

# Multiobjective Optimal Design of High Frequency Transformers Using Genetic Algorithm

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## Abstract

This paper deals with the multiobjective optimization (MO) design of high frequency (HF) transformers using genetic algorithms (GAs). In its most general form, the design problem requires minimizing the mass or overall dimensions of the core and windings as well as the loss of the transformer while ensuring the satisfaction of a number of constraints. In this contribution, the area product (i.e. the product of the core cross section and the winding area) and the power loss are used as objective functions whereas the operating frequency and the maximum flux density are chosen as optimization variables. The constraints include, as for them, appropriate limits on efficiency, maximum surface temperature rise and maximum ratio no-load/full load current. The area product is optimized in place of weight or volume of the transformer because these two quantities can be easily expressed in terms of area product. It is an elegant mean to limit the number of objective functions. The major advantage of the suggested design procedure is that it proposes to the designer a set of optimal transformers – instead of a single solution – so he can choose *a posteriori* which of them best fits the application under consideration. Finally, in order to illustrate the design procedure, the optimal design of a transformer supplied with square voltage waveform is performed and the results are discussed.

## Introduction

Magnetic components (inductors and transformers) are one of the most important components in switch mode power supplies (SMPS). Moreover the transformer is the major contributor in size of any SMPS since it typically determines about 25% of the overall volume and more than 30% of the overall weight [1]. Therefore, in recent years, several researchers have developed and computerized many optimization routines for transformers [1], [2], [3], [4] in which the objective functions are to minimize the power loss, volume or weight.

In [1], the total transformer power loss is used as the objective function. In that paper, the thermal modeling of the transformer has received a considerable attention. Indeed, a detailed thermal model is used to determine the core and winding temperature rises. In [2], an optimization routine to design HF transformer is discussed in which the core selection is based on the optimum throughput of energy with minimum losses. Such a design methodology has the major advantage to take full account of the current and voltage waveforms and HF skin and proximity effects. It is therefore very interesting to design magnetic components of SMPS in which the switching-type waveforms are far from sinusoidal waveforms. A simple and efficient routine is then presented in [3] where the product of power loss, core cross section and winding area is chosen as the objective function. The procedure consists of two stages. First, the optimum core size is determined and, at the second stage, the windings are designed

in order to minimize the power loss. Finally, in [4], the author presents an application of Geometric Programming [5] to the transformer design. In that contribution, the design problem requires minimizing the total mass or the cost of the core and wire material.

The main criticism about all those procedures is that they take into account only one objective function (minimizing the power loss, the volume, the weight or, sometimes, a weighted sum or product of two of them). Yet, it is well-known that the reduction in size and weight of magnetic components is achieved by operating at higher frequencies [2]. This raise in frequency leads to higher power losses which have to be minimized. Therefore, it is impossible to obtain a solution which minimizes all the objectives simultaneously and so, the optimal design of HF transformers appears as a MO problem. In this case, it is better to apply MO techniques.

The present contribution explores the optimal design of HF transformers using a multiobjective and constrained optimization procedure based on Evolutionary Algorithms (EAs). Among the several approaches to evolutionary optimization, GAs has been chosen and the so-called Elitist Non-Dominated Sorting Genetic Algorithm (NSGA-II) [6] is used to perform the search and optimization procedures. The area product (i.e. the product of the core cross section and the winding area), which was first introduced by McLyman [7], and the power loss are used as the objective functions whereas the operating frequency and the maximum flux density in the core cross section are chosen as design variables. Note that we have chosen to minimize the so-called area product in place of weight or volume of the transformer because these two quantities can be easily expressed in terms of area product [7]. It is an elegant mean to limit the number of objective functions.

The remainder of this contribution is organized as follows. First, the objective functions, i.e. the area product and the power loss, as well as the thermal model of the transformer are described in Section 2. Then, in Section 3, the MO technique based on GAs and the proposed optimization routine are presented. Finally, in Section 4, a design example is presented and the results are discussed.

