Optimization of Model Parameters

Genetic Algorithm on Theis Equation

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Optimization Problems

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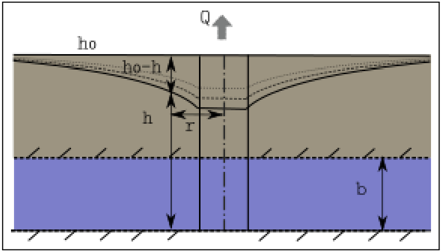
Solution Techniques:

Linear:	Linear Programming, Inverse Matrix
Non-Linear:	Gradient Based Methods, Particle Swarm, Ant Colony, **Genetic Algorithm**



Provided Data:

$$
\begin{aligned}
\text{Pumping Rate, } Q &= 8.155 ft^3/min \\
\text{Distance to OW, r} &= 582 ft \\
\text{Storativity, } S &= ? \\
\text{Transmissivity, } T &= ?
\end{aligned}
$$

Observed Data for Drawdown vs time: (Provided by Dr. Reza Soltanian)

SN (i)	time (t_i min)	drawdown (s_i ft)	SN (i)	time (t_i min)	drawdown (s_i ft)
1	1.93	0.11	11	14.73	0.31
2	2.98	0.14	12	17.75	0.34
3	4.00	0.15	13	35.22	0.40
⋮	⋮	⋮	⋮	⋮	⋮
10	12.73	0.29	20	280.46	0.64

Theis Solution

Theis equation is used as an analytical solution in the field of transient groundwater hydraulics, it can give us the drawdown (s) in terms of distance from the well (r) and the time (t).

$$s(r, t) = \frac{Q}{4\pi T} W(u) \tag{1}$$

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$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy = -\log(u) - E + \frac{u^1}{1 * 1!} - \frac{u^2}{2 * 2!} + \frac{u^3}{3 * 3!} - \dots \tag{2}$$

Where,

$$u = \frac{r^2 S}{4Tt} \tag{3}$$

$$E = 0.57721 \tag{4}$$

Optimization Model

Model Parameters:

1. Aquifer Storativity, S
2. Aquifer Transmissivity, T

Constraints:

1. $S \geq 1 \times 10^{-14}$, $S \leq 0.01$
2. $T \geq 0.001$, $T \leq 1000.0$

Objective Function:

Minimize mean absolute difference between predicted value and actual measurement.

$$Z = \frac{1}{N} \sum_{i} |s_i - s(r, t_i)| \tag{5}$$

Where,
s_i is the observed drawdown, and
$s(r, t_i)$ is the drawdown calculated using Equation (1).

Genetic Algorithm

Genetic Algorithm (GA) is among the algorithms that were designed to imitate the Nature. Since the nature already have so many complex things going on as a result of simple governing rules. GA tries to imitate it to solve problems or to understand the natural systems using the system of **Organisms**, their **Genes** and the **Evolution of Population**.

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Why?

1. We need Non-Linear technique,
2. It gives Global Solution,
3. Doesn't depend on initial guess,
4. I really wanted to try this.

