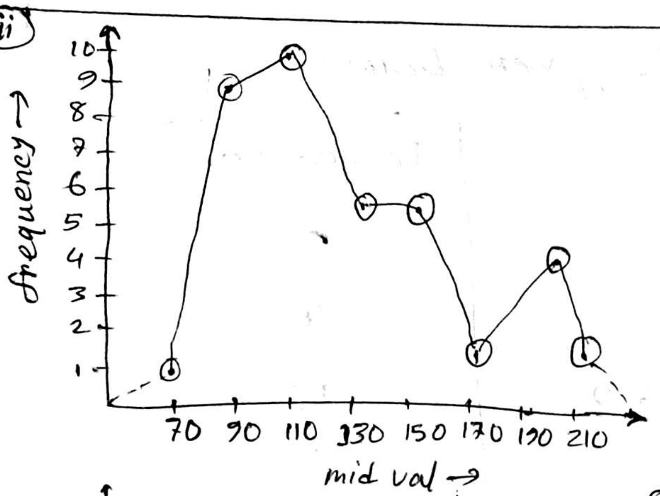
let us consider, class interval = 20

Table for calculation

class internal	Tally	Frequency	Mid value	Cumulative frequency
60 - 80	I	1	70	1
80 - 100	卌 IIII	9	90	10
100 - 120	卌 卌	10	110	20
120 - 140	卌 I	6	130	26
140 - 160	卌 I	6	150	32
160 - 180	II	2	170	34
180 - 200	IIII	4	190	38
200 - 220	II	2	210	40

***
frequency curve বললে
• বিন্দুগুলো হাত দিয়ে যোগ করতে হবে
• ডানে বামে ফাঁকা

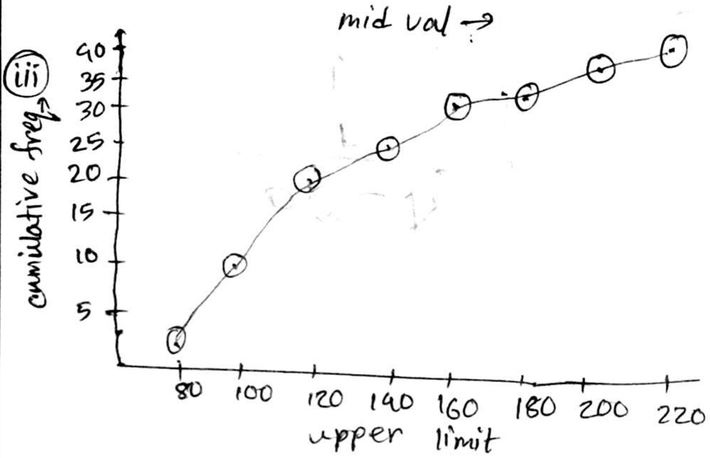
(ii)



frequency polygon

• বিন্দুগুলো স্কেল দিয়ে যোগ করতে হবে
• ডানে বামে (...) দিয়ে বন্ধ করে দিতে হবে

(iii)



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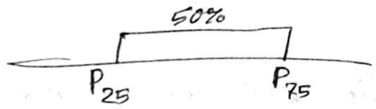
(iv)



Location of $P_{25} = \frac{N \times 25}{100}$ th value

$= \frac{40 \times 25}{100}$ th value

$= 10$-th value

Here, P_{25} class $= 80 - 100$

$P_{25} = L + \frac{\frac{N \times 25}{100} - F.C}{fp} \times c$

$=$

| $L = 80$
| $F.C = 1$
| $fp = 9$
| $C = 20$

Location of $P_{75} = \frac{75 \times N}{100}$ th value $= \frac{75 \times 40}{100}$ th value $= 30$th value

Here, P_{75} class $= 140 - 160$

$P_{75} = L + \frac{\frac{N \times 75}{100} - F.c}{fp} \times c$

$= 140 + \frac{30 - 26}{6} \times 20$

$= 153.33 \approx 153$

| $L = 140$
| $F.c = 26$
| $fp = 6$
| $c = 20$

The limits of wages of central ~~88~~ 50% of the workers

$(P_{25} - P_{75})$

$= (100 - 153)$

$P_{75} - P_{25}$

$(153 - 100)$

(iv)(b) The number of workers whose wages lie between Tk 120 → 160 is
$6+6 = 12$

** যদি বলত TK 115 and TK 175

$\frac{15}{20} \times 10 + 6 + 6 + \frac{15}{20} \times 2$

$= 7.5 + 12 + 1.5$

$= 8 + 12 + 2$

$= 22$

Linear Algebra - Palash sir

(i) Matrix (ii) Determinant (iii) System of linear equation (iv) Linear equation with condition (v) Eigen value and Eigen vector (vi) Vector space.

Matrix ① Definition ② Theorem ③ Inverse matrix ④ Rank of a matrix

Matrix: A matrix is a rectanguale array of numbers (Real or complex) enclosed by a pair of brackets (or double vertical rolls) and the numbers in the array are called the entrices of elements of the matrix.

Ex: $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}]$

Row matrix: A matrix that consists of only one row is called row matrix.

Ex: $A = [a_{11} \; a_{12} \; a_{13} \cdots a_{1n}]_{1\times n}$

Column matrix: A matrix that consists of only one column is called column matrix

Ex: $A = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{n1} \end{bmatrix}_{n\times 1}$

Square matrix: A matrix with the same number of row and column is called square matrix. Ex: $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times 3}$

Rectangular matrix: A matrix whose row and column are not equal then it is called rectangular matrix. $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times 3}$

***Diagonal matrix:** A square matrix whose all elements are zero except the main diagonal then it is called diagonal matrix.

Ex: $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}_{3\times 3}$

Zero matrix; A matrix whose all elements are zero is called zero matrix.
Null matrix

Ex: $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$

Identity matrix/Unit matrix: A square matrix whose elements $a_{ij} = 0$ if $i \neq j$ and $a_{ij} = 1$ if $i = 0$ is called identity matrix

Ex: $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

17/10/2024 - Discrete Math - Palash sir

tion Chapter 1 : Logic, set & function

Proposition: A proposition is a declarative sentence that is either true or false, but not both. Ex: (i) 4+4=8 is true is proposition
(ii) chittagong is the capital of banglades is false is proposition.

Compound Proposition: A proposition is constructed by two or more proposition is called compound proposition.
Ex: Dhaka is the capital of BD and Colombo is in Srilanka.

Conjunction: Let p and q be propositions. The conjuction of p and q denoted by $p \wedge q$ is the proposition "p and q". The conjuction $p \wedge q$ is true when both p and q are true and is false otherwise.

Truth table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

matrix. **Disjunction:** Let p and q be propositions. The disjunction of p and q denoted by $p \vee q$ is the proposition "p or q". The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation: Let p be a proposition. The negation of p denoted by $\neg p$ is the statement "it is not the case that p". If p is true then negation p is false. Again if p is false then negation p is true.

p	$\neg p$
T	F
F	T

Exclusive or: Let p and q be propositions. The exclusive or of p and q denoted by $p \oplus q$ is the proposition that is true when exactly one of p and q is true and is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

matrix.

07/10/2024 – Digital Sys. Design

Deci → Oct

$$\begin{array}{r|l} 8 & 175 \\ 8 & 21 \quad 7 \\ 8 & 2 \quad 5 \\ & 0 \quad 2 \end{array}$$

$\Rightarrow (175)_{10} = (257)_8$

$(.15)_{10} = (?)_8$

.15 × 8 = 1.2
.2 × 8 = 1.6
.6 × 8 = 4.8
.8 × 8 = 6.4
.4 × 8 = 3.2

(.11463..)

$(206.64)_8 = (?)_{10}$

$= 2 \times 8^2 + 0 \times 8^1 + 6 \times 8^0 + 6 \times 8^{-1} + 4 \times 8^{-2}$

$= 2 \times 64 + 0 \times 8 + 6 \times 1 + 6 \times \frac{1}{8} + 4 \times \frac{1}{64}$

$= 128 + 0 + 6 + 0.75 + 0.0625$

$= 134.8125$

Deci → Hex

$(2479)_{10} = (?)_{16}$

$$\begin{array}{r|l} 16 & 2479 \\ 16 & 154 \quad 15(F) \\ 16 & 9 \quad 10(A) \\ & 0 \quad 9 \end{array}$$

$= (9AF)_{16}$

.50 × 16 = 8.0

$= (.8)_{16}$

$(247)_{10} = (?)_8$

$$\begin{array}{r|l} 8 & 247 \\ 8 & 30 \quad 7 \\ 8 & 3 \quad 6 \\ & 0 \quad 3 \end{array}$$

$= (367)_8$

$(247)_{10} = (?)_{16}$

$$\begin{array}{r|l} 16 & 247 \\ 16 & 15 \quad 7 \\ & 0 \quad 15(F) \end{array}$$

$= (F7)_{16}$

Bin → Oct

$(110101)_2 = (?)_8$

$\underbrace{110}_{6}\ \underbrace{101}_{5}$

$(65)_8$

Bin → Hex

$(\underbrace{1011}_{5}\ \underbrace{0110}_{B}\ \underbrace{0101}_{5})_2 = (?)_{16}$

$(5B5)_{16}$

in → Dec

$(1101)_2 = (?)_{10}$

$= 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0$

$= 8 + 4 + 0 + 1$

$= (13)_{10}$

$(1101101)_2 = (?)_{10}$

$= 1\times2^6 + 1\times2^5 + 0\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$= 64 + 32 + 0 + 8 + 4 + 0 + 1$

$= (109)_{10}$

1(b) Highest value: 88

Lowest value: 51

~~∴ Range = H.V − L.V = 88 − 51 = 37~~

Stem	leaf	frequecy
5	1	1
6	3 5	2
7	9 9 0 3 7	5
8	8 5	2
		N = 16

(c) Highest value: 48

Lowest value: 16

∴ Range, R = H.V − L.V = 48 − 16 = 32

we know number of classes lie between 5 to 25

class interval, $c = \dfrac{R}{1 + 3.222 \log N}$ $[N = 24]$

$= \dfrac{32}{1 + 3.222 \log 24}$

$= 5.87$

$= 5$

Class interval	tally	frequency
15 − 20	II	2
20 − 25	III	3
25 − 30	~~IIII~~ I	6
30 − 35	~~IIII~~	5
35 − 40	IIII	4
40 − 45	III	3
45 − 50	I	1
		N = 24

Transpose of a matrix: A matrix whose row are change into column and column are change into row, then the new change matrix is called transpose of the original matrix.
If A is a matrix then transpose of matrix A is denoted by A^T or A'

Ex: $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times3}$ $A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3\times2}$

Symmetric matrix: A square matrix A is called symmetric matrix if it satisfy the following condition: $A^T = -A$
$\Rightarrow A = -A^T$

Ex: $A = \begin{bmatrix} 0 & b & -c \\ -b & 0 & e \\ c & -e & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & -b & c \\ b & 0 & -e \\ -c & e & 0 \end{bmatrix}$

Complex matrix: A matrix whose at least one element is a complex number or pure imaginary number, then this matrix is called complex matrix.

Ex: $A = \begin{bmatrix} 2+3i & 4 \\ 3i & 5-i \end{bmatrix}$

Conjugate of a complex matrix: A complex matrix is said to be a conjugate of a complex matrix if change their conjugate sign. If A is a complex matrix then conjugate of the complex matrix A is denoted by $\bar{A}$.

Ex: $A = \begin{bmatrix} 2+3i & -4i \\ 5 & 1-2i \end{bmatrix}$ $\bar{A} = \begin{bmatrix} 2-3i & 4i \\ 5 & 1+2i \end{bmatrix}$

Hermitian matrix: A square complex matrix is said to be a hermitian matrix if it satisfies the condition: $A = \bar{A}^T$

Ex: $A = \begin{bmatrix} 2 & 2+3i \\ 2-3i & 4 \end{bmatrix}$, $\bar{A} = \begin{bmatrix} 2 & 2-3i \\ 2+3i & 4 \end{bmatrix}$, $\bar{A}^T = \begin{bmatrix} 2 & 2+3i \\ 2-3i & 4 \end{bmatrix}$

Q: Show that $A = \begin{bmatrix} 2 & 2+i & 3i \\ 2-i & 5 & 5-2i \\ -3i & 5+2i & 7 \end{bmatrix}$ is a Hermitian matrix.

A:

$\Rightarrow \bar{A} = \begin{bmatrix} 2 & 2-i & -3i \\ 2+i & 5 & 5+2i \\ 3i & 5-2i & 7 \end{bmatrix}$

$\Rightarrow \bar{A}^T = \begin{bmatrix} 2 & 2+i & 3i \\ 2-i & 5 & 5-2i \\ -3i & 5+2i & 7 \end{bmatrix}$

$\therefore A = \bar{A}^T$ then it is a Hermitian matrix

[shown]

09/10/24 - Discrete Math

Implication/Conditional statement: Let p and q be propositions. The conditional statement $p \to q$ is the proposition "if p then q". The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \to q$, p is called the hypothesis and q is called the conclusion.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi-conditional statement: Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q". The biconditional statement $p \leftrightarrow q$ is true when p and q both have same truth values and false otherwise. Biconditional statement is also called bi-implication.

p	q	$p \leftrightarrow$
T	T	T
T	F	F
F	T	F
F	F	T

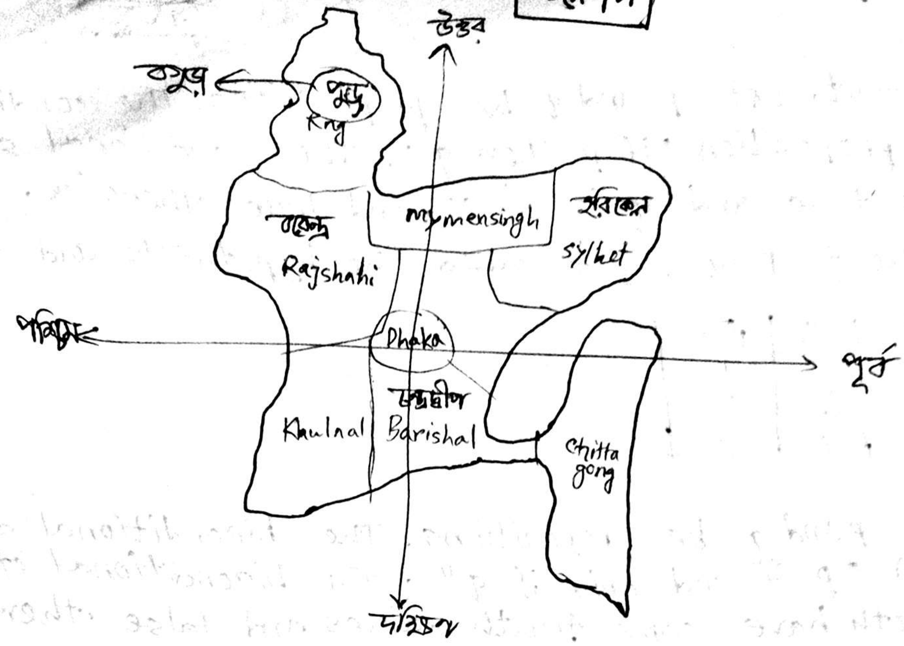
Construct the truth table for the compound proposition $(P \vee \neg Q) \to (P \wedge Q)$