## Transformer modeling

### Area Product

A two windings transformer with voltage  $V_1$  and current  $I_1$  in the primary and voltage  $V_2$  and current  $I_2$  in the secondary is considered in this paper. The number of turns in the primary and secondary are  $N_1$  and  $N_2$ , respectively.

The required area product  $A_p$  (defined in Fig. 1) of the two windings transformer, which is an indication of the core size, is expressed as the product of the core cross section  $A_c$  and window area  $A_w$ :

$$A_p = A_c \cdot A_w \quad (1)$$

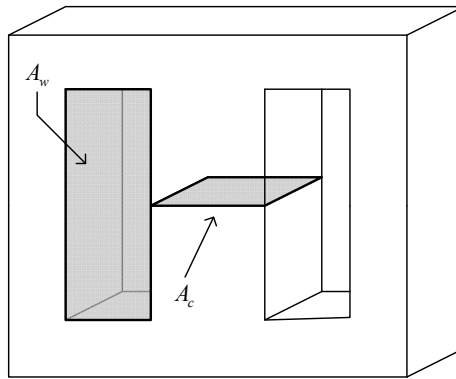


Fig. 1: Definition of area product

On the one hand, the required core cross section  $A_c$  of the core can be obtained by Faraday's law [8]:

$$A_c = \frac{V_1}{K_f \cdot f \cdot N_1 \cdot B_m} \quad (2)$$

where  $f$  is the operating frequency,  $K_f$  is the waveform factor ( $K_f = 4.44$  for a sinusoidal waveform and  $K_f = 4$  for a square waveform) and  $B_m$  is the maximum flux density. On the other hand, with the assumption that  $J_w$  is the current density in each winding (i.e. in primary and secondary windings), the required window area  $A_w$  is [8]:

$$A_w = \frac{N_1 \cdot I_1 + N_2 \cdot I_2}{K_u \cdot J_w} \quad (3)$$

where  $K_u$  is the window utilization factor by copper which is the ratio of the total copper area with respect to the total window area. Practical values of this factor range from 0.3 for Litz wire to 0.5 – 0.6 for round conductors [9].

Finally, using (1), (2) and (3), the area product is expressed as follows:

$$A_p = \frac{V_1 \cdot I_1 + V_2 \cdot I_2}{K_f \cdot K_u \cdot f \cdot J_w \cdot B_m} \quad (4)$$

Since the winding area is orthogonal to the core cross section, the volume and weight of the transformer are uniquely determined once the area product is known [7]. In the optimization routine (hereafter discussed), the area product will be used for purpose of the core selection.

### Power loss

The total loss of a transformer  $P_{loss}$  consists of two basic parts: core loss  $P_{core}$  and copper loss  $P_{copper}$ .

In general, core loss for ferrite cores is given in  $\text{mW}/\text{cm}^3$ , so that for a core of volume  $V_c$  [9]:

$$P_{core} = K_c \cdot V_c \cdot f^\alpha \cdot B_m^\beta \quad (5)$$

where  $K_c$ ,  $\alpha$  and  $\beta$  are constants which can be established from the manufacturer's datasheet by curve fitting methods. The core loss estimated by (5) includes hysteresis and eddy current losses [2].

On the other hand, the copper loss is the sum of the resistive losses of the two windings. According to [2], it can be expressed as:

$$P_{copper} = \sum_{i=1}^2 F_R(X_i) \cdot R_{dc,i} \cdot I_i^2 = \sum_{i=1}^2 F_R(X_i) \frac{\rho_w \cdot N_i \cdot MLT}{A_{w,i}} I_i^2 \quad (6)$$

where  $F_R$  is the eddy current loss factor,  $R_{dc}$  is the dc resistance,  $\rho_w$  is the resistivity of the conductors,  $MLT$  is the mean length of a turn in the winding and  $A_w$  is the wire conduction area.

In HF transformers, eddy current losses in windings, i.e. the losses due to skin and proximity effects, cannot be ignored. In this paper, the model developed by Dowell for a winding made of copper foil [10] is used to estimate the ac resistance and, so, to compute the eddy current factor  $F_R$ . This model is chosen because it is in a closed form, hence very convenient to use, and well accepted among researchers. Moreover, it is easily extensible on different windings structures.