Understanding Nature

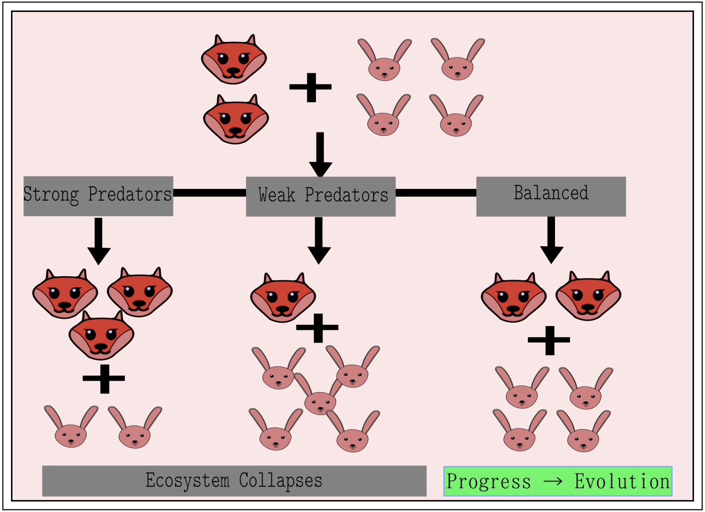


Figure: Interaction between Fox and Rabbit Population

Evolution of Population

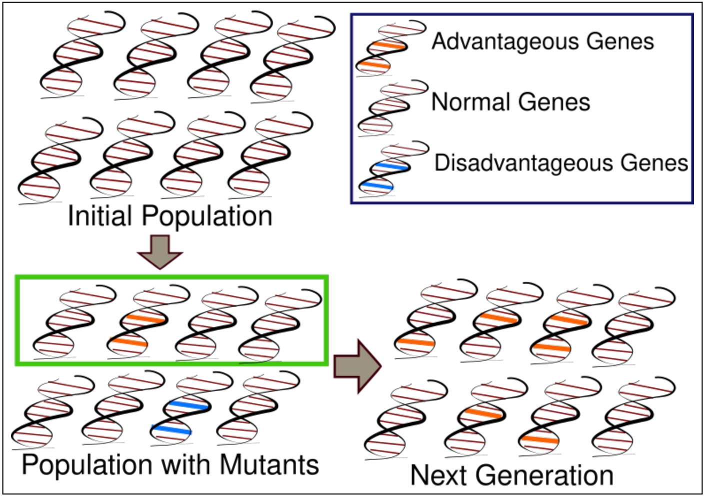


Figure: How Advantageous Genes are Passed

Algorithm

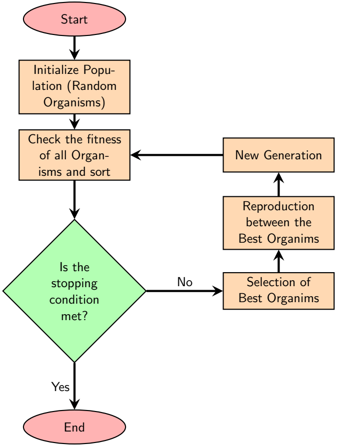


Figure: Flowchart for Genetic Algorithm

Algorithm

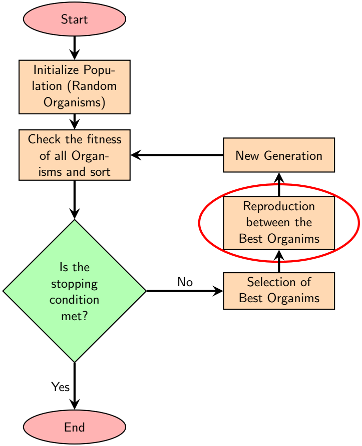


Figure: Flowchart for Genetic Algorithm

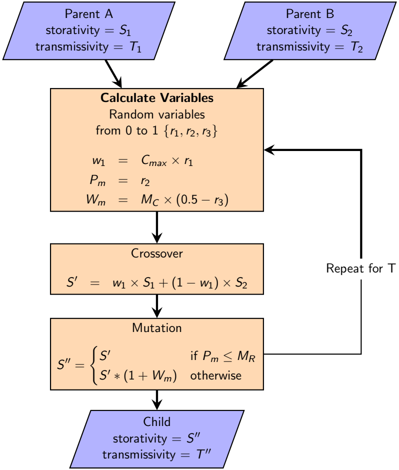