P	Q	$\neg Q$	$P \vee \neg Q$	$P \wedge Q$	$(P \vee \neg Q) \to (P \wedge Q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Construct the truth table for the compound proposition: $(P \vee Q) \leftrightarrow (\neg P \wedge Q)$

P	Q	$\neg P$	$P \vee Q$	$\neg P \wedge Q$	$(P \vee Q) \leftrightarrow (\neg P \wedge Q)$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	T

জনপদ



~~$(508.691)_8 = (?)_{10}$~~

~~$= 5\times8^2 + 0\times8^1 + 8\times8^0 + 6\times8^{-1} + 9\times8^{-2} + 1\times8^{-3}$~~

~~$= 320 + 0 + 8 + 0.75 + 0.140625 +$~~

$(507.621)_8 = (?)_{10}$

$= 5\times8^2 + 0\times8^1 + 7\times8^0 + 6\times8^{-1} + 2\times8^{-2} + 1\times8^{-3}$

$= 320 + 0 + 7 + 0.75 + 0.03125 + 0.001953125$

$= 327.7832032$

Oct → Hex

$(334.567)_8 = (?)_{16}$

3	3	4	.	5	6	7
011	011	100		101	110	111

$\underbrace{1101}_{D}\underbrace{1100}_{C} . \underbrace{1011}_{B}\underbrace{1011}_{B}\underbrace{1000}_{8}$

Statistics / 20/10/24 / ~~202~~ 2012

2(b) highest value = 75
lowest value = 5

Stem	leaf	frequency
0	5	1
1		0
2	4 8 4	3
3	7 9 5 3 3	5
4	2 5 8 9 1	5
5	6 0 8	3
6	5 2 7 9 1	5
7	0 5 2	3
		N = 25

2(c) (i) highest value = 86
lowest value = 31

Range, R = h.v. − l.v = 86 − 31 = 55

we know number of classes lies between 5 to 25

$\therefore$ class interval, $c = \frac{R}{1+3.222 \log N} = \frac{55}{1+3.222 \log 30} = 9.5$ ≈ 10

class interval	tally	frequency
30 - 40	~~IIII~~	5
40 - 50	~~IIII~~ III	8
50 - 60	~~IIII~~ I	6
60 - 70	IIII	4
70 - 80	IIII	4
80 - 90	III	3
		N = 30

(ii)

class interval	frequency	relative frequency	cumulative frequency	relative cumulative frequency	frequency density
30 - 40	5	0.167	5	16.667	0.5
40 - 50	8	0.267	13	43.33	0.8
50 - 60	6	0.2	19	63.33	0.6
60 - 70	4	0.133	23	76.67	0.4
70 - 80	4	0.133	27	90	0.4
80 - 90	3	0.1	30	100	0.3

***

$$\text{frequency density} = \frac{\text{frequency of a certain class}}{\text{class interval of that specific class}}$$

$$\text{relative frequency} = \frac{\text{frequency of a certain class}}{\text{total frequency}}$$

$$\text{relative cumulative frequency} = \frac{\text{cumulative freq. of a certain class}}{\text{total frequency}} \times 10$$

$$\text{percentage frequency of a class} = \frac{\text{freq. of a certain class}}{\text{total frequency}} \times 100$$

22/10/24 - Statistics
previous

20/10/24 — Linear algebra

Anti/Be/Skew Hermitian Matrix: A square complex matrix is said to be skew Hermitian matrix if it satisfies the following condition: $A = -\bar{A}^T$

Ex: $A = \begin{bmatrix} 2i & -2+i & 3i \\ 2+i & 5i & 5-2i \\ 3i & -5-2i & 0 \end{bmatrix}$ $\bar{A} = \begin{bmatrix} -2i & -2-i & -3i \\ 2-i & -5i & 5+2i \\ -3i & -5+2i & 0 \end{bmatrix}$ $\bar{A}^T = \begin{bmatrix} -2i & 2-i & -3i \\ -2-i & -5i & 5+2i \\ -3i & 5+2i & 0 \end{bmatrix}$

$\Rightarrow -\bar{A}^T = \begin{bmatrix} 2i & -2+i & 3i \\ 2+i & 5i & 5-2i \\ 3i & -5-2i & 0 \end{bmatrix} = A$

Idempotent matrix: A square matrix A is said to be an idempotent matrix if it satisfies the following condition: $A^2 = A$.

Ex: $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ $A^2 = \begin{bmatrix} 1+3-5 & -3-9+15 & -5-15+25 \\ -1-3+5 & 3+9-15 & 5+15-25 \\ 1+3-5 & -3-9+15 & -5-15+25 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$.

Show that the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ is idempotent.

$\Rightarrow A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} = \begin{bmatrix} 4+2-4 & -2-3+4 & 2+2-3 \\ -4-6+8 & 2+9-8 & -2-6+6 \\ -8-8+12 & 4+12-12 & -4-8+9 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

Nilpotent matrix: A square matrix is said to be a nilpotent matrix if it is satisfies the following condition: $A^2 = 0$

Ex: $A = \begin{bmatrix} -1 & 3 & 4 \\ 1 & -3 & -4 \\ -1 & 3 & 4 \end{bmatrix}$

Singular matrix: Let D be the determinant of matrix A. Then if $D=0$ the matrix is called singular matrix.

Ex: $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $D = |A| = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4-4 = 0$;

Non-singular matrix:

relative

21/10/24 - Discrete math

Q: Which of the following ~~sta~~ sentences are proposition? What are the truth values of those that are proposition?

1. Comilla is the capital of Bangladesh. (Proposition - false)
2. Are you sick? (Not proposition)
3. $x + 1 = 3$ ~~(Proposition)~~

Q: 1. Washington D.C is the capital of the USA.
2. Torronto is the capital of Canada.
3. $1 + 1 = 2$
4. $2+4=5$

A: All of the statements are propositions because they are statements which are either true or false. ~~Statement 1 and 2 are true propositions~~.

1. The truth value of statement 1 is true
2. " " " " " 2 is false
3. " " " " " 3 is true
4. " " " " " 4 is false

Q: 1. What time is it?
2. Read this carefully.
3. $x + 1 = 2$
4. $x + y = z$

A: None of the statements are propositions.

Q: 1. Sum of two and three are five.
2. What is your name?
3. $x+5=5$
4. $3+4=7$

A: Statement 1 and 4 are propositions because they are statements which are either true or false. Both of them are true propositions. But statement 2 and 3 are not propositions.

22/10/24 - Statistics

table from previous

class interval	frequency	mid value	cumulative frequency (less than)	cumulative frequency (more than)
30 – 40	5	35	5	30
40 - 50	8	45	13	25
50 - 60	6	55	19	17
60 - 70	4	65	23	11
70 - 80	4	75	27	7
80 - 90	3	85	30	3
	N = 30			

(i) Histogram:

(ii) frequency curve:

(iii) frequency polygon:

(iv) ogive / ogive (less than ogive):

(v) ogive (more than ogive):

22/10/24 – Dig. Sys. Des.

Code: Computer system using every character, number, special characters, symbol are represented by using 0 and 1 with unique thats called code.

Types of code: (i) Octal code (ii) Hexadecimal code (iii) BCD code (iv) Alphanumeric code (v) ASCII code (vi) coure code (vii) EBCDIC code (viii) gray code (ix) Unicode

BCD | Binary Coded Decimal | Decimal equivalent binary numbers are called BCD code. BCD is a 4 bit code

Example:

8	7	4	← Decimal
1000	0111	0100	← BCD

ASCII | American Standard Code for Information Interchange | This code is a 7bit code. So it has $2^7 = 128$ possible codes

Binary addition

A	B	O/P	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Binary subtraction

A	B	O/P	borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Binary multiplication

A	B	O/P
0	0	0
0	1	0
1	0	0
1	1	1

```
     1001
     1101
   ------
     1001
    00000
   100100
  1001000
 --------
 10100101
```

$-2^{31} \rightarrow 2^{31}-1$

□ 00

(1)
(2)
(3)

প্রাথমিক

0	000
1	001
2	010
3	011
-0	100
-1	101
-2	110
-3	111

$5+(-3)$

0	000
1	001
2	010
3	011
-1	100
-2	101
-3	100
-0	111

0	000
1	001
2	010
3	011
-1	111
-2	110
-3	101
-4	100

22/10/24 – Linear Algebra

Inverse matrix: Suppose A be a square matrix and $|A| \neq 0$ then the adjoint matrix of A is divided by the determinant of the matrix A. Then A is called inverse matrix. It is denoted by $A^{-1} = \frac{\text{adj } A}{|A|}$

Adjoint matrix: ~~Suppose~~ If we transpose a square matrix which is formed by the co-factor elements of the given matrix then it is called adjoint matrix of the given matrix. It is denoted by adj(A)

Ex: $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ then $\text{adj}(A) = \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}$

Q: If $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ prove that $A^2 - 3A + 2I = 0$

A:

$$A^2 = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 9-2 & 6+0 \\ -3-0 & -2+0 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ -3 & 0 \end{bmatrix}$$

$$2I = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^2 - 3A = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix} - \begin{bmatrix} 9 & 6 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A^2 - 3A + 2I = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(Proved)

Q: Show that $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$ is a nilpotent matrix.

A: Condition for nilpotent matrix: $A^2 = 0$

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 1+2-3 & 2+4-6 & 3+6-9 \\ 1+2-3 & 2+4-6 & 3+6-9 \\ -1-2+3 & -2-4+6 & -3-6+9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

[Shown]

Q: Show that $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is an idempotent matrix.

Condition for idempotent matrix: $A^2 = A$

$$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = A$$

(Shown)

22/10/24 - Dig. Sys. Des.

Orthogonal matrix: A square matrix A is called orthogonal matrix if it satisfies the following condition $AA^T = A^T A = I$

Ex:

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \therefore A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^T A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Q: Show that matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ is idempotent. (2022).

A: Condition for idempotent matrix: $A^2 = A$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} = \begin{bmatrix} 4+2-4 & -2-3+4 & 2+2-3 \\ -4-6+8 & 2+9-8 & -2-6+6 \\ -8-8+12 & 4+12-12 & -4-8+9 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} = A$$

Q: ~~Show that matrix~~ if $A = \begin{bmatrix} 2 & -2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then prove that $A^3 + A^2 - 21A - 45I = 0$

A:

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} 2 & -2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 4-4+3 & -4-2+6 & -6+12-0 \\ 4+2+6 & -4+1+12 & -6-6-0 \\ -2-4-0 & 2-2+0 & 3+12+0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 6 \\ 12 & 9 & -12 \\ -6 & 0 & 15 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 3 & 0 & 6 \\ 12 & 9 & -12 \\ -6 & 0 & 15 \end{bmatrix} \begin{bmatrix} 2 & -2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 6+0-6 & -6+0-12 & -9-0+0 \\ 24+18+12 & -24+9+24 & -36-54-0 \\ -12+0-15 & 12+0-30 & 18-0+0 \end{bmatrix} = \begin{bmatrix} 0 & -18 & \\ 54 & 9 & \\ -27 & -18 & \end{bmatrix}$$

$$21A = 21 \begin{bmatrix} 2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 42 & 42 & -63 \\ 42 & 21 & -126 \\ -21 & -42 & 0 \end{bmatrix}$$

$$45I = 45 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 45 & 0 & 0 \\ 0 & 45 & 0 \\ 0 & 0 & 45 \end{bmatrix}$$

$$A^3 + A^2 - 21A - 45I = \begin{bmatrix} 0 & -18 & -9 \\ 54 & 9 & -90 \\ -27 & -18 & 18 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 6 \\ 12 & 9 & -12 \\ -6 & 0 & 15 \end{bmatrix} - \begin{bmatrix} 42 & -42 & -63 \\ 42 & 21 & -126 \\ -21 & -42 & 0 \end{bmatrix} - \begin{bmatrix} 45 & 0 & \\ 0 & 45 & \\ 0 & 0 & 45 \end{bmatrix}$$

$$= \Big[$$

23/10/24 Discrete Math

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Tautology: A compound proposition that is always true (no matter what the truth values of the proposition that occur in it) is called tautology.

Truth table of a tautology

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Contradiction: A compound proposition that is always false is called a contradiction.

Truth table of a contradiction

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Contigency: A compound proposition that is neither a tautology nor a contradiction is called a contigency.

Truth table of a contigency

p	q	$p \to q$
T	F	F
F	T	T

Logically equivalent: The propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \Leftrightarrow q$ denotes that p and q are logically equivalent

p	q	$\neg p$	P $\neg p \vee q$	Q $p \to q$	$P \leftrightarrow Q$
F	F	T	T	T	
F	T	T	T	T	
T	F	F	F	F	
T	T	F	T	T	

Q: Show that $\neg(p \to q) \to p$ is a tautology.

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
F	F	T	F	T
F	T	T	F	T
T	F	F	T	T
T	T	T	F	T

Write down the bitwise OR, AND, XOR operation.

Soln: Truth table for the bit operators OR, AND, and XOR

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

24/10/24 - Statistics

Measures of central tendency

(i) Arithmetic Mean (A.M)
(ii) Geometric Mean (G.M)
(iii) Harmonic Mean (H.M)
(iv) Median (Me)
(v) Mode

Arithmetic Mean (A.M): The sum of all observations in a set of data and divided by their number.

For ungrouped data: let, x variable having n values $x_1, x_2, x_3 \cdots x_n$ with their arithmetic mean $\bar{x}$ then the arithmetic mean is defined as:

$$\bar{x} = \frac{\sum x_i}{n} \quad / \quad \bar{x} = \frac{x_1 + x_2 + x_3 \cdots x_n}{n}$$

for grouped data: Let, x variable having n values $x_1, x_2, x_3 \cdots x_n$ with their corresponding frequencies $f_1, f_2, f_3 \cdots f_n$ where, $N = \sum f_i$, then the arithmetic mean is defined as:

$$\bar{x} = \frac{\sum f_i x_i}{N} \quad / \quad \bar{x} = \frac{f_1 x_1 + f_2 x_2 \cdots + f_n x_n}{N}$$

Geometric Mean (G.M): The geometric mean of n non-zero positive observations is the nth root of their product.

for ungrouped data: Let, x variable having n non-zero positive observations then the geometric mean is defined as:

$$G.M = (x_1 x_2 x_3 \cdots x_n)^{1/n} \quad / \quad G.M. = \sqrt[n]{x_1 x_2 x_3 \cdots x_n}$$

for grouped data: let, x variable having n non-zero positive values $x_1, x_2, x_3, \cdots x_n$ with their corresponding frequencies $f_1, f_2, f_3, \cdots f_n$ where $\sum f_i = N$ then the geometric mean is defined as

$$G.M. = (x_1^{f_1} x_2^{f_2} x_3^{f_3} \cdots x_n^{f_n})^{1/N}$$

(i) Prove that, $G.M = \text{Antilog}\left(\frac{\sum \log x_i}{n}\right)$

(ii) Prove that, $G.M = \text{Antilog}\left(\frac{\sum f_i \log x_i}{N}\right)$

Proof (i) let, x variable having n non-zero positive values $x_1, x_2, x_3 \cdots x_n$ then $G.M. = (x_1 x_2 x_3 \cdots x_n)^{1/n}$

$$\Rightarrow \log G.M = \frac{1}{n} \log (x_1 x_2 x_3 \cdots x_n)$$

$$\Rightarrow \log G.M = \frac{\sum \log x_i}{n}$$

$$\Rightarrow G.M. = \log^{-1}\left(\frac{\sum \log x_i}{n}\right)$$

$$= \text{Antilog}\left(\frac{\sum \log x_i}{n}\right)$$

Proved

-1/10/24 - Dig. Sys. Des. | Logic Gate |

Logic gate: A logic gate is a simple switching circuit that determines wheather an input pulse can pass through to output in digital circuit.