The eddy current loss factor obtained by Dowell's model for rectangular conductors is given as follows [8]:

$$F_R(X, M_L) = \frac{R_{ac}}{R_{dc}} = X \left[ \frac{\sinh(2X) + \sin(2X)}{\cosh(2X) - \cos(2X)} + \frac{2(M_L^2 - 1)}{3} \frac{\sinh(X) - \sin(X)}{\cosh(X) + \cos(X)} \right] \quad (7)$$

where  $M_L$  is the number of layers counted from zero to maximum magnetomotive force and  $X$  is the normalized conductor thickness:

$$X = \frac{h}{\delta} \sqrt{K_{layer}} = \frac{h}{\delta} \sqrt{\frac{N_L \cdot b}{b_w}} \quad (8)$$

In (8),  $h$  is the conductor thickness,  $\delta$  is the skin depth,  $K_{layer}$  is the layer utilization factor,  $N_L$  is the number of turns per layer,  $b$  is the height of a conductor and  $b_w$  is the height of the window (see Fig. 2(a)). If round wire is used, it is converted to an equivalent rectangular conductor with the same cross section area. So, for a round wire of diameter  $d$ ,  $h = 0.886d$ . The relationship  $F_R(X, M_L)$  given from (7) is graphically represented in Fig. 2(b).

One can conclude from Fig. 2(b) that the eddy current loss factor, and so the winding loss, grows very strongly both with the number of layers  $M_L$  and the normalized conductor thickness  $X$ . It follows that at HF, associated with a small skin depth, the conductor thickness should be kept as small as possible. In order to respect this condition, Litz wire is commonly used. A Litz wire is a bunch of several isolated strands, each of them occupying every position in the wire and carrying the same external current. When Litz wire is used, equations (7) and (8) must be modified as follows [8]:

$$F_R(X, M_L, N_s) = \frac{R_{ac}}{R_{dc}} = X \left[ \frac{\sinh(2X) + \sin(2X)}{\cosh(2X) - \cos(2X)} + \frac{2(M_L^2 N_s - 1)}{3} \frac{\sinh(X) - \sin(X)}{\cosh(X) + \cos(X)} \right] \quad (9)$$

$$X = \frac{d_s}{2\delta} \sqrt{\pi K_{layer}} = \frac{d_s}{2\delta} \sqrt{\pi \frac{N_s \cdot d_s}{2D_L}} \quad (10)$$

where  $N_s$  is the number of strands,  $d_s$  is the strand diameter and  $D_L$  is the overall diameter of a Litz wire (see Fig. 2(c)). Note that the value of  $K_{layer}$  usually ranges from 0.5 to 0.8 depending on the Litz wire and size [8].

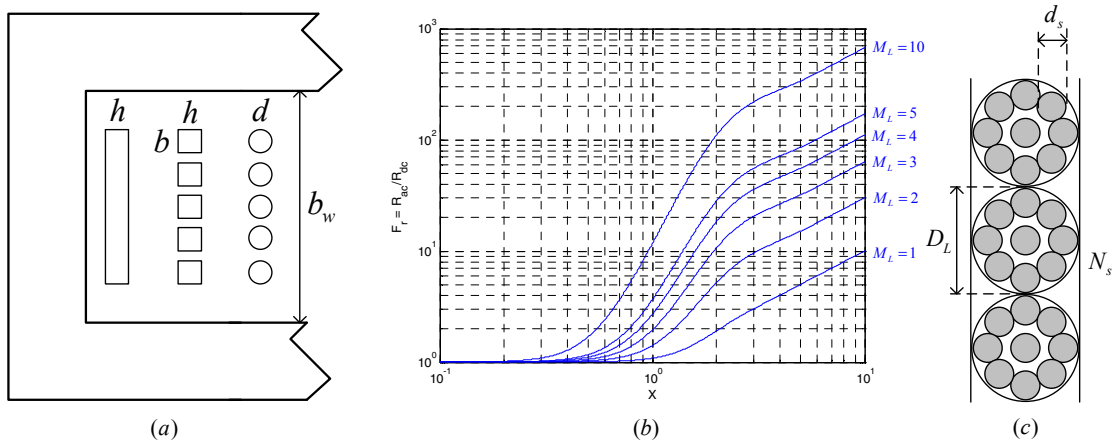


Fig. 2: (a) Winding layout, (b) ac/dc resistance ratio for a  $M_L$  layers winding and (c) Litz wire