Figure: Reproduction

Overall Model

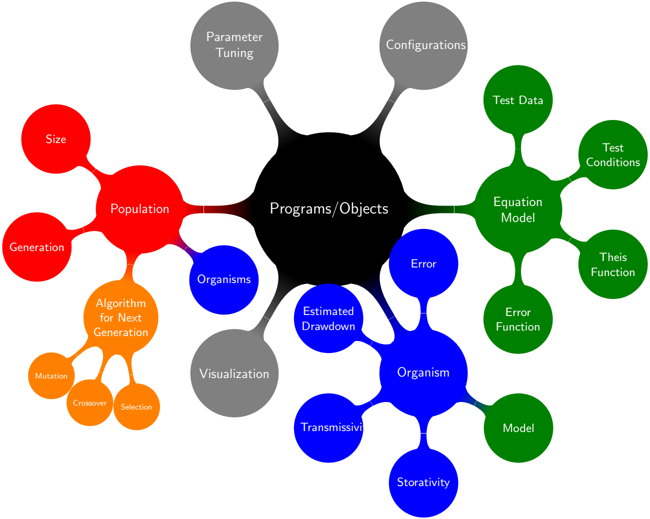


Figure: Objects Model for GA

First 3 Generations

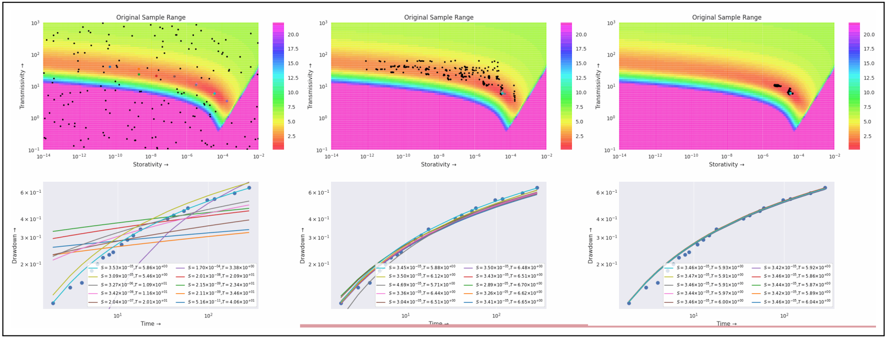


Figure: First 3 Generations of the Simulation

Parameter	Generation 1	Generation 2	Generation 3
Error	0.00988603	0.00988603	0.00930710
S_{best}	$3.53180773 \times 10^{-05}$	$3.53180773 \times 10^{-05}$	$3.80385357 \times 10^{-05}$
T_{best}	5.85535578	5.85535578	5.64879303