Types of logic gate:
1. Basic Gate: (i) AND (ii) OR (iii) NOT
2. Universal Gate: (i) NAND (ii) NOR
3. Exclusive Gate: (i) X-OR (ii) X-NOR

Boolean Algebra

Addition	Multiplication	Complement
$0+0=0$	$0\cdot0=0$	$\bar{0}=1$
$0+1=1$	$0\cdot1=0$	$\bar{1}=0$
$1+0=1$	$1\cdot0=0$	
$1+1=1$	$1\cdot1=1$	

27/10/24 -

Proof of De-Morgan's law

De-Morgan's law: (i) $\overline{A+B} = \bar{A}\cdot\bar{B}$

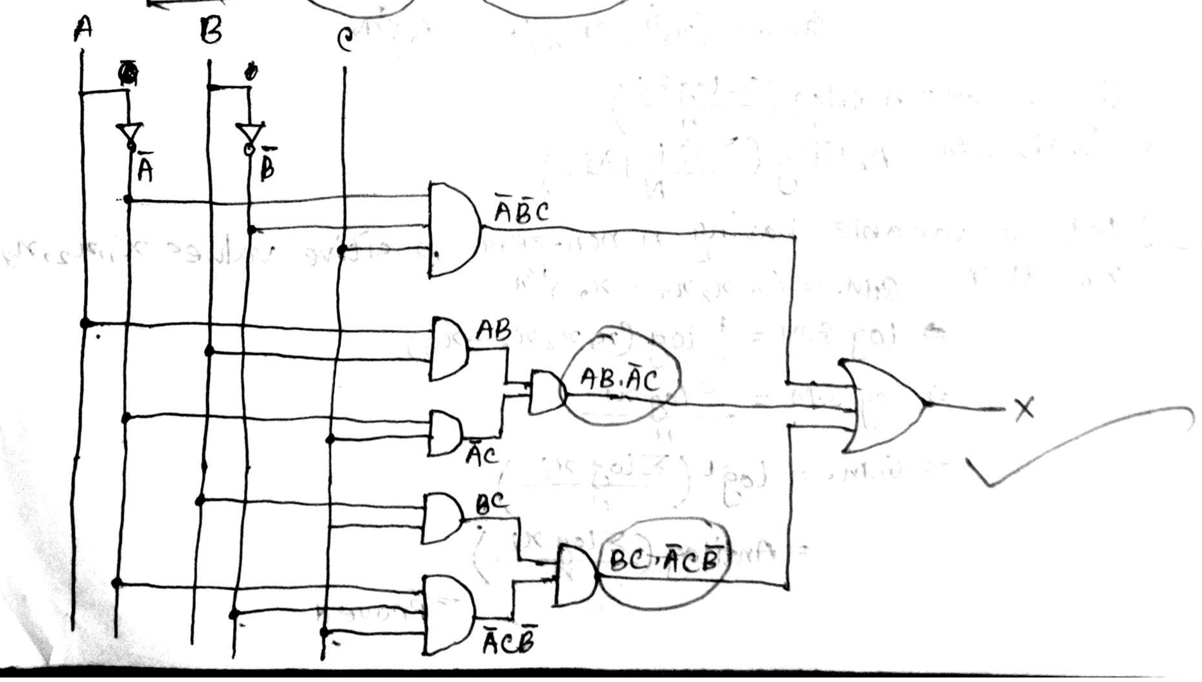
(ii) $\overline{AB} = \bar{A}+\bar{B}$

A	B	$\bar{A}$	$\bar{B}$	$A+B$	$\overline{A+B}$	$\bar{A}\cdot\bar{B}$	AB	$\overline{AB}$	$\bar{A}+\bar{B}$
0	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	0	1	1	0	0	0	1	1
1	1	0	0	1	0	0	1	0	0

law (i) law (ii)

Universality of NAND/NOR

Ex. $X = \boxed{\bar{A}\bar{B}C} + (AB\cdot\bar{A}C) + (BC\cdot\bar{A}C\bar{B})$



24/10/24

Linear Algebra

& $\quad T \quad \sim$

$\cancel{A\bar{A}} + AB + \bar{A}B + \cancel{B\bar{B}}$

1 (i) $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\quad A^{-1} = ?$

Solⁿ: $|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 1(0-0) + 1(0-0) + 1(0+1) = 1 \neq 0$

Now, we have to find cofactor elements of $|A|$

$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = 1$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = 2$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = (-1+2) = 1$

$\therefore$ co-factor matrix, $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

$\therefore \text{adj } A = M^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{\text{adj } A}{|A|} = \dfrac{1}{1} \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

Proof (ii) | Let $x_1, x_2, \cdots x_n$ having n number non-zero positive observations with corresponding frequencies $f_1, f_2, f_3 \cdots f_n$ where $\Sigma f_i = N$

$$\log G.M = \frac{1}{N}\log(x_1^{f_1} x_2^{f_2} \cdots x_n^{f_n})$$
$$= \frac{\log x_1^{f_1} + \log x_2^{f_2} + \cdots \log x_n^{f_n}}{N}$$
$$= \frac{f_1 \log x_1 + f_2 \log x_2 + \cdots + f_n \log x_n}{N}$$
$$= \frac{\Sigma f_i \log x_i}{N}$$
$$\Rightarrow GM = \log^{-1}\left(\frac{\Sigma f_i \log x_i}{N}\right)$$
$$\Rightarrow GM = \text{Antilog}\left(\frac{\Sigma f_i \log x_i}{N}\right)$$

[Proved]

2022/2(b) $\boxed{}$ $\log G = \dfrac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}$

Proof: Let, x variable having n_1 values $x_1, x_2, \cdots x_{n_1}$ with geometric mean G_1.

$$\therefore G_1 = (x_1 x_2 \cdots x_{n_1})^{\frac{1}{n_1}}$$
$$\Rightarrow \log G_1 = \log (x_1 x_2 \cdots x_{n_1})^{1/n_1}$$
$$\Rightarrow \log G_1 = \frac{1}{n_1}\log(x_1 x_2 \cdots x_{n_1})$$
$$\Rightarrow \log(x_1 x_2 \cdots x_{n_1}) = n_1 \log G_1 \quad \text{---(1)}$$

Let, y variable having n_2 values $y_1, y_2, \cdots y_{n_2}$ with geometric mean G

$$\cancel{\therefore G_2 = (y_1 y_2 \cdots y_{n_2})^{1/n_2}}$$
$$\cancel{\Rightarrow \log G_2 = \log (}$$

Similarly, we can prove that: $\log(y_1 y_2 \cdots y_{n_2}) = n_2 \log G_2$ —

Now, $n_1 + n_2$

$$G = (x_1 x_2 \cdots x_{n_1} y_1 y_2 \cdots y_{n_2})^{\frac{1}{n_1+n_2}}$$
$$\Rightarrow \log G = \frac{1}{n_1+n_2}\log(x_1 x_2 \cdots x_{n_1} y_1 y_2 \cdots y_{n_2})$$
$$\Rightarrow \log G = \frac{\log(x_1 x_2 \cdots x_{n_1}) + \log(y_1 y_2 \cdots y_{n_2})}{n_1 + n_2}$$
$$\Rightarrow \log G = \frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}$$

[Proved]

Harmonic Mean (H.M): The harmonic mean of a set of non-zero observations in a series is the reciprocal of the arithmetic mean of the reciprocals.
(তৌলিক গড় / অনুবর্তী গড়)

for ungrouped data: Let, x variable having n non-zero values $x_1, x_2, \ldots x_n$ then the harmonic mean is defined as

$$H.M = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

$$\therefore H.M = \frac{n}{\sum \frac{1}{x_i}}$$

for grouped data: Let, x variable having n non-zero values $x_1, x_2, \ldots x_n$ with corresponding frequencies $f_1, f_2, \ldots f_n$ where $\sum f_i = N$

$$H.M = \frac{N}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots + \frac{f_n}{x_n}} = \frac{N}{\sum \frac{f_i}{x_i}}$$

CamScanner

(2) $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$\therefore |A| = 3(-3+4) + 3(2-0) + 4(-2+0) = 3+6-8 = 1 \neq 0$

Now, we have to find cofactor elements,

$A_{11} = (-1)^{1+1} \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = 1$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = -2$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = -2$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -1$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = 3$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -3 \\ 0 & -1 \end{vmatrix} = 3$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} = 0$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = -4$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = -3$

$\therefore$ co-factor matrix, $M = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$

$\therefore \text{adj } A = M^T = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{\text{adj}}{|A|} = \dfrac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 9-6+0 & -9+9-4 & 12-12+4 \\ 6-6+0 & -6+9-4 & 8-12+4 \\ 0-2+0 & 0+3-1 & 0-4+1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 9-8+0 & -9+12-4 & 12-16+4 \\ 0-2+0 & 0+3-0 & 0-4+0 \\ -6+4-0 & 6-6+3 & -8+8-3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$\therefore A^3 = A^{-1}$

[Proved]

29/10/24 - Statistics

CSE 2021 : 2(c): For two non-zero positive values prove that $A.M \times H.M = (G.M)^2$

CSE 2020: 3(c): For two non-zero positive values prove that $A.M \geq G.M \geq H.M$

Proof: 2(c) let, x_1 and x_2 are two non-zero and positive values
we have,

$$A.M = \frac{x_1 + x_2}{2}$$

$$G.M = \sqrt{x_1 x_2}$$

$$H.M = \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}}$$

$$\therefore A.M \times H.M = \frac{x_1 + x_2}{2} \times \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}}$$

$$= \frac{x_1 + x_2}{\frac{x_2 + x_1}{x_1 x_2}}$$

$$= \frac{x_1 + x_2}{x_1 + x_2}(x_1 x_2)$$

$$= x_1 x_2$$

$$= (\sqrt{x_1 x_2})^2$$

$$\therefore A.M \times H.M = (G.M)^2$$

[Proved]

Proof: 3(c) let, x_1 and x_2 are two non-zero and positive values

$$A.M = \frac{x_1 + x_2}{2}$$

$$G.M = \sqrt{x_1 x_2}$$

$$H.M = \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}}$$

we know, $(x_1 - x_2)^2 \geq 0$

$$\Rightarrow (x_1 + x_2)^2 - 4x_1 x_2 \geq 0$$

$$\Rightarrow (x_1 + x_2)^2 \geq 4 x_1 x_2$$

$$\Rightarrow (x_1 + x_2) \geq 2\sqrt{x_1 x_2}$$

$$\Rightarrow \frac{x_1 + x_2}{2} \geq \sqrt{x_1 x_2} \quad \text{---} (i)$$

$$\therefore A.M \geq G.M \quad \text{---} (ii)$$

Again, $\left(\frac{1}{x_1} - \frac{1}{x_2}\right)^2 \geq 0$

$$\Rightarrow \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 - 4\frac{1}{x_1}\frac{1}{x_2} \geq 0$$

$$\Rightarrow \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 \geq \frac{4}{x_1 x_2}$$

$$\Rightarrow \left(\frac{1}{x_1} + \frac{1}{x_2}\right) \geq \frac{2}{\sqrt{x_1 x_2}}$$

$$\Rightarrow \sqrt{x_1 x_2} \geq \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}}$$

$$\therefore G.M \geq H.M \quad \text{---} (iii)$$

from eq (ii) and (iii) :

$$A.M \geq G.M \geq H.M$$

[Proved]

V 10/24 – Dig. Sys. Des. | Logic Gate |

CSE: 2021 : 2(d)

Given that,

$n = 540$

$\bar{x} = 60$

We know, $\bar{x} = \frac{\sum x_i}{n}$

$\Rightarrow \sum x = \bar{x} \cdot n = 540 \times 60 = 32400$

$\therefore$ incorrected total, $\sum x = 32400$

$\therefore$ corrected total, $\sum x' = \sum x$ – (wrong data) + (real data)

$= 32400 - (77+67) + (47+57)$

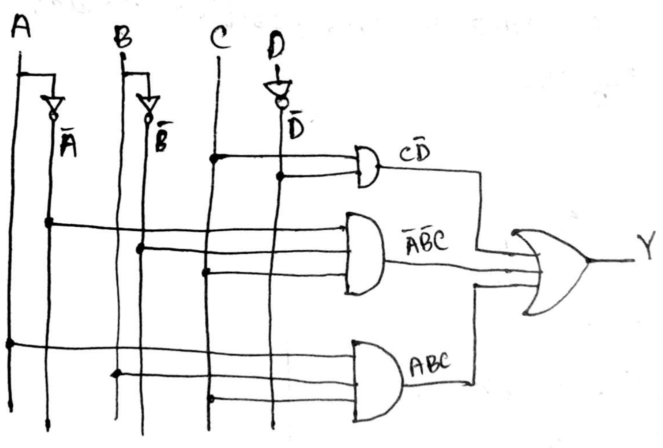
$= 32360$

$\therefore$ corrected mean, $\bar{x}' = \frac{\sum x'}{n}$

$= \frac{32360}{540}$

$= 59.92$

$Y = C\bar{D} + \bar{A}\bar{B}C + ABC$



$(a+b) = \square$

$a - b = \square$

$a, b = 1$

$(a+b)^2 = (a-b)^2 + 4ab$

$(a-b)^2 = (a+b)(a-b)$

$C\bar{D}(\bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB)$

$= C\bar{D}(\bar{A}(\bar{B}+B)^{1} + A(B+\bar{B}))$

$= C\bar{D}(\bar{A}+A)$

$= C\bar{D}$

$\bar{A}\bar{B}(CD + C\bar{D})$

$= \bar{A}\bar{B}(C(D+\bar{D}))$

$= \bar{A}\bar{B}C$

$AB(CD + C\bar{D})$

$= AB(C(D+\bar{D}))$

$= ABC$

$\cancel{CD + C\bar{D}}$

$= \cancel{D(C+C}$

12/11/24 - Statistics

29/10/24 — Dig. Sys. Des.

Combinational Logic Circuit

A circuit whose output depends only on the instantaneous value of input thats called combinational logic circuit.