## Thermal model

Transformer temperature estimation is needed during the optimization process to verify that temperature specifications are not exceeded. In a transformer with natural air cooling, as considered in this paper, the dominant heat-transfer mechanism is by convection [4]. The Newton's equation of convection is therefore used to determine the temperature rise ( $\Delta T$ ) of the magnetic component [2]:

$$\Delta T = R_{th} \cdot P_{loss} \quad (11)$$

where  $R_{th}$  is the thermal resistance defined as the inverse of the product of the convection heat-transfer coefficient  $h$  and the external surface area of the core and windings  $A_t$ :

$$R_{th} = \frac{1}{h \cdot A_t} \quad (12)$$

An expression of the coefficient  $h$  is suggested in [11]:

$$h = 1.42 \left( \frac{\Delta T}{H} \right)^{0.25} \quad (13)$$

where  $H$  is the height of the magnetic component. A typical value of the convection heat-transfer coefficient for cores encountered in SMPS is  $h = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ .

A drawback of equation (11) is that it gives only the temperature rise at the surface of the magnetic component. The temperature of the hottest point of the device is not specified. However, the temperature difference between the hottest point and the surface of the magnetic component does not usually exceed a few degrees [3] (except for very large device). Therefore, in a first approach, (11) can be used with enough accuracy for the optimal design of HF transformers.

## Optimization routine

As mentioned previously, in this contribution, we adopt a MO technique based on EAs. Those are stochastic search techniques that mimic natural evolutionary principles to perform the search and optimization procedures [12]. Among the several approaches to evolutionary optimization, GAs have been chosen and the so-called NSGA-II [6] is used to perform the HF transformer design. It should be noticed that the presence of several conflicting objectives in a problem gives rise to a set of optimal solutions (known as Pareto-optimal solutions) instead of a single optimal solution. This set of Pareto-optimal solutions constitutes the Pareto front.

### NSGA-II

NSGA-II is a recent and efficient multiobjective EA using an elitist approach [13]. It relies on two main notions: non-dominated ranking and crowding distance. Non-dominated ranking is a way to sort individuals in non-dominated fronts whereas crowding distance is a parameter that permits to preserve diversity among solutions of the same non-dominated front.

NSGA-II has been implemented in Matlab with a real coding scheme for the optimization variables. Such coding scheme is used in this paper because the optimization variables, viz. the operating frequency and maximum flux density, are continuous ones. Since the variables are coded directly, the algorithm is flexible and eliminates the difficulties (Hamming cliff problem and difficulty to achieve arbitrary decision) of coding a continuous variable with a binary coding [13].

There are three fundamental operations used in GAs: (1) selection, (2) crossover and (3) mutation. The primary objective of the selection operator is to make duplicates of good solutions and eliminate bad solutions in a population, in keeping the population size constant. To do so, a tournament selection [12] based on non-dominated rank and crowding distance of each individual is used. Then, the

selected individuals generate offsprings from crossover and mutation operators. To cross and mutate the real coded variables, the Simulated Binary Crossover and Polynomial Mutation operators [6] are used in this contribution.

Finally, the constraints must be taken into account. Several ways exist to handle constraints in EAs. The easiest way to take them into account in NSGA-II is to replace the non-dominated ranking procedure by a constrained non-dominated ranking procedure as suggest by its authors elsewhere (see, e.g., [6]). The effect of using this constrained-domination principle is that any feasible solution has a better non-dominated rank than any infeasible solution.