Parameters Tuning

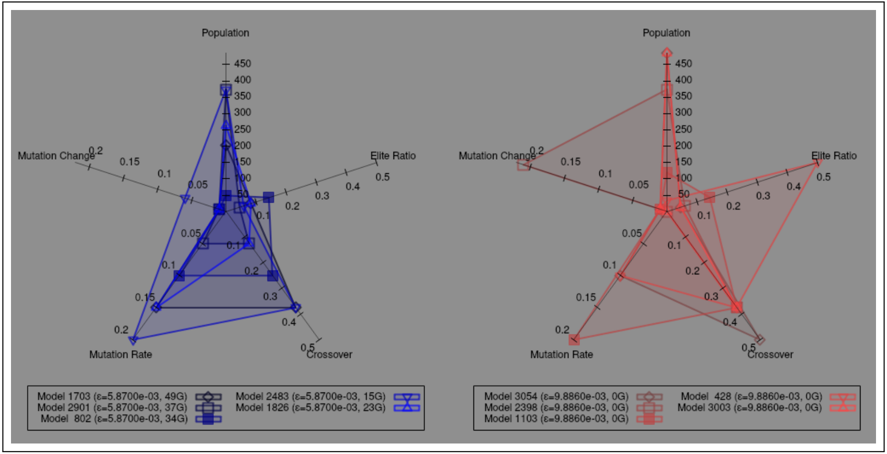


Figure: 5 Best and Worst Models and their GA parameters

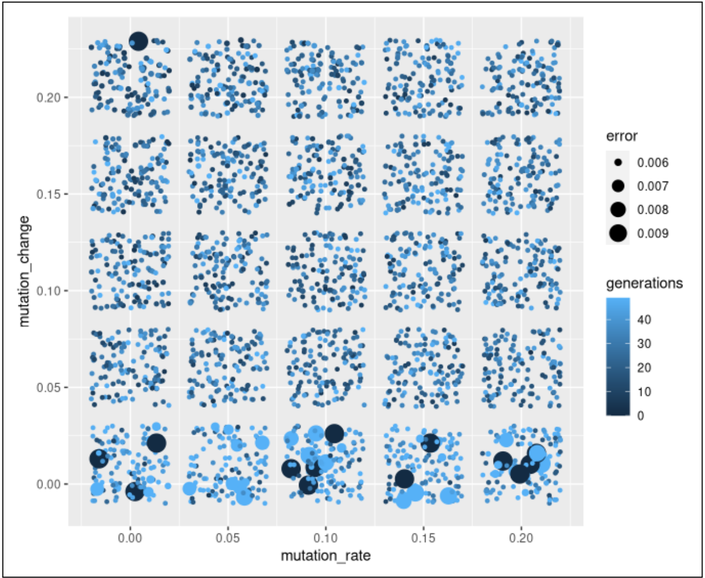
Final Results

We ran the model, got the results.

Then we solved the same problem using two different methods, then did a comparison with them (graphical method & aqtsolve).

	Parameters		Errors		
Method	Storativity	Transmissivity	M.A.E.	Mean	Std. Dev.
Graphical	3.8×10^{-05}	5.7020	8.12×10^{-03}	5.31×10^{-03}	8.63×10^{-03}
AQTsolve	4.2331×10^{-05}	5.5483	6.08×10^{-03}	-5.47×10^{-05}	7.86×10^{-03}
GA	4.2316×10^{-05}	5.5657	5.87×10^{-03}	-7.26×10^{-04}	7.87×10^{-03}

Importance of Mutation



Conclusions

- Aquifer Parameter Estimation can be solved using Genetic Algorithm,
- The results from GA is comparable or even more accurate than Industry Standard,
- GA takes more time and energy (on Computer) to solve the problem compared to specialized programs.

Thank You

Tools Used:

- Coding - Python
- Plotting - Python (matplotlib), R & Gnuplot
- Graphics - Inkscape
- Report/Presentation - $\LaTeX$
- Editing/scripting - emacs
- Laptop - 80E4 Lenovo G40-80
  - CPU - Intel(R) Core(TM) i5-5200U CPU 2.20GHz
  - OS - Arch Linux x86_64
  - Memory - 8 GB
- Codes: `https://github.com/Atreyagaurav/aquifer-properties-ga/`