Types of combinational logic circuit: (i) sum of products
(ii) products of sum

Example: (i) $ABC + A\bar{B}\bar{C} + AC$
(ii) $(A+B+C)(\bar{A}+\bar{B}C)(B+C)$

Simplify the expression:

(i) $Y = A\bar{B}D + A\bar{B}\bar{D}$
$= A\bar{B}(D+\bar{D})$
$= A\bar{B}\cdot 1$
$= A\bar{B}$

(ii) $(\bar{A}+B)(A+B)$
$= \bar{A}A + \bar{A}B + AB + BB$
$= 0 + \bar{A}B + AB + B$
$= B(\bar{A}+A+1)$
$= B$

(iii) $A\bar{B}\bar{C} + A\bar{B}C + ABC$
$\Rightarrow A\bar{B}(\bar{C}+C) + ABC$
$= A\bar{B}\cdot 1 + ABC$
$= A\bar{B} + ABC$
$= A(\bar{B}+BC)$
$= A(\bar{B}+B)(\bar{B}+C)$
$= A(\bar{B}+C)$

$\Rightarrow A\bar{B}\bar{C} + A\bar{B}C + A\bar{B}C + ABC$
$\Rightarrow A\bar{B}(\bar{C}+C) + AC(\bar{B}+B)$
$\Rightarrow A\bar{B} + AC$
$\Rightarrow A(\bar{B}+C)$

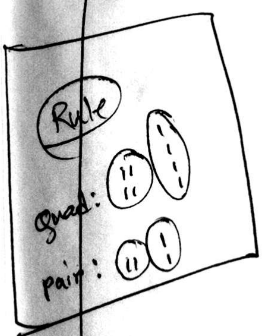
Karnaugh Map

if variable 'n' then its 2^n

Types of K-map: (i) Two variable K-map
(ii) Three " "
(iii) Four " "

Example: Using K-map solve this and draw the logic circuit

$Y = \bar{A}\bar{B}CD + ABCD + \bar{A}C\bar{D} + AC\bar{D} + ABC$
$= \bar{A}\bar{B}CD + ABCD + \bar{A}C\bar{D}(B+\bar{B}) + AC\bar{D}(B+\bar{B}) + ABC(D+\bar{D})$
$= \bar{A}\bar{B}CD + ABCD + \bar{A}BC\bar{D} + \bar{A}\bar{B}C\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + ABCD + ABC\bar{D}$



		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
0	$\bar{A}\bar{B}$			1	1
1	$\bar{A}B$				1
3	AB			1	1
2	$A\bar{B}$				1

$\therefore Y = C\bar{D} + \bar{A}\bar{B}C + ABC$

	$\bar{B}$	B
$\bar{A}$		
A		

	$\bar{C}$	C
$\bar{A}\bar{B}$		
$\bar{A}B$		
AB		
$A\bar{B}$		

$\sqrt{20/24}$ — Dig. Sys. Des.   Logic Gate

9] $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\theta(\cos\theta - 0) - \sin\theta(-\sin\theta - 0) = \cos^2\theta + \sin^2\theta$
$= 1 \neq 0$

Now, we have to find the co-factor elements of $|A|$

$A_{11} = \cos\theta$ | $A_{21} = -\sin\theta$ | $A_{31} = 0$
$A_{12} = \sin\theta$ | $A_{22} = \cos\theta$ | $A_{32} = 0$
$A_{13} = 0$ | $A_{23} = 0$ | $A_{33} = 1$

$\therefore$ cofactor matrix, $M = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \text{adj } A = M^T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9] $x + y + x - y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow 2x = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\Rightarrow x = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

$\therefore$ ~~x+y = [7]~~ $\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Rank: After reduce to echelon form the number of non-zero row defined the rank of a matrix.

Ex: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Reduce it to echelon form

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $R_1 = R_1$, $R_2' = 4R_1 - R_2$, $R_3' = 7R_1 - R_3$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & 6 & 12 \end{bmatrix}$ $R_1 = R_1$, $R_2 = R_2$, $R_3' = 2R_2 - R_3$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{bmatrix}$

Here the number of non-zero row = 2 $\therefore$ Rank, $R(A) = 2$

-2+3

-3+5

10 (i) $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$

Reduce it to echelon form

$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$ $R_1 = R_2$, $R_2' = 2R_1 - R_2$, $R_3' = 3R_1 - R_3$

$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & 3 & 1 \\ 0 & -4 & 6 & 2 \end{bmatrix}$ $R_1 = R_1$, $R_2 = R_2$, $R_3' = 2R_2 - R_3$

$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore$ Rank $\cancel{R}(A) = 2$

30/10/24 - Discrete math

Define bit string. Find the bitwise OR, bitwise AND, and bitwise XOR of the bit string 0110110110 and 1100011101.

Soln Bit-string: A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the ~~string~~.

The bitwise OR is obtained by taking the OR of the corresponding bits respectively.

```
  0110110110
  1100011101
  ----------
  1110111111
```

The bitwise AND is obtained by taking the AND of the corresponding bits respectively.

```
  0110110110
  1100011101
  ----------
  0100010100
```

The bitwise XOR is obtained by taking the XOR of the corresponding bits respectively.

```
  0110110110
  1100011101
  ----------
  1010101011
```

Find bitwise OR, AND, XOR of the bit strings: 10010011010 and 11100010110

OR:

```
  10010011010
  11100010110
  -----------
  11110011110
```

AND:

```
  10010011010
  11100010110
  -----------
  10000010010
```

XOR:

```
  10010011010
  11100010110
  -----------
  01110001100
```

OR:

```
  110010110011
  010001101001
  ------------
  110011111011
```

AND:

```
  110010110011
  010001101001
  ------------
  010000100001
```

XOR:

```
  110010110011
  010001101001
  ------------
  100011011010
```

Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
F	F	F	F	F	F	F	F
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	T	T	T	T	T	T	T

Show that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are logically eqivalent

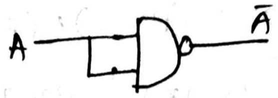
p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
F	F	F	F	F	F	F	F
F	F	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	T	T	T	F	F	F	F
T	F	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	T	F	T	T	T	F	T
T	T	T	T	T	T	T	T

30/10/24 – Dig. Sys. Des Lab

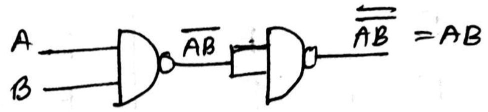
Experiment 01: Verify the operation of NAND Gate as universal gate.

NOT – 7404
AND – 7408
OR – 7432
NAND – 7400
NOR – 7402

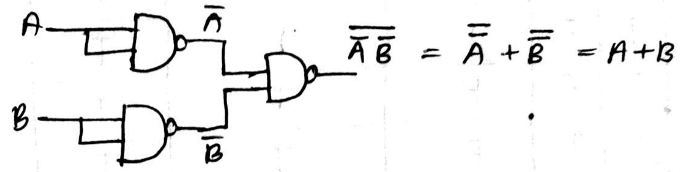
NAND to NOT :

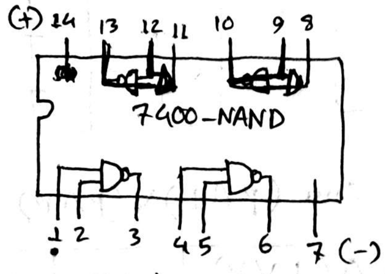


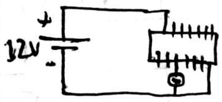
NAND to AND :

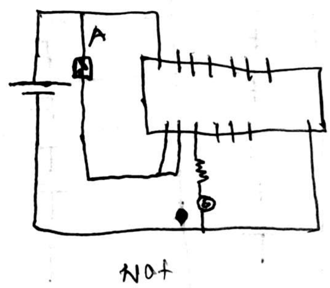


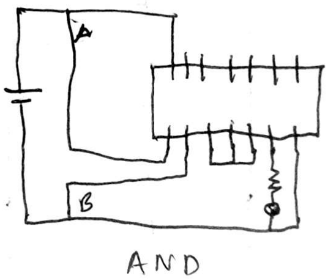
NAND to OR :

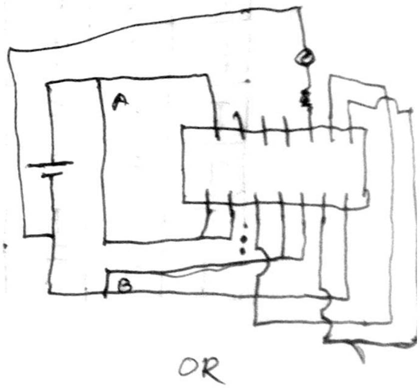












31/10/24 - Dig. Sys. Des.

Binary Substitution Using 2's complement

(−22) (+23)

03/11/24 - Statistics

CSE 2022 - 2(c):

Table for calculation

Profits	No. of Shops (f)	Mid value (x)	$d = \frac{x-A}{c}$	f.d	CF
0-100	12	50	-2	-24	12
100-200	18	150	-1	-18	30
200-300	27	250 → A	0	0	57
300-400	20	350	1	20	77
400-500	17	450	2	34	94
	N = 94			$\Sigma fd = 12$	

c = class interval

$\therefore$ mean, $\bar{x} = A + \frac{\Sigma fd}{N} \times c$

$= 250 + \frac{12}{94} \times 100$

$= 262.76$

location of median $= \frac{N}{2}$ th value

$= \frac{94}{2}$ th value

$= 47$ th value

Here, median class $= 200-300$

$\therefore$ median $= L + \frac{\frac{N}{2} - p.c.f}{f_m} \times c$

$= 200 + \frac{47 - 30}{27} \times 100$

$= 262.96$

L = lower limit of the median class

p.cf. = preceding cumulative frequency

f_m = frequency of the median clas

Here, modal class = 200-300

$\therefore$ mode $= L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$

$= 200 + \frac{9}{9+7} \times 100$

$= 256.25$

L = same as prev

Δ_1 = modal freq − prev. freq (27 − 18)

Δ_2 = modal freq − after. freq (27 − 20)

*** ব্যতিক্রমি হলে
Mode = 3·Median − 2·mean

CSE-2020-3(d)

Let, us consider the number of male employees $= n_1$

the average income of " " $= \overline{x}_1$

$\therefore \overline{x}_1 = 1000$

the number of female employees $= n_2$

the average income of " " $= \overline{x}_2$

$\therefore \overline{x}_2 = 800$

combined mean, $\overline{x}_c = 920$

we know,

$$\overline{x}_c = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$$

$$\Rightarrow 920 = \frac{n_1 \cdot 1000 + n_2 800}{n_1 + n_2}$$

$$\Rightarrow (n_1+n_2)920 = n_1 \cdot 1000 + n_2 \cdot 800$$

$$\Rightarrow 920n_1 + 920n_2 = 1000n_1 + 800n_2$$

$$\Rightarrow 1000n_1 - 920n_1 = 920n_2 - 800n_2$$

$$\Rightarrow 80n_1 = 120n_2$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{120}{80} = \frac{3}{2}$$

$\therefore$ the percentage of male employees $= \left(\frac{3}{3+2}\right) \times 100\% = 60\%$

$\therefore$ " " " female " $= \left(\frac{2}{3+2}\right) \times 100\% = 40\%$.

CS CamScanner

11/11/24 –

03/11/24 – Linear Algebra || Determinant

1) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

$$\text{L.H.S.} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} \quad \begin{matrix} c_1' = c_1 - c_2 \\ c_2' = c_2 - c_3 \end{matrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a^2-b^2 & b^2-c^2 & c^2 \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} a^2-b^2 & b^2-c^2 \\ a^3-b^3 & b^3-c^3 \end{vmatrix}$$

$$= \begin{vmatrix} (a+b)(a-b) & (b+c)(b-c) \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} a+b & b+c \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix} \quad c_2' = c_2 - c_1$$

$$= (a-b)(b-c)\begin{vmatrix} a+b & c-a \\ a^2+ab+b^2 & bc+c^2-a^2-ab \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} a+b & c-a \\ a^2+ab+b^2 & (c+a)(c-a)+b(c-a) \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} a+b & c-a \\ a^2+ab+b^2 & (c-a)(a+b+c) \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)\begin{vmatrix} a+b & 1 \\ a^2+ab+b^2 & a+b+c \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)(a^2+ab+ac+ab+b^2+bc-a^2-ab-b^2)$$

$$= (a-b)(b-c)(c-a)(ab+bc+ca)$$

(Proved)

2) L.H.S. $= \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} \quad \begin{matrix} C_1' = C_1 + C_2 \\ C_2' = C_2 + C_3 \end{matrix}$

$$= \begin{vmatrix} a+b & -(b+c) & -b \\ a+b & b+c & -a \\ -(a+b) & b+c & a+b+c \end{vmatrix}$$

$$= (a+b)(b+c)\begin{vmatrix} 1 & -1 & -b \\ 1 & 1 & -a \\ -1 & 1 & a+b+c \end{vmatrix} \quad c_1' = c_1 + c_2$$

$$= (a+b)(b+c)\begin{vmatrix} 0 & -1 & -b \\ 2 & 1 & -a \\ 0 & 1 & a+b+c \end{vmatrix}$$

CamScanner

$$= -2(a+b)(b+c)\begin{vmatrix} -1 & -b \\ 1 & a+b+c \end{vmatrix}$$

$$= -2(a+b)(b+c)(-a-b-c+b)$$

$$= -2(a+b)(b+c)(-a-c)$$

$$= 2(a+b)(b+c)(a+c)$$

[Shown]

③ $$\text{L.H.S} = \begin{vmatrix} a^2 & bc & ca+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} \quad C_1' = C_1 - C_2 - C_3$$

$$= abc\begin{vmatrix} -2c & c & a+c \\ 0 & b & a \\ -2c & b+c & c \end{vmatrix}$$

$$= (abc)(-2c)\begin{vmatrix} 1 & c & a+c \\ 0 & b & a \\ 1 & b+c & c \end{vmatrix} \quad R_1' = R_1 - R_3$$

$$= (abc)(-2c)\begin{vmatrix} 0 & -b & a \\ 0 & b & a \\ 1 & b+c & c \end{vmatrix}$$

$$= (abc)(-2c)\begin{vmatrix} -b & a \\ b & a \end{vmatrix}$$

$$= (abc)(-2c)(-ab-ab)$$

$$= (abc)(-2c)(-2ab)$$

$$= 4a^2b^2c^2$$

[Proved]

L.H.S.

$$\begin{vmatrix} x^2 & yz & zx+z^2 \\ x^2+xy & y^2 & zx \\ xy & y^2+yz & z^2 \end{vmatrix}$$

$$= xyz\begin{vmatrix} x & z & x+z \\ x+y & y & x \\ y & y+z & z \end{vmatrix} \quad C_1' = C_1 - C_2 -$$

$$= xyz\begin{vmatrix} -2z & z & x+z \\ 0 & y & x \\ -2z & y+z & z \end{vmatrix}$$

$$= (xyz)(-2z)\begin{vmatrix} 1 & z & x+z \\ 0 & y & x \\ 1 & y+z & z \end{vmatrix} \quad R_1' = R$$

$$= (xyz)(-2z)\begin{vmatrix} 0 & -y & x \\ 0 & y & x \\ 1 & y+z & z \end{vmatrix}$$

$$= (xyz)(-2z)\begin{vmatrix} -y & x \\ y & x \end{vmatrix}$$

$$= (xyz)(-2z)(-xy-xy)$$

$$= (xyz)(-2z)(-2xy)$$

$$= 4x^2y^2z^2$$

[Proved]

(4) $$L.H.S = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \qquad \begin{array}{l} c_1' = c_1 - b\cdot c_3 \\ c_2' = c_2 + a\cdot c_3 \end{array}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b+a^2b+b^3 & -a-a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \{1(1-a^2-b^2+2a^2) - 0 - 2b(0-b)\}$$

$$= (1+a^2+b^2)^2 (1+a^2-b^2+2b^2)$$

$$= (1+a^2+b^2)^2 (1+a^2+b^2)$$

$$= (1+a^2+b^2)^3 = R.H.S$$

[Proved]

(6) $$L.H.S = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} \qquad \begin{array}{l} c_1' = c_1 - c_2 \\ c_2' = c_2 - c_3 \end{array}$$

$$= xyz \begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix}$$

$$= xyz \begin{vmatrix} x-y & y-z \\ x^2-y^2 & y^2-z^2 \end{vmatrix}$$

$$= xyz\,(x-y)(y-z) \begin{vmatrix} 1 & 1 \\ x+y & y+z \end{vmatrix}$$

$$= xyz\,(x-y)(y-z)(y+z-x-y)$$

$$= xyz\,(x-y)(y-z)(z-x) = R.H.S$$

04/11/24 - Dis. Math

05/11/

T#1 4

\# Show, $\neg(p\vee(\neg p\wedge q))$ and $\neg p\wedge\neg q$ are logically equivalent.

p	q	$\neg p$	$\neg q$	$\neg p\wedge q$	$p\vee(\neg p\wedge q)$	A: $\neg(p\vee(\neg p\wedge q))$	B: $\neg p\wedge\neg q$	$A\Leftrightarrow B$
F	F	T	T	F	F	T	T	T
F	T	T	F	T	T	F	F	T
T	F	F	T	F	T	F	F	T
T	T	F	F	F	T	F	F	T

Since the biconditional of $\neg(p\vee(\neg p\wedge q))$ and $\neg p\wedge\neg q$ is tautology, so they are log. equiv.