It is important to emphasize that the GA must be properly configured. The size of the population is one of the important parameters of the GA so as the termination criterion. The size of the population  $N_{pop}$  is taken equal to 20 ( $10 \times$  number of optimization variables). It is important to note that, on one hand,  $N_{pop}$  should be large enough to discover small details of the Pareto front whereas, on the other hand,  $N_{pop}$  should not be too large to avoid long time optimization. The termination criterion consists in a pre-defined number of generations which is here arbitrarily fixed to 100. Finally, the crossover probability and the mutation probability are respectively chosen to be 0.85 and 0.015 as typically suggested in literature [12].

### Design procedure

The overall optimization procedure has been implemented in Matlab as shown in Fig. 3. First, a random initial population is generated. Then, the objective functions are evaluated based on the initial population and the above-described HF transformer model. A test convergence is performed to check for a termination criterion. If this criterion is not satisfied, the reproduction process using genetic operations starts. A new population is generated and the previous steps are repeated until the termination criterion is satisfied. Otherwise, the Pareto front, i.e. the non-dominated solutions within the entire search space, is plotted and the optimization process is finished.

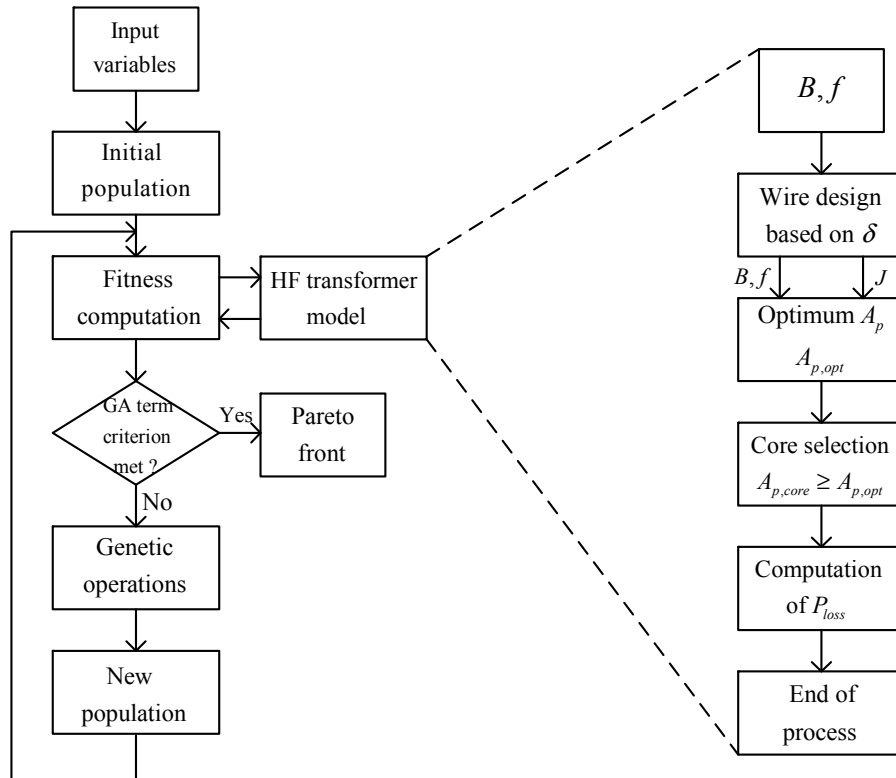


Fig. 3: Flowchart of the HF transformer design procedure based on GA

The use of the HF transformer model in the design procedure is now described. First the input variables, viz. the transformer specifications (input and output currents and voltages), the type of ferrite (a database contains the values of the constants  $\alpha$ ,  $\beta$ , and  $K_c$  of the most common ferrite materials), etc., as well as the constraints, e.g. the maximum allowable surface temperature rise, the efficiency, etc., must be stored in the computer memory. From these input data and the values of the optimization variables, i.e. the operating frequency and the maximum flux density, the windings are first designed based on the skin depth  $\delta$  and the adequate wire is selected from a database of standardized wires. At this stage, the minimum number of strands is derived, considering the diameter of strands equal to  $2\delta$ . In a second step, the optimum area product is computed using (4) and the standardized core (ETD-29, ETD-34, etc.) which has the closest area product (bigger than the optimum one) is selected for the end of the procedure. At this stage, the overall dimensions of the core, the ferrite specifications and the windings are designed. Therefore, the total power loss can be easily calculated using (5) and (6). Note that some parameters of the transformer are also calculated during the design such as the magnetizing inductance, the primary and secondary leakage inductances, etc. These parameters are computed according to a procedure explained elsewhere [4].