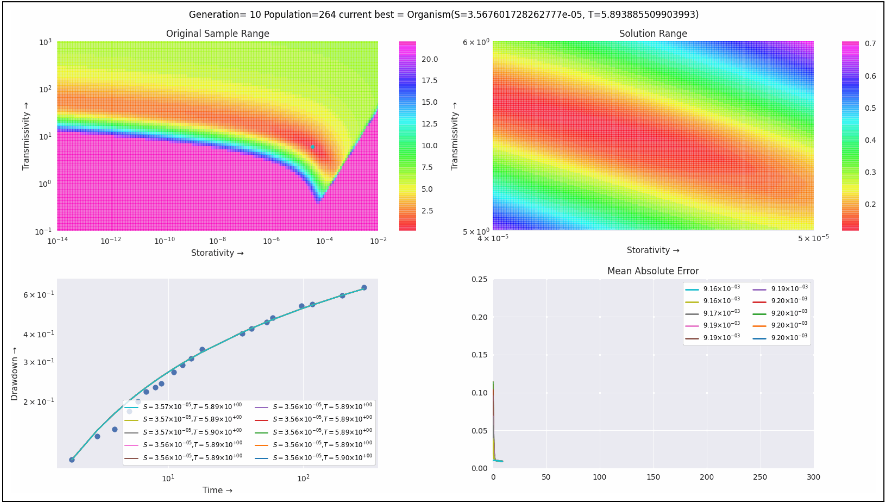
Supplementary Slides

5 Sample Generations
- 10, 20 & 30
- 100, 200 & 300

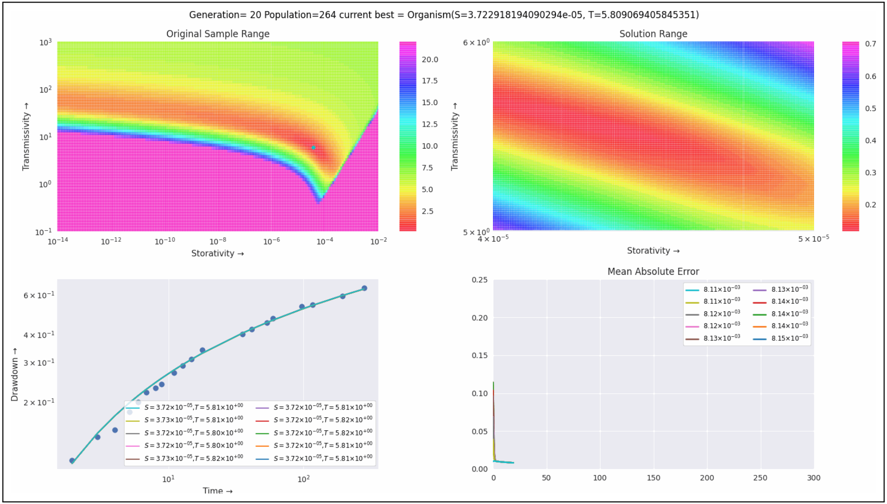
6 Other Methods
- AQTsolve
- Graphical

7 Other GA Methods

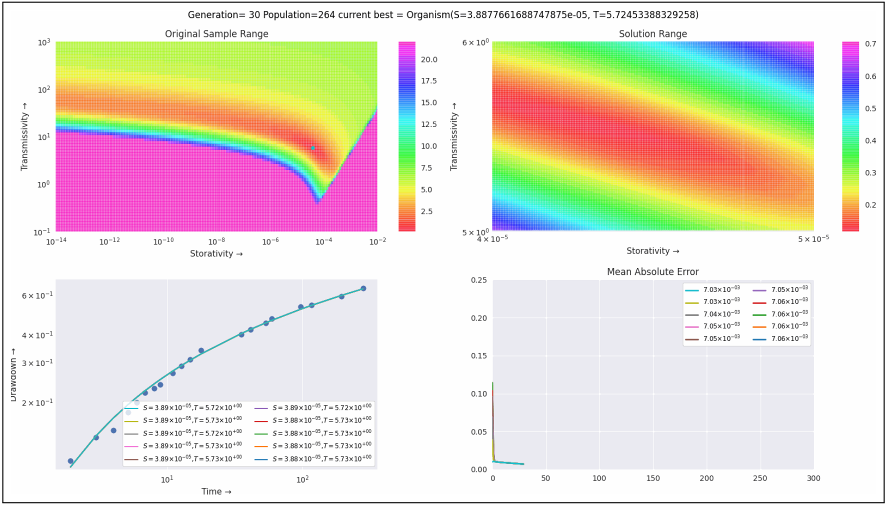
Generation 10



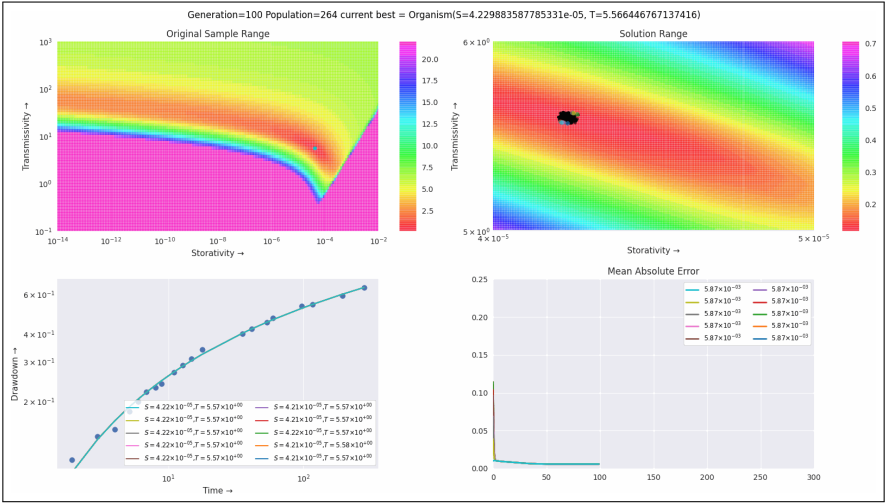
Generation 20



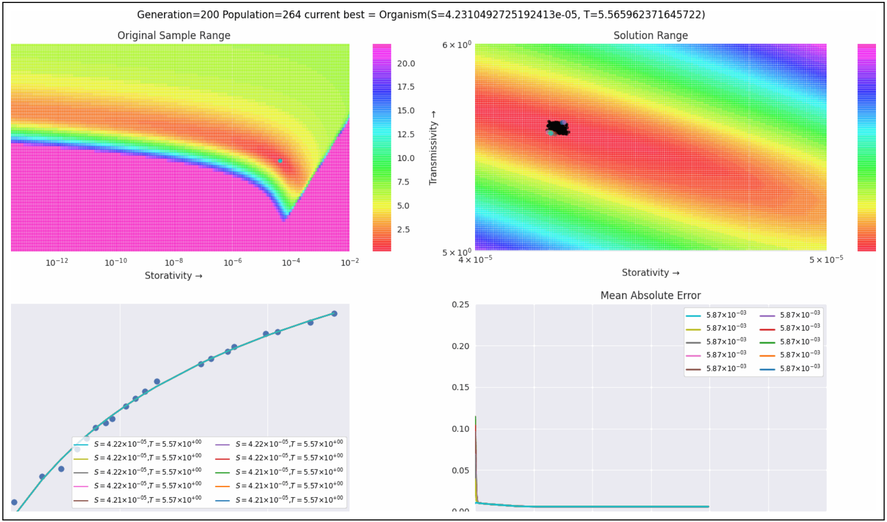
