Problem 3: Using bilinear transformation, design a high pass filter, monotonic in passband with cutoff frequency of 1000 Hz and down 10 dB at 350 Hz. The sampling frequency is 5000 Hz. Implement the using basic building blocks. Show the derivation for this filter. Demonstrate the filter's output for 5 different frequencies ranging from 100Hz to 10000Hz. Choose these frequencies smartly to demonstrate the filter working.

Given,

$$\alpha_p = 3 \text{ dB}$$

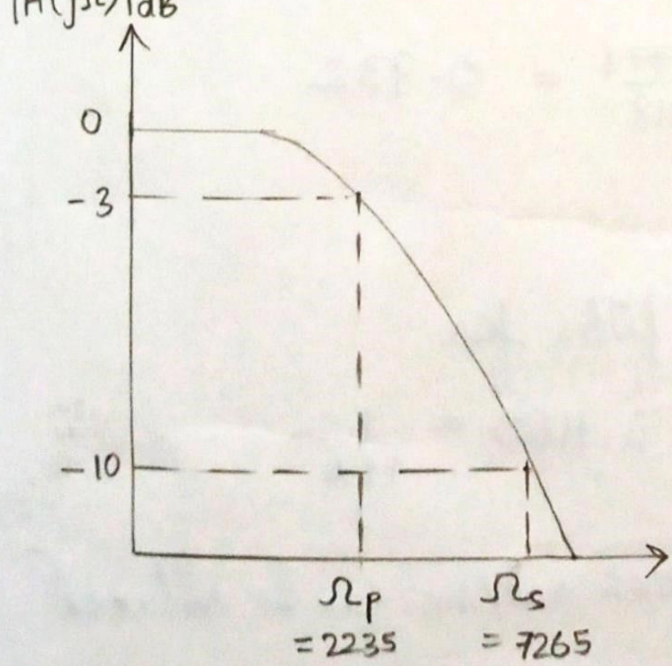
$$\alpha_s = 10 \text{ dB}$$

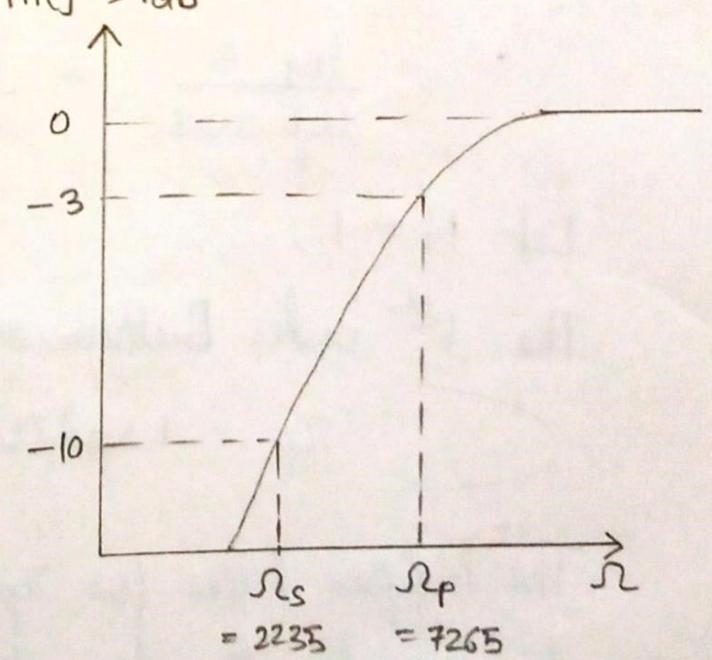
$$\omega_p = 2 \times \pi \times 1000 = 2000\pi \text{ rad/sec}$$

$$\omega_s = 2 \times \pi \times 350 = 700\pi \text{ rad/sec}$$

$$f_s = 5000 \text{ Hz}$$

$$T = \frac{1}{f_s} = \frac{1}{5000} = 2 \times 10^{-4} \text{ s}$$





Passband frequency of highpass filter is stopband frequency of lowpass filter and vice versa.

Highpass filter monotonic in passband is Butterworth filter

Prewarping the digital frequencies, we have,

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p T}{2}$$

$$= \frac{2}{2\times10^{-4}} \tan \frac{(2000\pi \times 2\times10^{-4})}{2}$$

$$= 10^4 \tan (0.2\pi)$$

$$= 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s T}{2}$$

$$= \frac{2}{2\times10^{-4}} \tan \frac{(700\pi \times 2\times10^{-4})}{2}$$

$$= 10^4 \tan (0.07\pi)$$

$$= 2235 \text{ rad/sec}$$

The order of the filter,

$$N = \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \frac{\Omega_s}{\Omega_p}} = \frac{\log \sqrt{\frac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log \frac{7265}{2235}}$$

$$= \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} = 0.932$$

Let $N = 1$;

The 1^{st} order Butterworth filter for

$\Omega_p = 1$ rad/sec is $H(s) = \frac{1}{1+s}$

The highpass filter for $\Omega_p = 7265$ rad/sec can be obtained by using the transformation

$$s \rightarrow \frac{\Omega_p}{s}$$

i.e $s \rightarrow \frac{7265}{s}$

The transfer function of highpass filter,

$$H(s) = \frac{1}{s+1}\Big|_{s = \frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation,

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Big|_{s=\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265}$$

$$= \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}}$$