## Design example

In order to illustrate the design procedure, a transformer supplied with a square voltage waveform is considered with the specifications presented in Table I. The lower and upper bounds of the optimization variables as well as the constraints are also reported.

The results, i.e. the Pareto front, of the design procedure are presented in Fig. 4. Each point of the Pareto front represents an optimal transformer that respects all the constraints. In Table II, we present in details the design of the seven points of the Pareto front, i.e. the seven possible transformers.

The major advantage of the proposed design procedure is that a set of solutions is proposed to the designer who can choose *a posteriori* which objective function to promote and then select one particular transformer. So, a degree of freedom is still available at the end of the optimization process. In industrial framework, the Pareto front can be confronted with additional criteria or engineer know-how not included in models. An additional criterion could be, in this paper, the maximisation of the efficiency of the transformer. It is a well-known fact that maximum efficiency is achieved when core loss and copper loss are made equal [13]. Therefore, as shown in Table II, transformer #3 best fits this additional criterion (the difference between the two values is less than 10%) and could be selected as the best choice for the application under consideration. Moreover, as can be easily seen in Fig. 4, this particular solution is a good compromise between the two objectives.

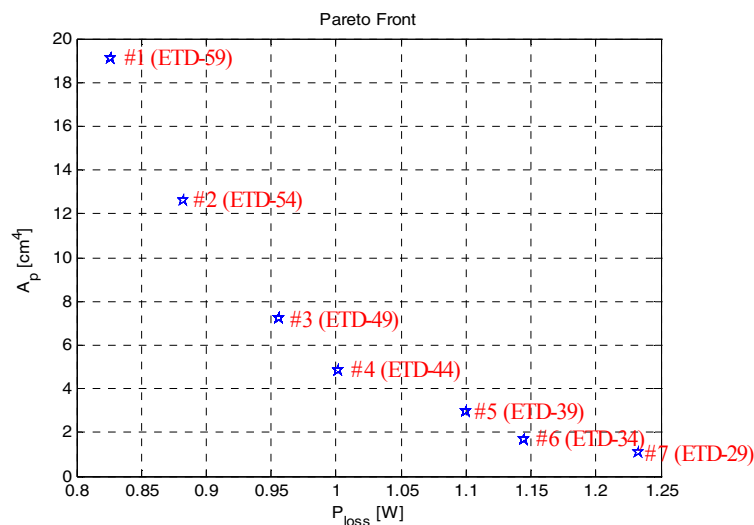


Fig. 4: Pareto front

**Table I: Transformer specifications, optimization variables and constraints**

Parameters	Values
Input voltage $V_I$	110 V
Input current $I_I$	0.9 A
Transformer turns ratio $m$ ( $m = N_2/N_1$ )	0.5
Core type	ETD
Core material	Ferrite 3F3
Operating frequency $f$	[1 ; 30] kHz
Maximum flux density $B_m$	[0.01 ; 0.33 ( $B_{sat}$ )] T
Maximum allowable surface temperature rise $\Delta T_{max}$	35 °C
Minimum efficiency $\eta_{min}$	90%
Maximum no-load/full load current ratio $k_\phi$	10%