Generation 30



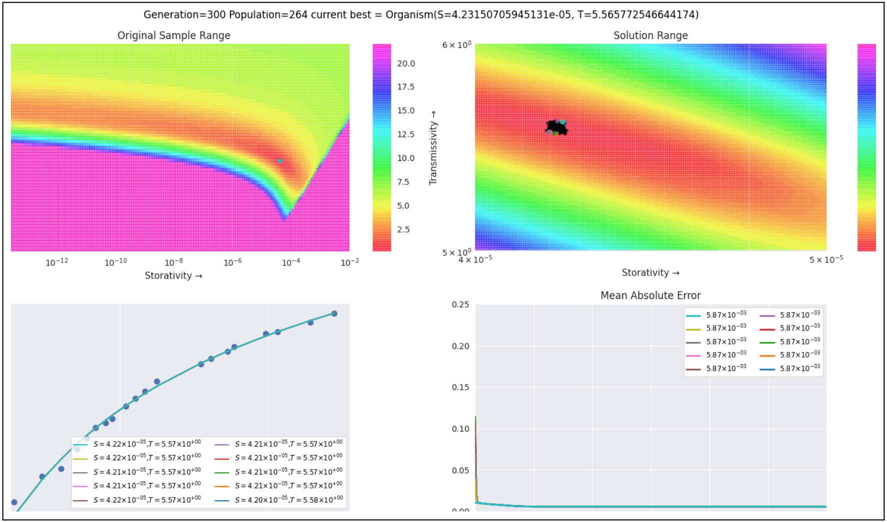
Generation 100



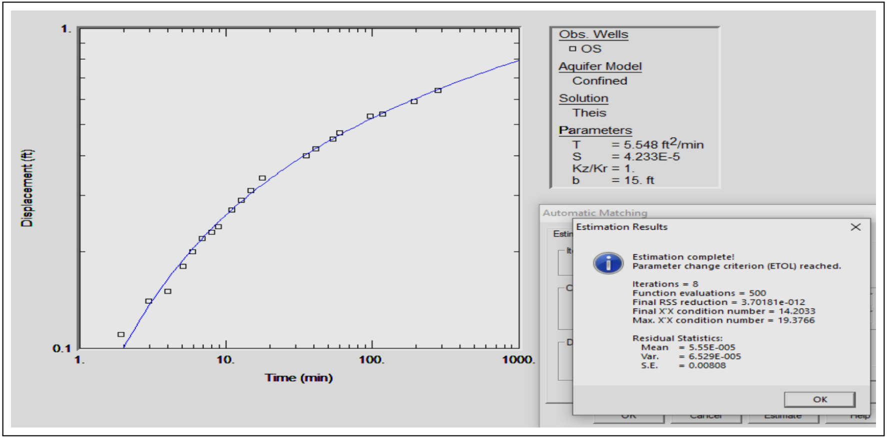
Generation 200



Generation 300

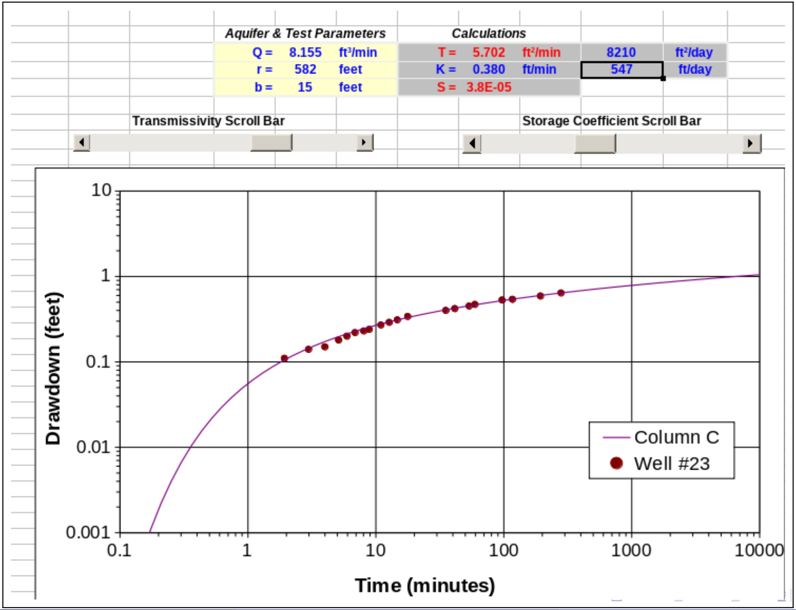


AQTsolve



1.
0.1
Displacement (ft)
1.
10.
100.
1000.
Time (min)
Obs. Wells
OS
Aquifer Model
Confined
Solution
Theis
Parameters
T = 5.548 ft²/min
S = 4.233E-5
Kz/Kr = 1.
b = 15. ft
Automatic Matching
Estimation Results
Estimation complete!
Parameter change criterion (ETOL) reached.
Iterations = 8
Function evaluations = 500
Final RSS reduction = 3.70181e-012
Final X'X condition number = 14.2033
Max. X'X condition number = 19.3766
Residual Statistics:
Mean = 5.55E-005
Var. = 6.529E-005
S.E. = 0.00808
OK

Graphical Method



Binary Genetic Algorithm