\# Show that, $\neg(p\wedge q)$ and $\neg p\vee\neg q$ are ~~a tautology~~ logically equivalent.

p	q	$\neg p$	$\neg q$	$p\wedge q$	A: $\neg(p\wedge q)$	B: $\neg p\vee\neg q$	$A\Leftrightarrow B$
F	F	T	T	F	T	T	T
F	T	T	F	F	T	T	T
T	F	F	T	F	T	T	T
T	T	F	F	T	F	F	T

\# Show that, $(p\wedge q)\rightarrow(p\vee q)$ is a tautology

p	q	$p\wedge q$	$p\vee q$	$(p\wedge q)\rightarrow(p\vee q)$
F	F	F	F	T
F	T	F	T	T
T	F	F	T	T
T	T	T	T	T

H.W.

$p\wedge(q\vee r)\leftrightarrow(p\wedge q)\vee(p\wedge r)$

\# Show that, $\neg(p\vee q)$ and $\neg p\wedge\neg q$ are logically equivalent. ~~and tautology~~

p	q	$\neg p$	$\neg q$	$(p\vee q)$	$\neg(p\vee q)$	$\neg p\wedge\neg q$	$\neg(p\vee q)\Leftrightarrow\neg p\wedge\neg q$
F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	T
T	F	F	T	T	F	F	T
T	T	F	F	T	F	F	T

\# Show that, $p\vee(q\wedge r)\Leftrightarrow(p\vee q)\wedge(p\vee r)$ is a tautology.

p	q	r	$q\wedge r$	$p\vee(q\wedge r)$	$p\vee q$	$p\vee r$	$(p\vee q)\wedge(p\vee r)$	$p\vee(q\wedge r)\Leftrightarrow(p\vee q)\wedge(p\vee r)$
F	F	F	F	F	F	F	F	T
F	F	T	F	F	F	T	F	T
F	T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	T	T	T	T	T	T	T	T

12/11/24 –

05/11/24 - D.S.D

CT#1 (4)

$Y = ABC\bar{D} + A\bar{B}D + ABC̄\bar{D} + CD$

$= ABC\bar{D} + AB\bar{C}\bar{D} + A\bar{B}D(C+\bar{C}) + CD(A+\bar{A})$

$= ABC\bar{D} + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}D + ACD + \bar{A}CD$

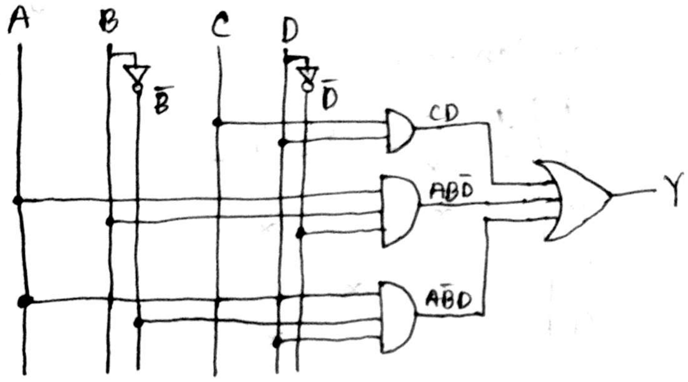
$= ABC\bar{D} + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}D + ACD(B+\bar{B}) + \bar{A}CD(B+\bar{B})$

$= ABC\bar{D} + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}D + ABCD + A\bar{B}CD + \bar{A}BCD + \bar{A}\bar{B}CD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$			1	
$\bar{A}B$			1	
AB	1		1	1
$A\bar{B}$		1	1	

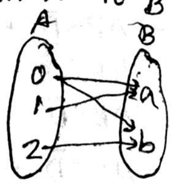
$\therefore$ simplification : $Y = CD(\bar{A}\bar{B} + \bar{A}B + AB + A\bar{B}) + AB(\bar{C}\bar{D} + C\bar{D}) + A\bar{B}(\bar{C}D + CD)$

$= CD + AB\bar{D} + A\bar{B}D$



08/11/24 - Discrete Math II Relation and their properties

Relation: Let, A and B be two sets. A binary relation from A to B is a subset of $A \times B$.



for example: Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$

then $A \times B = \{0, 1, 2\} \times \{a, b\}$

$= \{(0,a), (0,b), (1,a), (1,b), (2,a), (2,b)\}$

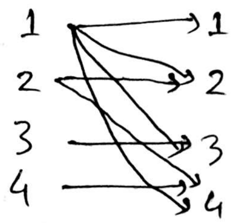
then $\{(a,a), (0,b), (1,a), (2,b)\}$ is a relation from A to B because it is the subset of $A \times B$.

Q# Let, A be the set $\{1,2,3,4\}$. Write down the relation $R = \{(a,b) \mid a$ divides by $b\}$ with graphical represen-tation and tabular form.

A# Since (a,b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b, so the relation is

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

Graphical representation



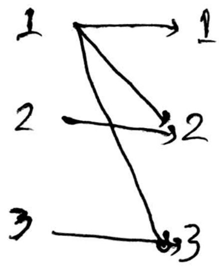
Tabular form

R	1	2	3	4
1	x	x	x	x
2		x		x
3			x	
4				x

Q# $\{1, 2, 3\}$

A# Since (a,b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b, so the relation is: $R = \{(1,1), (1,2), (1,3), (2,2), (3,3)\}$

Graphical representation

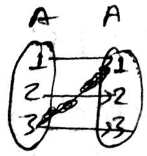


Tabular form

R	1	2	3
1	x	x	x
2		x	
3			x

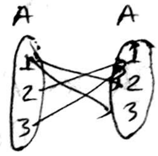
Reflexive Relation: A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

For example: the relation $R = \{(1,1), (2,2), (3,3)\}$ is reflexive on the set $A = \{1,2,3\}$



Symmetric Relation: A relation R on a set A is called symmetric if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

For example: the relation $R = \{(1,2), (2,1), (1,3), (3,1)\}$ is symmetric on the set $A = \{1,2,3\}$



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section A → 6-10
B → 16-20

Section A

6.

Alg

code

output

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Dept of CSE
DCSC

madam

5/2
= 2

0 1

643
100
10

rev = 0;
n = 346
(rev*10)+(n%10)

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

D = $\sqrt{b^2 - 4ac}$

(D > 0) {
(b + D)/(2a)
(b − D)/(2a)
}
(D == 0) {
b/2a
}
else {

$x^2 - 7x - 8$
$x^2 - 8x + x - 8$
$\Rightarrow x(x-8) + 1(x-8)$
$\Rightarrow (x-8)(x+1)$

~~(x-3)(x-3)~~

$x^2 + 4x - 2x - 8$

$x(x+4) - 2(x+4)$

Sequential Logic Circuit

* Digital System whose output does not depend only on the current input rather they are called sequential systems depending on current input and memory.

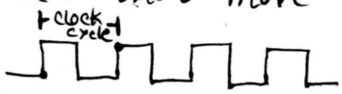
Example: Flip-flop, counter, shift register etc.

Characteristics of a sequential logic circuit:

* This method consists of an electronic memory.
* A combinational part can store some of the output signals in memory.
* The state of the output signal of sequential logic circuit depends on both the input signal and the memory.

Clock:

A clock is the sum of numbers of successive positive and negative pulses that move the square shape in time.



condition of clock pulse:

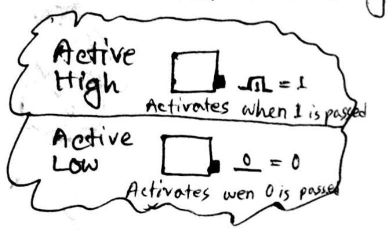
1. Must be periodic
2. Must be stable

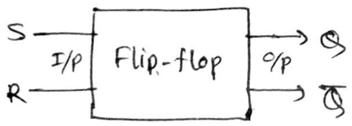
$$\boxed{\text{clock cycle time} = \frac{1}{\text{cycle/second}}}$$

Latch:

For circuit is 'on' latch will set 1 $(Q=1, \bar{Q}=0)$
For " " 'reset' " " " 0 $(Q=0, \bar{Q}=1)$

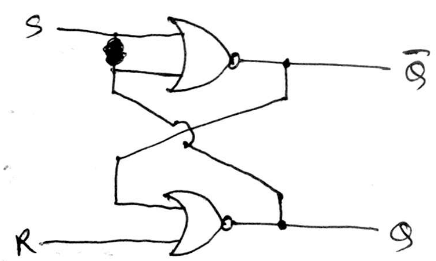
Latch is also called flip-flop circuit. To create binary latch using NOR and NAND gates:





S = set
R = reset
I/P = input
O/P = output

NOR Latch:

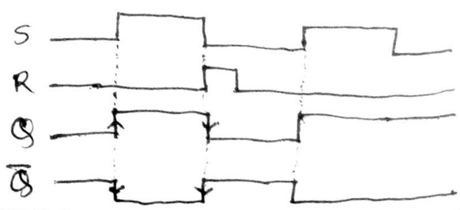


ckt of NOR Latch

S	R	Q	$\bar{Q}$
0	0	Not change	
0	1	1	0
1	0	0	1
1	1	Not usable	

NOR Latch working summary:

if S=1, R=0 then flip-flop is set $(Q=1, \bar{Q}=0)$
if S=0, R=1 " " " reset $(Q=0, \bar{Q}=1)$
if S=R=0 NOR Latch has no change
if S=R=1 " " " not usable



12/11/24

07/11/24:- Statistics

2018| 3(c)

Daily sales	Num of shops (f)	Mid value (x)	f.x	Log x	f log x	$\frac{f}{x}$
20 - 30	9	25	100	1.39	5.59	0.16
30 - 40	7	35	245	1.54	10.8	0.2
40 - 50	16	45	720	1.65	26.45	0.35
50 - 60	12	55	660	1.74	20.88	0.218
60 - 70	6	65	390	1.81	10.87	0.09
70 - 80	5	75	375	1.87	9.37	0.067
	N = 50		$\Sigma fx =$ 2490		$\Sigma f \log x$ = 83.96	$\Sigma \frac{f}{x}$ = 1.08

Arithmetic mean, $\bar{x} = \frac{\Sigma fx}{N}$

$= \frac{2490}{50}$

$= 49.8$ (Ans)

Geometric mean, $GM = \text{Antilog}\left(\frac{\Sigma f \log x}{N}\right)$

$= \text{Antilog}\left(\frac{83.96}{50}\right)$

$= \text{Antilog}(1.6792)$

$= 47.77$ (Ans)

Harmonic mean, $H.M = \frac{N}{\Sigma \frac{f}{x}}$

$= \frac{50}{1.085}$

$= 46.08$ (Ans)

2014 | 2(c)

Given that,

mean salary, $\bar{x} = 2800$

$n = 200$

we know, $\bar{x} = \frac{\Sigma x}{n}$

or, $\Sigma x = n\bar{x}$

incorrected total, $\Sigma x = 200 \times 2800$

$= 560000$

corrected total, $\Sigma x' = \Sigma x - (\text{wrong data}) + (\text{correct data})$

$= 560000 - (3200 + 3600) + (3400 + 3000)$

$= 559600$

correct mean salary, $\bar{x}' = \frac{\Sigma x'}{n}$

$= \frac{559600}{200}$

$= 2798$ (Ans)

2013 | 2(b)

Let us consider,

missing frequency $= f_1$

given that, $\bar{x} = 22$

Table for calculation

Central value (x)	Frequency (f)	fx
10	5	50
15	10	150
20	15	300
25	f_1	$25f_1$
30	5	150
	$N = 35 + f_1$	$\Sigma fx = 650 + 25f_1$

mean, $\bar{x} = \frac{\Sigma fx}{N}$

$\Rightarrow 22 = \frac{650 + 25f_1}{35 + f_1}$

$\Rightarrow 770 + 22f_1 = 650 + 25f_1$

$\Rightarrow 25f_1 - 22f_1 = 770 - 650$

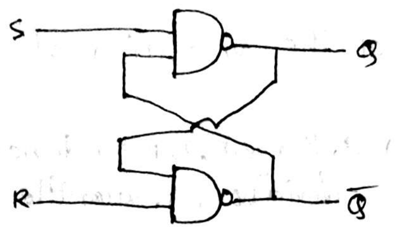
$\Rightarrow 3f_1 = 120$

$\Rightarrow f_1 = \frac{120}{3} = 40$

$\therefore$ missing frequency $= 40$ (Ans)

NAND Latch / SR Latch

10/11/24 - DSD



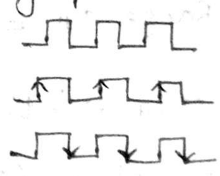
S	R	Q	$\bar{Q}$
1	1	No change	
1	0	0	1
0	1	1	0
0	0	Not usable	

if $S=1$, $R=0$ then flip-flop is reset ($Q=0$, $\bar{Q}=1$)
if $S=0$, $R=1$ " " " set ($Q=1$, $\bar{Q}=0$)
if $S=R=1$ NAND latch has no change
if $S=R=0$ " " is not usable

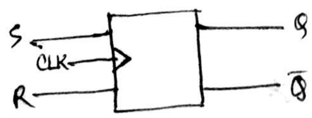
Difference between latch and flip-flop

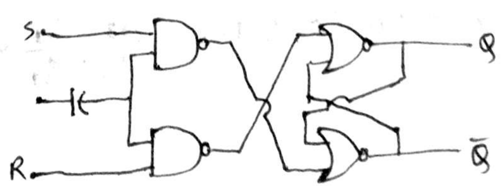
LATCH	FLIP-FLOP
(i) Its a device thats used as a switch	(i) It is used as storage element
(ii) It can transfer data until the switch is on	(ii) They change signal from high to low and from low to high and do not change their position until the next signal change.
(iii) This is used as a temporary buffer	(iii) They are used as a register
(iv) A combination of flip-flop is latch	(iv) It cannot work without clock signal
(v) It does not require clock pulse	(v) It requires clock pulse
(vi) It works without clock signal	

Triggering: Digital system operate asynchronous and synchronous. In asynchronous system, the output changes as soon as the input changes.
A signal known as a clock is used to change the output at the right time after changing in input of synchronous system.
The clock is usually a square pulse denoted by CLK, CK, CP
There are 3 types of triggering pulse:
(i) Level triggering pulse
(ii) Positive " "
(iii) Negative " "



SR flip-flop





S	R	Q_{n-1}	Q_n
0	0	0	0
0	0	1	1
1	0	0	1
1	0	1	1
0	1	0	0
0	1	1	0

* 8 bit flip-flop

10/11/24 - Statistics || Measures of Dispersion

Dispersion: Dispersion is the measure of the variation of the items.