**Table II: Optimization results**

	$B_m$ [T]	$f$ [Hz]	$P_{loss}$ [W]	$P_{core}$ [W]	$P_{copper}$ [W]	$A_p$ [cm <sup>4</sup> ]	$\eta$ [%]	$\Delta T$ [°C]	$k_\phi$ [%]
#1	0.15	6809.6	0.83	0.24	0.59	19.07	99.1	5.1	4.5
#2	0.20	6809.6	0.88	0.55	0.33	12.61	98.9	6	9
#3	0.21	8965.9	0.96	0.52	0.44	7.25	98.8	7.9	8
#4	0.21	11197.2	1	0.36	0.64	4.86	98.8	11.5	4.9
#5	0.22	13945.2	1.1	0.35	0.75	2.93	98.7	15.8	4.2
#6	0.25	17448.2	1.15	0.42	0.73	1.67	98.7	21.8	4.4
#7	0.25	21519.6	1.23	0.4	0.83	1.08	98.6	29.2	3.9

In order to analyze the optimization results presented in Table II, a study of the correlation level between the optimization variables and the objective functions is performed. The results are graphically presented in Fig. 5 and discussed below. Note that the influence of the optimization variables on the two basic parts of the power loss, viz. core loss and copper loss, are also considered.

From Fig. 5, it can be concluded that both optimization variables have a significant influence on the two objective functions. Indeed, the correlation coefficients between each variable and each objective functions are, in absolute value, bigger than 0.8. The correlations are therefore strong.

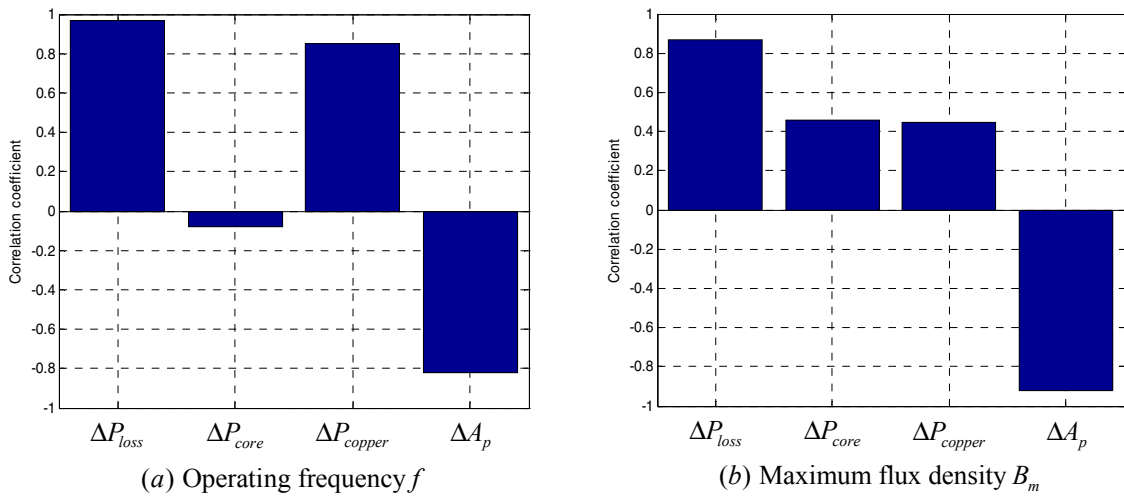


Fig. 5: Correlation between the objective functions and (a) the operating frequency  $f$  and (b) the maximum flux density  $B_m$



However, the operating frequency  $f$  has a greater influence on the variation of the power loss  $\Delta P_{loss}$  than the maximum flux density. Indeed, the correlation coefficient between  $f$  and  $\Delta P_{loss}$  is 0.97 against 0.87 between  $B_m$  and  $\Delta P_{loss}$ . The positive values of these two correlation coefficients indicate that the power loss grows when the operating frequency or maximum flux density grows (as expected in view of (5) and (6)).

The maximum flux density  $B_m$  has, in turn, a greater influence on the variation of the area product  $\Delta A_p$ . Indeed, the correlation coefficients between  $B_m$  and  $\Delta A_p$  is -0.92 against -0.82 between  $f$  and  $\Delta A_p$ . The negative values of these two correlation coefficients indicate that the area product reduces when the operating frequency or maximum flux density grows (as expected in view of (4)).

