中国科学院大学

2021 年招收攻读硕士学位研究生入学统一考试试题答案

——公式参考中科大信号与系统

试题答案仅供参考

一、选择

1. A
2. B
3. B
4. B
5. 解析:

$$y_{uzs}(t)=\int_{t-2}^{+\infty}e^{t-\tau}x(\tau-1)d\tau=x(t-1)*e^{t}u(-t+2)=x(t)*e^{t-1}u(-t+3)$$

$$x(t-1)*e^{t}u(-t+2)=\int_{t-2}^{\infty}x(\tau-1)u(\tau-t+2)d\tau$$

$$h(t)=e^{t-1}u(-t+3)$$

6. 无题目
7. 解析:

$$X(s)=\frac{2}{(s+1)^2+4}$$

8. A

解析：输入能谱为 1，即频谱为 1

$$Y(w)=X(w)H(w)=X(w)R_2(w)=X(w)[u(w+1)-u(w-1)]$$

$$E=|Y(w)|^2=u(w+1)-u(w-1)$$

9. 解析:

因果、稳定

$$\sum_{n=0}^{\infty}|h[n]|=1+1+\frac{1}{2!}+\frac{1}{3!}+\cdots=e<\infty$$

10. 0

解析：$\sin wt+\cos wt=\sqrt{2}\cos(wt-\frac{\pi}{4})$

二、填空

1. 全通、是一条水平线
2. 解析:

$$\sum_{k=0}^{n-1}x[k]=x[n]*u[n-1]$$

$$nu[n]*(-1)^n u[n]=\sum_{k=0}^{n-1}x[k]=x[n]*u[n-1]\xleftrightarrow{Z}X(z)\frac{z^{-1}}{1-z^{-1}}$$

$$nu[n]=u[n]*u[n]-u[n]\xleftrightarrow{Z}\frac{1}{(1-z^{-1})^2}-\frac{1}{1-z^{-1}}=\frac{z^{-1}}{(1-z^{-1})^2}$$

$$(-1)^n u[n] \xleftrightarrow{Z} \frac{1}{1+z^{-1}}$$

$$\frac{z^{-1}}{(1-z^{-1})^2}\cdot\frac{1}{1+z^{-1}} = X(z)\frac{z^{-1}}{1-z^{-1}} \Rightarrow X(z) = \frac{1}{(1-z^{-1})(1+z^{-1})}$$

3. 解析:

$$\int_{-\infty}^{\infty} f'(t)dt = 0 \quad \int_{-\infty}^{t} f(\tau)d\tau \xleftrightarrow{CFT} \frac{F(w)}{jw} + \pi F(0)\delta(w)$$

$$F(w) = \frac{1}{jw}[Sa(\frac{w\tau}{2}) - e^{-jw\frac{\tau}{2}}]$$

4. 解析:

$$f(t) = \frac{e^{-9t}-1}{t}u(t)$$

5. 解析:

$$f(t) = f_1(t) * f_2(t) = \cos w(t+1) - \cos w(t-1) = -2\sin wt \sin w$$

6. /

7. $h(t) = \frac{3}{2}\delta(t) + (e^{-2t} + 8e^{3t})u(t)$

8. $H(s) = \frac{cs+a}{s+b} \quad \mathrm{Re}\{s\} > -b$

三、判断

1. ✓
2. ×
3. ✓

四、计算

利用单边拉普拉斯变换求解

$$f_2(t) = \sin \pi t u(t) \xleftrightarrow{L_u} \frac{\pi}{s^2+\pi^2}$$

$$f_1(t) = [u(t) - u(t-2)] * \sum_{k=0}^{\infty}\delta(t-3k) \xleftrightarrow{L_u} \frac{1}{s}(1-e^{-2s})\frac{1}{1-e^{-3s}}$$

$$f(t) = f_1(t) * f_2(t) \xleftrightarrow{L_u} \frac{\pi}{s^2+\pi^2}\cdot\frac{1}{s}(1-e^{-2s})\frac{1}{1-e^{-3s}} = [\frac{-\frac{1}{\pi}s}{s^2+\pi^2} + \frac{\frac{1}{\pi}}{s}](1-e^{-2s})\frac{1}{1-e^{-3s}}$$

$$f(t) = \{[-\frac{1}{\pi}\cos \pi t + \frac{1}{\pi}]u(t) - [-\frac{1}{\pi}\cos \pi t + \frac{1}{\pi}]u(t-2)\} * \sum_{k=0}^{\infty}\delta(t-3k)$$