Types of dispersion

(i) Absolute measures of dispersion → 1. Range(R), 2. Mean deviation (M.D), 3. Standard deviation, 4. Quartile deviation

(ii) Relative " " "

(i)-① Range (R) = H.V. − L.V

(i)-② Mean deviation (M.D): Let, x variable having n values $x_1, x_2, \dots x_n$ with mean deviation M.D.

then $$M.D = \frac{\Sigma |x_i - \bar{x}|}{n}$$

$$M.D = \frac{\Sigma |x_i - Me|}{n} \qquad Me = \text{Median}$$

$$M.D. = \frac{\Sigma |x_i - Mo|}{n} \qquad Mo = \text{Mode}$$

for grouped data: Let, x variable having n values $x_1, x_2, \dots x_n$ with corresponding frequencies $f_1, f_2, \dots f_n$ where $\Sigma f_i = N$ and arithmetic mean $\bar{x}$

$$\therefore M.D. = \frac{\Sigma f_i |x_i - \bar{x}|}{N} \qquad \left(\because \bar{x} = \frac{\Sigma f_i x_i}{N}\right)$$

(i)-③ Standard deviation: The standard deviation is the positive square root of the mean of ~~sq~~ the squared deviation from their mean of a set of values.

Let, x variable having n values $x_1, x_2, \dots x_n$ with their mean;

$$\therefore S.D. = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}} \quad \Bigg\| \quad SD = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$$

(grouped data)

(i)-④ Quartile deviation:

1 — Q_1 (1st quartile) — Q_2 (Median) — Q_3 (3rd quartile) — 100

$$Q.D = \frac{Q_3 - Q_1}{2}$$

(ii)-① Coefficient of Range (C.R) $= \frac{H.V - L.V}{H.V + L.V} \times 100\%$

(ii)-② Coefficient of mean deviation (C.M.D.) $= \frac{M.D}{\bar{x}} \times 100\%$

(ii)-③ Coefficient of variation (C.V.) $= \frac{S.D}{\bar{x}} \times 100\%$

(ii)-④ Coefficient of quartile deviation (C.Q.D) $= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100\%$

18/11/24 - Statistics

30/11/24 - Linear Algebra

(6) LHS $= \begin{vmatrix} -a & -b & c & d \\ b & -a & -d & c \\ c & -d & a & -b \\ d & c & b & a \end{vmatrix}$

$= \dfrac{1}{abcd} \begin{vmatrix} -ad & -bc & bc & ad \\ bd & -ac & -bd & ac \\ cd & -ad & -ab & -ab \\ d^2 & c^2 & b^2 & a^2 \end{vmatrix}$

$= \dfrac{1}{abcd} \begin{vmatrix} 0 & -bc & bc & ad \\ 0 & -ac & -bd & ac \\ 0 & -cd & ab & -ab \\ a^2+b^2+c^2+d^2 & c^2 & b^2 & a^2 \end{vmatrix}$

$= \dfrac{-1}{abcd}(a^2+b^2+c^2+d^2) \begin{vmatrix} -bc & bc & ad \\ -ac & -bd & ac \\ -cd & ab & -ab \end{vmatrix}$

$= \dfrac{-(a^2+b^2+c^2+d^2)\,abc}{abcd} \begin{vmatrix} -b & c & d \\ -a & -d & c \\ -d & a & -b \end{vmatrix}$

$= \dfrac{-(a^2+b^2+c^2+d^2)}{d} \{-b(bd-ac) - c(ab+cd) + d(-a^2-d^2)\}$

$= \dfrac{-(a^2+b^2+c^2+d^2)}{d} \{-b^2d + abc - abc - c^2d - a^2d - d^3\}$

$= \dfrac{-(a^2+b^2+c^2+d^2)(-d)}{d} (a^2+b^2+c^2+d^2)$

$= (a^2+b^2+c^2+d^2)^2$

[Proved]

(7) L.H.S $= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ $\quad c_1' = c_1 - c_2$, $c_2' = c_2 - c_3$

$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$

$= \begin{vmatrix} a-b & b-c \\ a^3-b^3 & b^3-c^3 \end{vmatrix}$

$= \begin{vmatrix} a-b & b-c \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) \end{vmatrix}$

$= (a-b)(b-c) \begin{vmatrix} 1 & 1 \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}$

$= (a-b)(b-c)(b^2+bc+c^2-a^2-ab-b^2)$

$= (a-b)(b-c)(c^2-a^2+bc-ab)$

$= (a-b)(b-c)\{(c+a)(c-a) + b(c-a)\}$

$= (a-b)(b-c)(c-a)(c+a+b)$

[Prove]

(8) L.H.S. $= \begin{vmatrix} 1+x_1 & x_2 & x_3 \\ x_1 & 1+x_2 & x_3 \\ x_1 & x_2 & 1+x_3 \end{vmatrix}$ $c_1' = c_1 + c_2 + c_3$

$$= \begin{vmatrix} 1+x_1+x_2+x_3 & x_2 & x_3 \\ 1+x_1+x_2+x_3 & 1+x_2 & x_3 \\ 1+x_1+x_2+x_3 & x_2 & 1+x_3 \end{vmatrix}$$

$= (1+x_1+x_2+x_3) \begin{vmatrix} 1 & x_2 & x_3 \\ 1 & 1+x_2 & x_3 \\ 1 & x_2 & 1+x_3 \end{vmatrix}$ $R_1' = R_1 - R_2$, $R_2' = R_2 - R_3$

$$= (1+x_1+x_2+x_3) \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & x_2 & 1+x_3 \end{vmatrix}$$

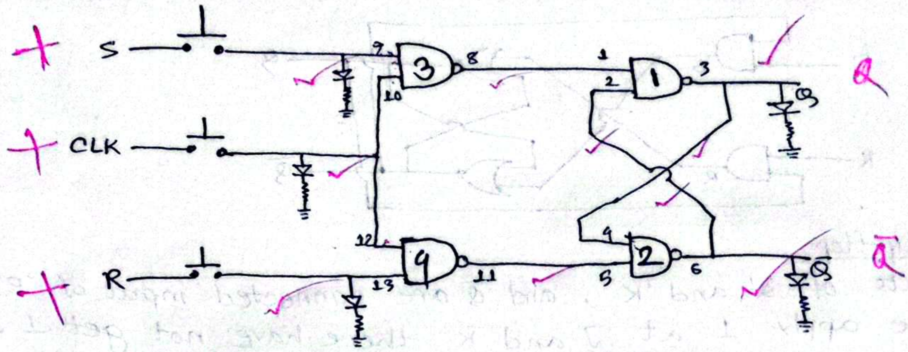
$$= (1+x_1+x_2+x_3) \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix}$$

$$= (1+x_1+x_2+x_3)(1-0)$$

$$= 1+x_1+x_2+x_3$$

sys ready
Enter
edit
compile
link
execute

Experiment 02: Design and implement S-R flip-flop



CLK	S	R	Q	$\bar{Q}$
0	X	X	Memory	
1	0	0	Memory	
1	1	0	1	0
1	0	1	0	1
1	1	1	Invalid	

S	R	Q	$\bar{Q}$
0	0	Invalid	
0	1	1	0
1	0	0	1
1	1	Memory	

S⇄R

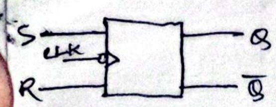
SR
11 Invalid
00 $\bar{Q}$
00

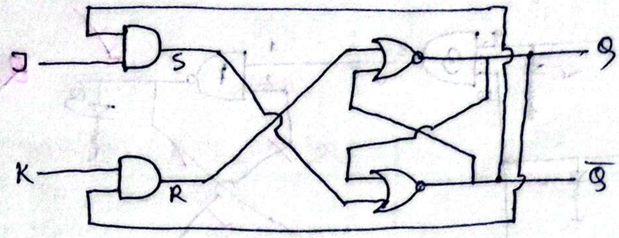
Q
$\bar{Q}$

S ⟷ R

13/11/24 - JK Flip-flop

res
rase
race





Connections of JK-flip-flop

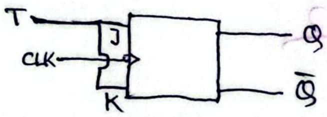
J and $\bar{Q}$ are inputs of 'S' and 'K', and $\bar{Q}$ are connected input of 'R'. That's why if we apply 1 at J and K there have not get 1 at a time in S and R.

So, there are no res. condition.

for J=K=1 input and output is Q is change that means Q = 1 then Q = 0 otherwise Q = 1 then Q = 1

J	K	$S=J\bar{Q}$	$R=KQ$	Q_{n-1}	Q_n
0	0	0	0	X	X
1	0	0	0	1	X
0	1	0	1	0	X
1	1	0	1	0	1
1	1	1	0	1	0

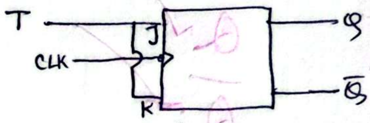
T- flip-flop



Negative edge trigger

CLK	T	Q
X	0	Nc
↑	X	Nc
1	X	Nc
↓	1	Toggle

(-)



Positive edge trigger

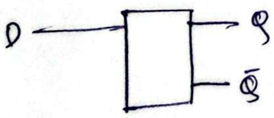
CLK	T	Q
X	0	Nc
↓	X	Nc
1	X	Nc
↑	1	Toggle

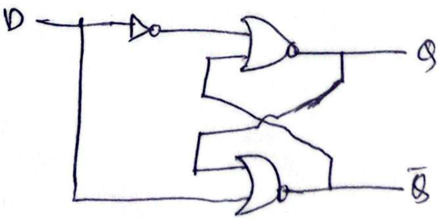
(+)

nc = no change
↑ = pos. trig
↓ = neg. trig
X = any inp
Toggle = (-)/(+)

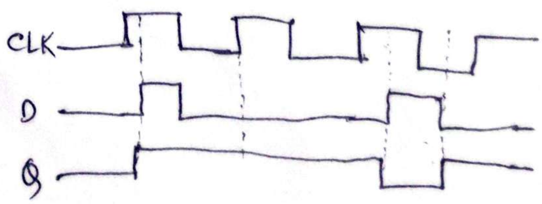
D - flip-flop

In the SR flip-flop both input is 1 there have create race condition. For eliminat is situation in the flip-flop we use D flip-flop. It's also called D-latch





CLK	D	Q
0	X	Nc
1	0	0
1	1	1



12/11/24 - Statistics

45. Factory A : $n_1 = 1500$, $\bar{x}_1 = 6000$, variance, $\sigma_1^2 = 444$

Factory B : $n_2 = 1700$, $\bar{x}_2 = 5000$, variance, $\sigma_2^2 = 520$

(i) For factory A

$\bar{x}_1 = \frac{\sum x_1}{n_1}$

$\Rightarrow \sum x_1 = n_1 \bar{x}_1$

$= 1500 \times 6000$

$= 9000000$

For factory B

$\bar{x}_2 = \frac{\sum x_2}{n_2}$

$\Rightarrow \sum x_2 = n_2 \bar{x}_2$

$= 1700 \times 5000$

$= 8500000$

$\therefore$ Factory A pays larger wages bill.

(ii) for factory A

$C.V = \frac{\sigma_1}{\bar{x}_1} \times 100$

$= \frac{21.07}{6000} \times 100$

$= 0.35$

$\sigma_1^2 = 444$
$\Rightarrow \sigma_1 = \sqrt{444}$
$= 21.07$

for factory B

$C.V = \frac{\sigma_2}{\bar{x}_1} \times 100$

$= \frac{22.80}{5000} \times 100$

$= 0.45$

$\sigma_2^2 = 520$
$\Rightarrow \sigma_2 = \sqrt{520}$
$= 22.80$

$\therefore$ factory B is greater variability in individual wage.