It can also be concluded from Fig. 5 that the maximum flux density has approximately the same influence on core loss and copper loss (see Fig. 5(b)). One can also remark that the correlation between the operating frequency and the copper loss is strong (see Fig. 5(a)). This is due to the eddy current losses in windings due to HF effects, i.e. skin and proximity effects. Finally, it should be emphasized that the very weak value of the correlation coefficient between the operating frequency and the core loss (-0.08) does not permit to conclude that these quantities are uncorrelated. Indeed, a nonlinear relationship exists between them.

## Conclusion

The problem of the MO design of HF transformers using GAs has been addressed. The area product and the power loss (including HF losses due to skin and proximity effects) have been chosen as objective functions whereas the operating frequency and maximum flux density have been chosen as optimization variables. A new multiobjective optimal design procedure, based on the so-called NSGA-II, has been proposed. Its major advantage is that it proposes to the designer a set of optimal solutions – instead of a single optimal one – so that the designer can choose *a posteriori* which transformer best fits the under consideration application. Moreover, in industrial framework, this set of solutions can be confronted with additional criteria or engineer know-how not included in models. Finally, in order to illustrate the design procedure, the optimal design of a transformer supplied with a square voltage waveform has been performed and the results have been discussed with the help of a correlation level study between the optimization variables and the objective functions.

Future work aims at treating the core dimensions as well as the current density in windings as optimization variables, in addition to those considered in this contribution.

## References

- [1] R. Petkov, "Optimum Design of a High-Power High-Frequency Transformer", *IEEE Transactions on Power Electronics*, Vol. 11, No. 1, pp. 33-42, January 1996.
- [2] W. G. Hurley, W. H. Wölfe and J. G. Breslin, "Optimized Transformer Design: Inclusive of High-Frequency Effects," *IEEE Transactions on Power Electronics*, Vol. 13, No. 4, pp. 651-659, July 1998.
- [3] S. Farhangi, "A Simple and efficient Optimization Routine for Design of High Frequency Power Transformers", *Proceedings of the 8<sup>th</sup> Power Electronics and Application Conference*, EPE 1999, Lausanne, September 1999.
- [4] R. A. Jabr, "Application of Geometric Programming to Transformer Design", *IEEE Transactions on Magnetics*, Vol. 41, No. 11, pp. 4261-4269, November 2005.
- [5] S. P. Boyd, S. J. Kim, L. Vandenbergh and A. Hassibi, "A tutorial on geometric programming", *Optimization Engineering*, Vol. 8, No. 1, pp. 67-127, 2007.
- [6] K. Deb, A. Pratap, S. Agarwal and T. Meyarivan, "A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II", *IEEE Transactions on Evolutionary Computation*, Vol. 6, No. 2, pp. 182-197, April 2002.
- [7] Wm. T. McLyman, "Transformer and Inductor Design Handbook", 3<sup>rd</sup> Edition, Marcel Dekker Inc., 2004.
- [8] W.-J. Gu and R. Liu, "A study of Volume and Weight vs. Frequency for High-Frequency Transformers", *Records of the 24<sup>th</sup> Annual IEEE Power Electronics Specialists Conference*, PESC 1993, June 1993.
- [9] N. Mohan, T. M. Undeland and W. P. Robbins, "Power Electronics: Converters, Applications and Design", John Wiley & Sons, Inc., New Jersey, 2003.

- [10] P. Dowell, "Effect of eddy currents in transformer windings", *IEEE Proc.*, Vol. 113, pp. 1387-1394, 1966.
- [11] A. Van den Bossche, V. Valchev and J. Melkebeek, "Improved Thermal Modelling of Magnetic Components for Power Electronics", *EPE Journal*, Vol. 12, No. 2, pp. 7-13, May 2002.
- [12] K. Deb, "Multi-Objective Optimization using Evolutionary Algorithms", John Wiley & Sons, Inc., New Jersey, 2001.
- [13] K. Deb and M. Goyal, "Optimizing Engineering Designs Using a Combined Genetic Search", *Proceedings of the 7<sup>th</sup> International Conference on Genetic Algorithms*, USA, 1997.