MSA 8190 Statistical Foundations

Homework 5

Issued: October 18, 2021

Due: October 31, 2021, 11:59 PM

Note: For this HW submit a single R or R Markdown file. If you need to explain any part you can add a comment in your R file. You also need to record a 10-minute presentation and submit its link. Data for Problems 4 and 7 are uploaded in the HW5_data.xlsx MS Excel file.

Problem 1
A manufacturer produces piston rings for an automobile engine. It is known that ring diameter is normally distributed with $\sigma = 0.001$ millimeters. A random sample of 15 rings has a mean diameter of $\bar{x} = 74.036$ millimeters.

(a) Construct a 99% two-sided confidence interval on the mean piston ring diameter.

(b) Construct a 95% lower-confidence bound on the mean piston ring diameter.

Problem 2
A textile fiber manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of 12 kilograms with a standard deviation of 0.5 kilograms. The company wishes to test the hypothesis $H_0 : \mu = 12$ against $H_1 : \mu < 12$ using a random sample of four specimens.

(a) What is the type I error probability if the critical region is defined as $\bar{x} < 11.5$ kilograms?

(b) Find β for the case where the true mean elongation is 11.25 kilograms.

Problem 3
The heat evolved in calories per gram of a cement mixture is approximately normally distributed. The mean is thought to be 100 and the standard deviation is 2. We wish to test $H_0 : \ \mu = 100$ versus $H_1 : \ \mu \neq 100$ with a sample of $n = 9$ specimens.

(a) If acceptance region is defined as $98.5 \leq \bar{x} \leq 101.5$, find the type I error probability α.

(b) Find β for the case where the true mean heat evolved is 103.

(c) Find β for the case where the true mean heat evolved is 105. This value of β is smaller than the one found in part (b) above. Why?

Problem 4
The compressive strength of concrete is being tested by a civil engineer. He tests 12 specimens and obtains the following data.

2216	2237	2225	2301	2318	2255
2249	2204	2281	2263	2275	2295

(a) Is there evidence to support the assumption that compressive strength is normally distributed? Does this data set support your point of view? Include a graphical display in your answer.

(b) Test the normality of the data by Shapiro-Wilk test?

(c) Construct a 95% two-sided confidence interval on the mean strength.

(d) Construct a 95% lower-confidence bound on the mean strength.

Problem 5

A manufacturer produces crankshafts for an automobile engine. The wear of the crankshaft after 100,000 miles (0.0001 inch) is of interest because it is likely to have an impact on warranty claims. A random sample of $n = 15$ shafts is tested and $\bar{x} = 2.78$. It is known that $\sigma = 0.9$ and that wear is normally distributed.

(a) Test $H_0 : \ \mu = 3$ versus $H_1 : \ \mu \neq 3$ using $\alpha = 0.05$

(b) What is the power of this test if $\mu = 3.25$?

(c) What sample size would be required to detect a true mean of 3.75 if we wanted the power to be at least 0.9?

(d) Explain how the question in part (a) could be answered by using p-value and confidence interval.

Problem 6

Suppose three radar guns are set up along a stretch of road to catch people driving over the speed limit of 40 miles per hour. Each radar gun is known to have a normal measurement error modeled on $N(0, 5^2)$. For a passing car, let $\bar{x}$ be the average of the three readings. Our default assumption for a car is that it is not speeding.

(a) Write down the null and alternative hypotheses.

(b) The police would like to set a threshold on $\bar{x}$ for issuing tickets so that no more that 4% of the tickets are given in error.

(c) What is the power of this test with the alternative hypothesis that the car is traveling at 45 miles per hour? How many cameras are needed to achieve a power of .9 with $\alpha = 0.04$?

Problem 7

The rainfall in acre-feet from 20 clouds that were selected at random and seeded with silver nitrate follows:

18.0	30.7	19.8	27.1	22.3	18.8	31.8	23.4	21.2	27.9
31.9	27.1	25.0	24.7	26.9	21.8	29.2	34.8	26.7	31.6

(a) Can you support a claim that mean rainfall from seeded clouds exceeds 25 acre-feet? Use $\alpha = 0.01$.

(b) Is there evidence that rainfall is normally distributed?

(c) Compute the power of the test if the true mean rainfall is 27 acre-feet.

(d) What sample size would be required to detect a true mean rainfall of 27.5 acre-feet if we wanted the power of the test to be at least 0.9?

(e) Explain how the question in part (a) could be answered by constructing a one-sided confidence bound on the mean diameter.

(f) Explain how the question in part (a) could be answered by using p-value.

Problem 8

We perform a t-test for the null hypothesis $H_0 : \mu = 10$ at significance level $\alpha = 0.05$ by means of a dataset consisting of $n = 16$ elements with sample mean 11 and sample variance 4.

(a) Should we reject the null hypothesis in favor of $H_1 : \mu \neq 10$?

(b) What if we test against $H_1 > 10$?

Problem 9

The sugar content of the syrup in canned peaches is normally distributed. Suppose that the variance is thought to be $\sigma^2 = 18$ (milligrams)^2. A random sample of $n = 10$ cans yields a sample standard deviation of $s = 4.8$ milligrams.

(a) Test the hypothesis $H_0 : \ \sigma^2 = 18$ versus $H_1 : \ \sigma^2 \neq 18$ using $\alpha = 0.05$.

(b) What is the P-value for this test?

(c) Find a 95% two-sided confidence interval for σ.

(d) Find a 90% lower confidence bound for σ.

(e) Discuss how part (a) could be answered by constructing a 95% two-sided confidence interval for σ.

(f) Suppose that the true variance is $\sigma^2 = 40$. How large a sample would be required to detect this difference with probability at least 0.90?