(iii) combined mean, $\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$= \frac{1500 \times 6000 + 1700 \times 5000}{1500 + 1700}$

$= 5468.75$

≈ 5469

CSE-2020
4(d)

factory A

$\bar{x}_1 = 1560$, $\sigma_1 = 90$, $n_1 = 200$

factory B

$\bar{x}_2 = 1580$, $\sigma_2 = 70$, $n_2 = 160$

$\therefore$ combined mean, $\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$= \frac{200 \times 1560 + 160 \times 1580}{200 + 160}$

$= 1568.89 \approx 1569$

$\therefore$ combined standard deviation, $\sigma = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$

$= \sqrt{\frac{200(8100+81) + 160(4900+121)}{200+160}}$

$= 82.32$

$\sigma_1^2 = 90^2 = 8100$
$\sigma_2^2 = 70^2 = 4900$
$d_1^2 = (\bar{x}_1 - \bar{x}_c)^2$
$= (1560 - 1569)^2$
$= 81$
$d_2^2 = (\bar{x}_2 - \bar{x}_c)^2$
$= (1580 - 1569)^2$
$= 121$

Oct 2022
3 (c)

For two unequal observations,

Prove that, $M.D = S.D = \frac{R}{2}$

Proof: Let, x_1 and x_2 are two observations where $x_1 > x_2$

By definition,

$$R = |x_1 - x_2|$$

$$\therefore M.D = \frac{\Sigma |x_i - \bar{x}|}{n}$$

$$= \frac{\Sigma |x_i - \bar{x}|}{2}$$

$$= \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}|}{2}$$

$n = 2$

$\bar{x} = \frac{x_1 + x_2}{2}$

$$= \frac{\left|x_1 - \frac{x_1+x_2}{2}\right| + \left|x_2 - \frac{x_1+x_2}{2}\right|}{2}$$

$$= \frac{\left|\frac{2x_1 - x_1 + x_2}{2}\right| + \left|\frac{2x_2 - x_1 - x_2}{2}\right|}{2}$$

$$= \frac{\left|\frac{x_1 - x_2}{2}\right| + \left|\frac{x_1 - x_2}{2}\right|}{2}$$

$$= \frac{2\left|\frac{x_1 - x_2}{2}\right|}{2}$$

$$\Rightarrow M.D = \left|\frac{x_1 - x_2}{2}\right|$$

$$\Rightarrow MD = \frac{R}{2} \quad \text{—— (i)}$$

$$\therefore SD = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{2}}$$

$$= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2}{2}}$$

$$= \sqrt{\frac{\left(x_1 - \frac{x_1+x_2}{2}\right)^2 + \left(x_2 - \frac{x_1+x_2}{2}\right)^2}{2}}$$

$$= \sqrt{\frac{\left(\frac{2x_1 - x_1 - x_2}{2}\right)^2 + \left(\frac{2x_2 - x_1 - x_2}{2}\right)^2}{2}}$$

$$= \sqrt{\frac{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{x_1 - x_2}{2}\right)^2}{2}}$$

$$= \sqrt{\frac{2\left(\frac{x_1 - x_2}{2}\right)^2}{2}}$$

$$= \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2}$$

$$= \frac{x_1 - x_2}{2}$$

$$\Rightarrow SD = \frac{R}{2} \quad \text{—— (ii)}$$

$$\therefore MD = SD = \frac{R}{2}$$

12/11/24 - linear

(9) L.H.S = $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$

$= \frac{1}{xy} \begin{vmatrix} ax & by & ax+by \\ bx & cy & bx+cy \\ ax^2+bxy & bxy+cy^2 & 0 \end{vmatrix}$ $C_3' = C_3 - C_1 - C_2$

$= \frac{1}{xy} \begin{vmatrix} ax & by & 0 \\ bx & cy & 0 \\ ax^2+bxy & bxy+cy^2 & -(ax^2+2bxy+cy^2) \end{vmatrix}$

$= \frac{-(ax^2+2bxy+cy^2)}{xy} \begin{vmatrix} ax & by \\ bx & cy \end{vmatrix}$

$= \frac{-(ax^2+2bxy+cy^2)}{xy} (acxy - b^2xy)$

$= \frac{-(ax^2+2bxy+cy^2)\, xy(ac-b^2)}{xy}$

$= -(ax^2+2bxy+cy^2)(ac-b^2)$

$= (b^2-ac)(ax^2+2bxy+cy^2)$

(10) L.H.S = $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$ $C_1' = C_1 + C_2 + C_3$

$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$

$= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$ $R_1' = R_1 - R_2$, $R_2' = R_2 - R_3$

$= 2(a+b+c) \begin{vmatrix} 0 & -(a+b+c) & 0 \\ 0 & a+b+c & -(a+b+c) \\ 1 & a & c+a+2b \end{vmatrix}$

$= 2(a+b+c) \begin{vmatrix} -(a+b+c) & 0 \\ a+b+c & -(a+b+c) \end{vmatrix}$

$= 2(a+b+c)\{(a+b+c)^2 - 0\}$

$= 2(a+b+c)^3$

(11) L.H.S = $\begin{vmatrix} (b+c)^2 & a^2 & 1 \\ (c+a)^2 & b^2 & 1 \\ (a+b)^2 & c^2 & 1 \end{vmatrix}$ $R_1' = R_1 - R_2$, $R_2' = R_2 - R_3$

$$= \begin{vmatrix} (b+c)^2-(c+a)^2 & a^2-b^2 & 0 \\ (c+a)^2-(a+b)^2 & b^2-c^2 & 0 \\ (a+b)^2 & c^2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c)^2-(c+a)^2 & a^2-b^2 \\ (c+a)^2-(a+b)^2 & b^2-c^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c+c+a)(b-a) & (a+b)(a-b) \\ (c+a+a+b)(c-b) & (b+c)(b-c) \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} -(a+b+2c) & a+b \\ -(2a+b+c) & b+c \end{vmatrix} \quad R_1' = R_1 - R_2$$

$$= (a-b)(b-c) \begin{vmatrix} a-c & a-c \\ -(2a+b+c) & b+c \end{vmatrix}$$

$$= (a-b)(b-c)(a-c) \begin{vmatrix} 1 & 1 \\ -(2a+b+c) & b+c \end{vmatrix}$$

$$= (a-b)(b-c)(a-c)(b+c+2a+b+c)$$

$$= (a-b)(b-c)(a-c)(2a+2b+2c)$$

$$= -2(a+b+c)(a-b)(b-c)(c-a)$$

(13) L.H.S = $\begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$ $C_1' = C_1 - C_2$, $C_2' = C_2 - C_3$

$$= \begin{vmatrix} \log x - \log y & \log y - \log z & \log z \\ \log 2x - \log 2y & \log 2y - \log 2z & \log 2z \\ \log 3x - \log 3y & \log 3y - \log 3z & \log 3z \end{vmatrix}$$

$$= \begin{vmatrix} \log \frac{x}{y} & \log \frac{y}{z} & \log z \\ \log \frac{2x}{2y} & \log \frac{2y}{2z} & \log 2z \\ \log \frac{3x}{3y} & \log \frac{3y}{3z} & \log 3z \end{vmatrix}$$

$$= \log \frac{x}{y} \log \frac{y}{z} \begin{vmatrix} 1 & 1 & \log z \\ 1 & 1 & \log 2z \\ 1 & 1 & \log 3z \end{vmatrix}$$

$= 0$ [$\because$ two dumns are same]

(14) $$L.H.S. = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} \quad \begin{matrix} c_1' = c_1 - c_2 \\ c_2' = c_2 - c_3 \end{matrix}$$

$\cancel{\left(\frac{n}{1}\right)}$

$\cancel{(A-1)(1-n)}$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x^2-y^2 & y^2-z^2 & z^2 \\ x^3-y^3 & y^3-z^3 & z^3 \end{vmatrix}$$

$$= \begin{vmatrix} x^2-y^2 & y^2-z^2 \\ x^3-y^3 & y^3-z^3 \end{vmatrix}$$

$$= \begin{vmatrix} (x+y)(x-y) & (y+z)(y-z) \\ (x-y)(x^2+xy+y^2) & (y-z)(y^2+yz+z^2) \end{vmatrix}$$

$$= (x-y)(y-z)\begin{vmatrix} x+y & y+z \\ x^2+xy+y^2 & y^2+yz+z^2 \end{vmatrix} \quad c_1' = c_1 - c_2$$

$$= (x-y)(y-z)\begin{vmatrix} x-z & y+z \\ x^2+xy-yz-z^2 & y^2+yz+z^2 \end{vmatrix}$$

$$= (x-y)(y-z)\begin{vmatrix} x-z & y+z \\ (x+z)(x-z)+y(x-z) & y^2+yz+z^2 \end{vmatrix}$$

$$= (x-y)(y-z)(x-z)\begin{vmatrix} 1 & y+z \\ x+y+z & y^2+yz+z^2 \end{vmatrix}$$

$$= (x-y)(y-z)(x-z)\left(\cancel{y^2}+\cancel{yz}+z^2-xy-\cancel{y^2}-\cancel{yz}-xz-yz-\cancel{z^2}\right)$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx)$$

18/11/24 – Discrete Math

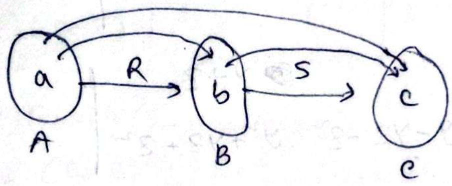
Anti-symmetric: A relation R on a set A is called antisymmetric if $(a,b) \in R$ and $(b,a) \in R$ only if $a=b$ for $a,b \in A$.

For example: The relation $R = \{(1,2),(2,3),(1,1)\}$ is antisymmetric on the set $\{1,2,3\}$

Transitive: A relation R on a set A is called transitive if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for $a,b,c \in A$.

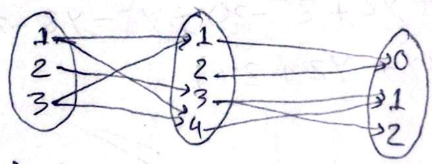
For example: The relation $R = \{(1,2),(2,3),(1,3)\}$ is transitive on the set $\{1,2,3\}$

Composite Relation: Let, R be a relation from a set A to a set B and S a relation from B to a set C. The composite of R and S is the relation consisting of ordered pairs (a,c) where $a \in A$, $c \in C$, and for which there exists an element $b \in B$ such that $(a,b) \in R$ and $(b,c) \in S$. We denote the composite of R and S by $S \circ R$.



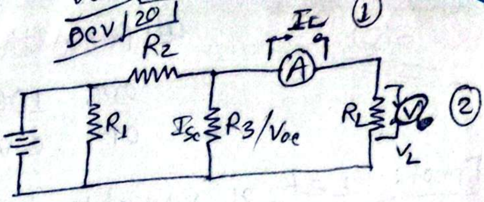
Q: What is the composite relations R and S, where R is the relation from $\{1,2,3\}$ to $\{1,2,3,4\}$ with $R = \{(1,1),(1,4),(2,3),(3,1),(3,4)\}$ and S is the relation from $\{1,2,3,4\}$ to $\{0,1,2\}$ with $S = \{(1,0),(2,0),(3,1),(3,2),(4,1)\}$.

Soln:



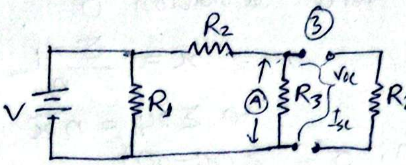
$R \circ S = \{(1,0),(1,1),(2,1),(2,2),(3,0),(3,1)\}$

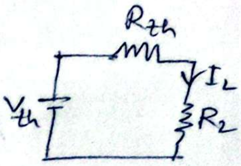
Experiment 4: Verify thevenin's theorem



Here, $R_1 = 100\Omega$
$R_2 = 150\Omega$
$R_3 = 200\Omega$
$R_L = 3.1\ k\Omega$

$R_{th} = \frac{V_{oc} = V_{th}}{I_{se}} = \frac{4.67}{60} = 0.7783\,v$ $\quad V_{oc} = 4.67\,V$ $\quad I_{sc} = 60\,mA$





$I_L = \frac{V_{oc}}{R} = \frac{V_{oc}}{R_{th} + R_2} = \frac{4.67}{0.7783 + 3.1} = 1.204$

$V_L = IR = I_L \times R_L = 1.20 \times 3.1 = 3.73$

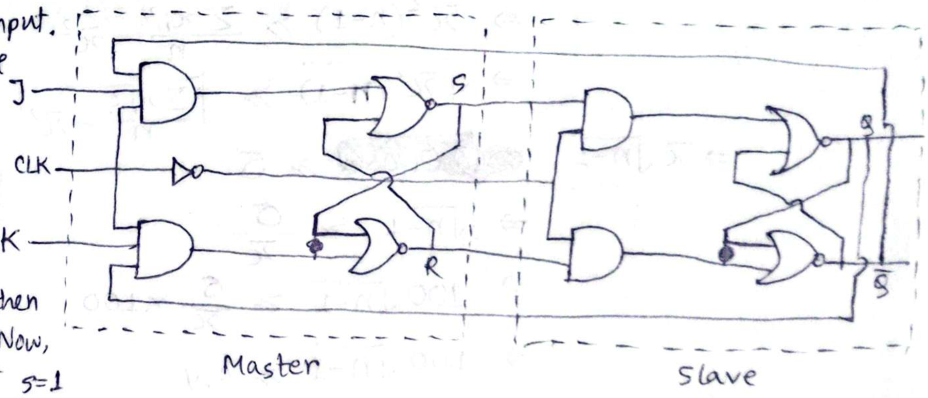
No. of object	Load Resistance R_L (kΩ)	Thevenin equivalent ckt		Original ckt		Error	
		V_L (v)	I_L (mA)	V_L (v)	I_L (mA)	V_L(v)	I_L(mA)
01		3.73	10.20	5.20	1.8		
02							

14/11/24 - D.S.D.

JK Master Slave flip-flop

Master slave flip-flop made by using two clocked flip-flop. One of the flip-flop is called master and other is called slave.

Master output acts as the slave input. Note here, master clock is positive and slave clock is negative. When clock is 1 then master is active and slave is inactive. Again when clock is 0 then master is inactive and slave is active.



Suppose, initially at $Q=0$ and $\bar{Q}=1$ then apply 1 and 0 at J and K input. Now, the clock is 1 then master output $S=1$ and $R=0$ at this time, slave output is not changed. Again, if clock is 0 then master output is not changed but slave output $S=1$ and $R=0$ for input changes $Q=1$ and $\bar{Q}=0$. Similarly for input $J=0$, $K=0$ output will be $\bar{Q}=0$, $Q=1$. If $J=K=1$ then clock pulse will change with output toggle.

Master slave flip-flop works like a negative edge trigger JK flip-flop. This flip-flop was popular before the development of the negative edge trigger JK flip-flop. Now it is no longer used, it is replaced by the negativ edge trigger JK flip-flop.

CLK	J	K	Q
X	0	0	NC
⎍	0	1	0
⎍	1	0	1
⎍	1	1	toggle

CSE 2014
3(c)

Prove that, $100\sqrt{n-1} \geq c.v$ (for n positive observations)

or, $100\sqrt{n-1} \geq c.v$

or, $c.v \leq 100\sqrt{n-1}$

Proof: Let, x variable having n number positive observations with mean $\bar{x}$ and standard deviation σ. then we have,

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\Rightarrow \sum x_i = n\bar{x} \quad \text{--- (i)}$$

Again, $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

$$\Rightarrow \sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\Rightarrow \sigma = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} \quad \text{--- (ii)}$$

We know,

$$(\sum x_i)^2 = \sum x_i^2 + 2\sum\sum x_i x_j$$

$$\Rightarrow (\sum x_i)^2 \geq \sum x_i^2 \quad [\because 2\sum\sum x_i x_j > 0]$$

$$\Rightarrow (n\bar{x})^2 \geq \sum x_i^2 \quad [\text{from eq (i)}]$$

$$\Rightarrow n^2\bar{x}^2 \geq \sum x_i^2$$

$$\Rightarrow n\bar{x}^2 \geq \frac{\sum x_i^2}{n}$$

$$\Rightarrow n\bar{x}^2 - \bar{x}^2 \geq \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\Rightarrow \bar{x}^2(n-1) \geq \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\Rightarrow \sqrt{\bar{x}^2(n-1)} \geq \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2}$$

$$\Rightarrow \bar{x}\sqrt{n-1} \geq \sigma \quad [\text{from eq (ii)}]$$

$$\Rightarrow \sqrt{n-1} \geq \frac{\sigma}{\bar{x}}$$

$$\Rightarrow 100\sqrt{n-1} \geq \frac{\sigma}{\bar{x}} \times 100$$

$$\Rightarrow 100\sqrt{n-1} \geq c.v \quad \left[\because c.v. = \frac{\sigma}{\bar{x}} \times 100\right]$$

[Proved]

H.W/Exam
'22: 3(c)
'14: 3(c)

#2019 (3(b))

H.W

Let us consider,

City A $= x$
city B $= y$

Table for calculation

x	$(x-\bar{x})^2$	y	$(y-\bar{y})^2$
⋮	⋮	⋮	⋮
Σx	$\Sigma(x-\bar{x})^2$	Σy	$\Sigma(y-\bar{y})^2$

for city A

$$C.V = \frac{\sigma_A}{\bar{x}} \times 100 \qquad \sigma_A = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$=$

for city B

$$C.V = \frac{\sigma_B}{\bar{y}} \times 100 \qquad \sigma_B = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

$=$

(C.V কম হলে বেশি stable)

17/11/24 – D.S.D

Counter

Counter is a sequential circuit that can ~~be used~~ count input pulse. The number of digits a counter can count that called modulus (MOD number).

