

ECE 240 Formula Sheet

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1. Time-domain signals

A continuous-time signal takes the form

$$x(t + nT) = x(t) \quad n \in \mathbb{Z}.$$

A signal $z(t) = \alpha x(t + aT_1) + \beta x(t + bT_2)$ will be periodic if

$$\frac{T_1}{T_2} = \frac{a}{b}$$

for some $a, b \in \mathbb{Z}$.

Let $x(t)$ be some signal.

- The *total energy* is given by

$$E = \lim_{T \rightarrow \infty} \int_{-L}^L |x(t)|^2 dt.$$

- The *average power* is given by

$$P = \lim_{T \rightarrow \infty} \frac{1}{2L} \int_{-L}^L |x(t)|^2 dt.$$

- If $x(t)$ is periodic,

$$P = \frac{1}{T} \int_0^T |x(t)|^2 dt.$$

E finite \rightarrow **Energy signal** $\rightarrow P = 0$.

E infinite and P finite \rightarrow **Power signal**.

Periodic signal \rightarrow **Power signal**.

Let $x(t)$ be some signal.

- A *time shift* is represented by

$$x(t - t_0).$$

- A *reflection* is represented by

$$x(-t).$$

The *unit step signal* is defined as

$$u(t) = \begin{cases} 1 & t > 0, \\ 0 & t < 0. \end{cases}$$

A *rectangular pulse* is represented as

$$\text{rect}\left(\frac{t}{T}\right) = u\left(t + \frac{T}{2}\right) - u\left(t - \frac{T}{2}\right).$$

A *ramp signal* is represented as

$$r(t) = tu(t) = \begin{cases} t & t \geq 0, \\ 0 & t < 0. \end{cases}$$

The *unit impulse* $\delta(t)$ (Dirac delta function) is defined as

$$\int_{t_1}^{t_2} x(t)\delta(t) dt = x(0) \quad t_1 < 0 < t_2.$$

It has the following properties:

- $\delta(t) = 0$ for $t \neq 0$,

- $\int_{-\infty}^{\infty} \delta(t) dt = 1$,
- $\delta(-t) = \delta(t)$, and
- $\delta(0) = \infty$.

Some $p(t)$ can be used as a model of a delta function if

- $p(t)$ is even,
- $\lim_{\epsilon \rightarrow 0^+} p(t) = +\infty$ for $t = 0$,
- $\lim_{\epsilon \rightarrow 0^+} p(t) = 0$ for $t \neq 0$, and
- $\int_{-\infty}^{\infty} p(t) dt = 1$ for all $\epsilon > 0$.

If these conditions are satisfied, then

$$\lim_{\epsilon \rightarrow 0^+} p(t) = \delta(t).$$

The *sifting property* is represented as

$$\int_{t_1}^{t_2} x(t)\delta(t - t_0) dt = \begin{cases} x(t_0) & t_1 < t_0 < t_2, \\ 0 & \text{otherwise.} \end{cases}$$

If $x(t)$ is continuous at $t = t_0$, the *sampling property* states that

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0).$$

The *scaling property* states that

$$\delta(at + b) = \frac{1}{|a|} \delta\left(t + \frac{b}{a}\right) \quad a \neq 0.$$

The derivative of $\delta(t)$ is defined as

$$\int_{t_1}^{t_2} x(t)\delta'(t - t_0) dt = -x'(t_0) \quad t_1 < t_0 < t_2.$$

It has the following properties:

- $x(t) * \delta'(t) = \int_{-\infty}^{\infty} x(\tau)\delta'(t - \tau) d\tau = x'(t)$,
- $x(t)\delta'(t - t_0) = x(t_0)\delta'(t - t_0) - x'(t_0)\delta(t - t_0)$,
- $\int_{-\infty}^t \delta'(\tau - t_0) d\tau = \delta(t - t_0)$, and
- $\delta'(-t) = -\delta'(t) \rightarrow \int_{-\infty}^{\infty} \delta'(t) dt = 0$.

2. Continuous-time systems

A system is *linear* if the superposition principle can be applied:

$$\alpha x_1(t) + \beta x_2(t) = \alpha y_1(t) + \beta y_2(t).$$

If we have $x(t) \rightarrow y(t)$, the system is *time-invariant* if

$$x(t - t_0) \rightarrow y(t - t_0).$$

A system is *memoryless* if the present output only depends on the present input.