五、计算

1.

$$y_{uzi}(t) = 4e^{-t}u(t) - 3e^{-2t}u(t)$$

$$y_{uzs}(t) = \frac{3}{2}u(t) - 2e^{-t}u(t) + \frac{1}{2}e^{-2t}u(t)$$

$$y(t)=y_{uzi}(t)+y_{uzs}(t)=\frac{3}{2}u(t)+2e^{-t}u(t)-\frac{5}{2}e^{-2t}u(t)$$

2. 强迫响应：$\frac{3}{2}u(t)$　　自由响应：$2e^{-t}u(t)-\frac{5}{2}e^{-2t}u(t)$

六、计算

1. 解

$$F(w)=\frac{E}{\tau_2}\tau_1 Sa(\frac{w\tau_1}{2})\tau_2 Sa(\frac{w\tau_2}{2})=E\tau_1 Sa(\frac{w\tau_1}{2})Sa(\frac{w\tau_2}{2})$$

$$=\frac{E(T+\tau)}{2}Sa(\frac{T+\tau}{4}w)Sa(\frac{T-\tau}{4}w)$$

2. 解

$$f(t)=f_0(t)*\sum_{k=-\infty}^{\infty}\delta(t-kT)$$

$$F(w)=F_0(w)\cdot\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(w-k\frac{2\pi}{T})=\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}F_0(k\frac{2\pi}{T})\delta(w-k\frac{2\pi}{T})$$

$$F(w)\xleftarrow{ICFT}f(t)=\frac{1}{T}\sum_{k=-\infty}^{\infty}F_0(k\frac{2\pi}{T})e^{jk\frac{2\pi}{T}t}$$

$$F_k=\frac{1}{T}F_0(k\frac{2\pi}{T})=\frac{1}{T}F(w)\Big|_{w=k\frac{2\pi}{T}}$$

3. $w_m=\frac{4\pi}{T+\tau}$　$T=\frac{2\pi}{2w_m}=\frac{T+\tau}{4}$

七、计算

1.

$$z_0^n\xrightarrow{Z}H(z)z_0^n\quad(-1)^n\xrightarrow{Z}\frac{7}{4}(-1)^n\quad z_0=-1\quad H(-1)=\frac{7}{4}\quad a=-9$$

$$H(z)=1-\frac{3z^{-1}}{1-\frac{1}{2}z^{-1}}+\frac{5}{3}\frac{z^{-1}}{1-\frac{1}{3}z^{-1}}$$

2.

$$H(z)=\frac{1-\frac{13}{6}z^{-1}+\frac{1}{3}z^{-2}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}=\frac{Y(z)}{X(z)}$$

$$\frac{1}{6}y[n-2]-\frac{5}{6}y[n-1]+y[n]=x[n]-\frac{13}{6}x[n-1]+\frac{1}{3}x[n-2]$$

3. $|z|>\frac{1}{2}$包含单位圆，因果稳定

八、计算

1. 解

$$s(t)=m(t)\cos w_c t$$

$$s_c(t) = m(t)\cos w_c t \cos(w_c t + \Delta_\varphi) = \frac{1}{2}m(t)\cos\Delta_\varphi + \frac{1}{2}m(t)\cos(2w_c t + \Delta_\varphi)$$

$$\frac{1}{2}m(t)\cos\Delta_\varphi \xleftrightarrow{CFT} \frac{1}{2}\cos\Delta_\varphi M(w)$$

2. 解

$$s(t) = m(t)\cos w_c t$$

$$s_c(t) = m(t)\cos w_c t \cos(w_c t + \Delta_\varphi t) = \frac{1}{2}m(t)\cos\Delta_\varphi t + \frac{1}{2}m(t)\cos(2w_c t + \Delta_\varphi t)$$

此时，存在频域混叠，因此不能够解调出 $m(t)$