Counter made by 2^n JK flip-flop can count $0 \rightarrow 2^n - 1$

Types of counter:

(i) Asynchronous counter / Ripple counter → 1. Ripple up counter (0, 1, 2, 3, 4) ; 2. Ripple down counter (4, 3, 2, 1, 0)

(ii) Synchronous counter

Ripple up counter:

3 bit counter $2^3 = 8$ numbers count

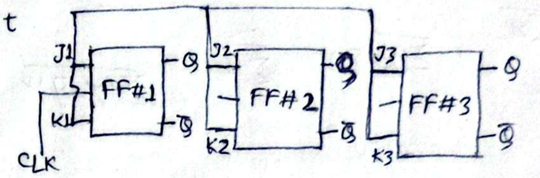
$0 \rightarrow \boxed{000}$

$7 \rightarrow \boxed{111} \rightarrow n$

Q + edge trigger → down

$\bar{Q}$ − edge trigger → down

$\bar{Q}$ → up



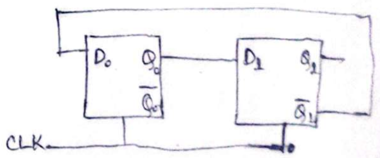
Q_2	Q_1	Q_0	
0	0	0	→ 0
0	0	1	→ 1
0	1	0	→ 2
0	1	1	→ 3
1	0	0	→ 4
1	0	1	→ 5
1	1	0	→ 6
1	1	1	→ 7

Synchronous counter:

The problems encountered with ripple counter are caused by the accumulated flip-flop propogation delay, started another way, the flip-flop don't change all state simultaneously in synchronism with input pulse. These limitations can be overcome with the use of synchronous or parallel counters in which all of the flip-flop are triggered simultaneously.

$0 \rightarrow 1 \rightarrow 3 \rightarrow 2$

Present state	Next state	output
0 0	0 1	0 1
0 1	1 1	1 1
1 0	0 0	0 0
1 1	1 0	1 0



$M_{R_1} \cup M_{R_2} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$M_{R_1} \cap M_{R_2} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \wedge \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

17/11/24 - Linear Algebra

(12) L.H.S.

$$= \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} \quad \begin{matrix} C_1' = C_1 - C_3 \\ C_2' = C_2 - C_3 \end{matrix}$$

$$= \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (c+a)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix} \quad R_3' = R_3 - R_1 - R_2$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac-a^2 & 0 & a^2 \\ 0 & bc+ab-b^2 & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix} \quad \begin{matrix} C_1' = C_1 + C_3 \\ C_2' = C_2 + C_3 \end{matrix}$$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac & a^2 & a^2 \\ b^2 & ab+bc & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$$

$$= \frac{(a+b+c)^2}{ab} 2ab \begin{vmatrix} ab+ac & a^2 \\ b^2 & ab+bc \end{vmatrix}$$

$$= 2(a+b+c)^2 ab \begin{vmatrix} b+c & a \\ b & a+c \end{vmatrix}$$

$$= 2ab(a+b+c)^2 (ab+ac+bc+c^2-ab)$$

$$= 2ab(a+b+c)^2 (ac+bc+c^2)$$

$$= 2ab(a+b+c)^2 c(a+b+c)$$

$$= 2abc(a+b+c)^3 = R.H.S.$$

(15)

$$\Delta = \begin{vmatrix} x & x^2 & x^3+1 \\ y & y^2 & y^3+1 \\ z & z^2 & z^3+1 \end{vmatrix}$$

$$= \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} + \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + 1 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= (xyz+1) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad \begin{matrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \end{matrix}$$

$$= (xyz+1) \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= (xyz+1) \begin{vmatrix} x-y & x^2-y^2 \\ y-z & y^2-z^2 \end{vmatrix}$$

$$= (xyz+1)(x-y)(y-z) \begin{vmatrix} 1 & x+y \\ 1 & y+z \end{vmatrix}$$

$$= (xyz+1)(x-y)(y-z)(y+z-x-y)$$

$$= (xyz+1)(x-y)(y-z)(z-x)$$

if $\Delta = 0$ then

$(xyz+1)(x-y)(y-z)(z-x) = 0$

According to the condition

$x \neq y \neq z$ then $xyz+1=0$

(16) L.H.S. $= \begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{vmatrix} \quad \begin{matrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \\ R_3' = R_3 - R_4 \end{matrix}$

$$= \begin{vmatrix} 0 & a-b & a^2-b^2 & a^3-b^3 \\ 0 & b-c & b^2-c^2 & b^3-c^3 \\ 0 & c-d & c^2-d^2 & c^3-d^3 \\ 1 & d & d^2 & d^3 \end{vmatrix}$$

$$= - \begin{vmatrix} a-b & a^2-b^2 & a^3-b^3 \\ b-c & b^2-c^2 & b^3-c^3 \\ c-d & c^2-d^2 & c^3-d^3 \end{vmatrix}$$

$$= -(a-b)(b-c)(c-d) \begin{vmatrix} 1 & a+b & a^2+ab+b^2 \\ 1 & b+c & b^2+bc+c^2 \\ 1 & c+d & c^2+cd+d^2 \end{vmatrix} \quad \begin{matrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \end{matrix}$$

$$= -(a-b)(b-c)(c-d) \begin{vmatrix} 0 & a-c & a^2+ab-bc-c^2 \\ 0 & b-d & b^2+bc-cd-d^2 \\ 1 & c+d & c^2+cd+d^2 \end{vmatrix}$$

$$= -(a-b)(b-c)(c-d) \begin{vmatrix} a-c & (a-c)(a+c)+b(a-c) \\ b-d & (b-d)(b+d)+c(b-d) \end{vmatrix}$$

$$= -(a-b)(b-c)(c-d) \begin{vmatrix} (a-c) & (a-c)(a+b+c) \\ (b-d) & (b-d)(b+c+d) \end{vmatrix}$$

$$= -(a-b)(b-c)(c-d)(a-c)(b-d) \begin{vmatrix} 1 & a+b+c \\ 1 & b+c+d \end{vmatrix}$$

$$= -(a-b)(b-c)(c-d)(a-c)(b-d)(b+c+d-a-b-c)$$

$$= -(a-b)(b-c)(c-d)(a-c)(b-d)(d-a)$$

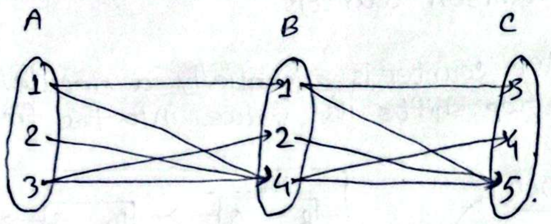
$$= (d-c)(d-b)(d-a)(c-b)(c-a)(b-a)$$

CamScanner

18/11/24 - D.M

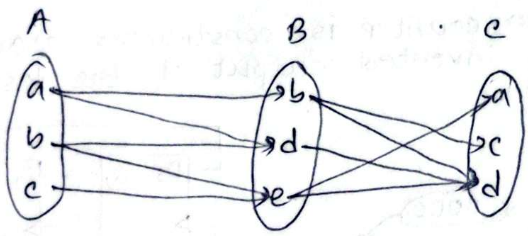
If $A=\{1,2,3\}$, $B=\{1,2,4\}$, $C=\{3,4,5\}$ and R is a relation between A and B $R=\{(1,1),(1,4),(2,4),(3,2),(3,4)\}$. Also S is a relation between B and C $S=\{(1,3),(1,5),(2,5),(4,4),(4,5)\}$ Find $R \circ S$

Ans//



$R \circ S = \{(1,3),(1,5),(1,4),(2,4),(2,5),(3,4),(3,5)\}$

If $A=\{a,b,c\}$, $B=\{b,d,e\}$, $C=\{a,c,d\}$
$R=\{(a,b),(a,d),(b,d),(c,e),(b,e)\}$; $S=\{(b,c),(d,d),(e,d),(b,d),(e,a)\}$



$R \circ S = \{(a,c),(a,d),(b,d),(b,a),(c,a),(c,d)\}$

If $A=\{1,2,3\}$ and $B=\{1,2\}$ then the relation $R=\{(2,1),(3,1),(3,2)\}$. What is the matrix representation of relation R

Soln// Given that, $A=\{1,2,3\}$; $B=\{1,2\}$; $R=\{(2,1),(3,1),(3,2)\}$

The matrix representation of relation R is,
$$M_R = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{array}$$

If $A=\{1,2,3,4\}$; $B=\{2,3,5\}$; $R=\{(1,2),(2,2),(3,2),(3,3),(4,5),(4,2)\}$

$$M_R = \begin{array}{c|ccc} & 2 & 3 & 5 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{array}$$

Suppose that the relation R_1 and R_2 on a set A are represented by the matrices $M_{R_1}=\begin{bmatrix}1&0&1\\1&0&0\\0&1&0\end{bmatrix}$ and $M_{R_2}=\begin{bmatrix}1&0&1\\0&1&1\\1&0&0\end{bmatrix}$. What is the matrices representation of $R_1 \cup R_2$ and $R_1 \cap R_2$

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix}1&0&1\\1&0&0\\0&1&0\end{bmatrix} \vee \begin{bmatrix}1&0&1\\0&1&1\\1&0&0\end{bmatrix} = \begin{bmatrix}1&0&1\\1&1&1\\1&1&0\end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix}1&0&1\\1&0&0\\0&1&0\end{bmatrix} \wedge \begin{bmatrix}1&0&1\\0&1&1\\1&0&0\end{bmatrix} = \begin{bmatrix}1&0&1\\0&0&0\\0&0&0\end{bmatrix}$$

19/11/24 - D.S.D.

Shift Register Counter

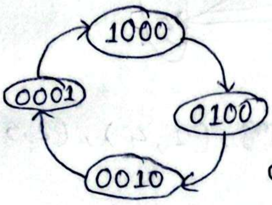
Shift register counter uses feedback, which means the output of the last flip-flop in the register is connected back to the first flip-flop in some way.

Types of shift register counter: (i) Ring counter
(ii) Johnson counter

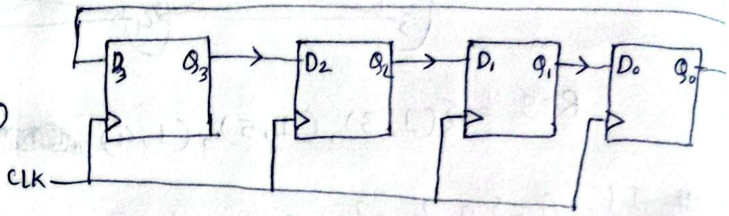
Ring counter: The simplest shift-register counter is essentially a circulating shift register connected so that the last flip-flop shifts its value into the first flip-flop.

Q_3	Q_2	Q_1	Q_0	CLK
1	0	0	0	0
0	1	0	0	1
0	0	1	0	2
0	0	0	1	3

state table



state diagram

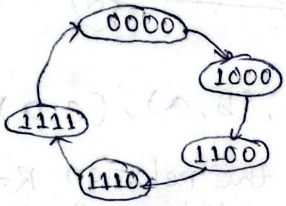


Block diagram

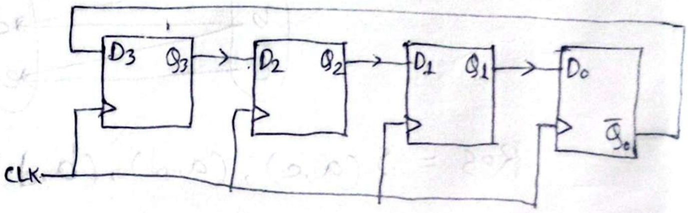
Johnson counter: The johnson counter is constructed exactly like a normal ring counter except that the inverted output of the last flip-flop is connected back to the first flip-flop.

Q_3	Q_2	Q_1	Q_0	CLK
0	0	0	0	0
1	0	0	0	1
1	1	0	0	2
1	1	1	0	3
1	1	1	1	4

state table



State diagram



block diagram

19/11/24 - Statistics

Correlation (সংশ্লেষ)

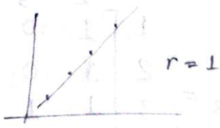
The relationship between two or more variable is called correlation.

Types of correlation: (i) Simple correlation (betn two var)
(ii) Multiple correlation (betn > two var)

(i)-(i) Perfect positive correlation:

উভয় চলকের মান সমহারে বৃদ্ধি পাবে

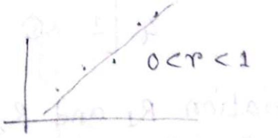
x: 2 4 6 8 10
y: 5 10 15 20 25



(ii) Partial positive correlation:

যেকোনো একটি বা সবকটি চলকের মান সমহারে বৃদ্ধি পাবে না

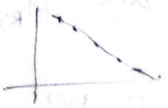
x: 2 4 6 8 10
y: 5 7 8 15 16



(iii) Perfect negative correlation:

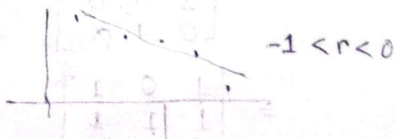
একটি চলকের মান সমহারে বৃদ্ধি পাবে অপরটি সমহারে হ্রাস পাবে

x: 2 4 6 8 10
y: 20 15 10 5 0



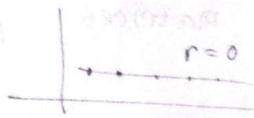
(iv) Partial negative correlation:

x: 2 4 6 8 10
y: 10 8 7 5 2



Assignment #1
- all logic gates (symbol + expression)
- number conversion (all one)

(v) zero correlation
x: 2 5 7 8
y: 2 2 2 2



Coefficient of correlation: Coefficient of correlation is the measure of the strength of the linear relationship between two variables.

Let, the two variables x and y having n pairs of observations (x_1, y_1), $(x_2, y_2), \ldots, (x_n, y_n)$ with their arithmetic mean $\bar{x}$ and $\bar{y}$, then the coefficient of correlation is defined as,

$$r = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma (x-\bar{x})^2 \, \Sigma (y-\bar{y})^2}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left\{\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right\}\left\{\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right\}}}$$

$(62.75)_{10} = (?)$

Dec → Bin, Oct, Hex

Other: Bin, Oct, Hex → Dec

Bin ⟷ Oct

Bin ⟷ Hex

Oct ⟷ Hex

m m yh

tosh