MAE 263F Homework #1

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***Abstract*— This document contains the three deliverables for HW #1 of MAE 263F, other than the MATLAB code for the solvers.**

I. Assignment 1: Three Spheres in viscous Fluid

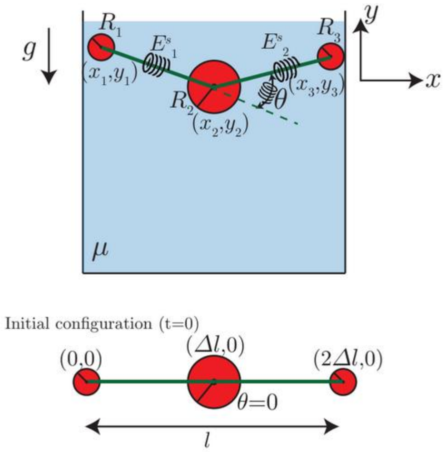

Figure 1: Three spheres in viscous fluid (Jawed & Lim).

This problem calls for both the implicit and explicit simulation of three spheres connected by an elastic beam falling in a viscous fluid over 10 seconds. Each sphere is represented as a node on the beam. The x- and y-positions of the spheres are contained in a position vector **q**, where

$$\boldsymbol{q} = \begin{bmatrix} x1 \\ y1 \\ x2 \\ \vdots \end{bmatrix}$$

The governing equation for the motion of these spheres is

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \frac{\partial \boldsymbol{E}^{elastic}}{\partial \boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} - \boldsymbol{W} = 0$$

where **M** is the mass matrix containing the masses of the spheres, $\mathbf{E}^{\text{elastic}}$ is the elastic energy of the beam (including stretching and bending), **C** is the damping matrix representing the viscosity of the fluid, and **W** is the weight of the spheres.

The implicit solver was based on Newton-Raphson method and employed functions by M. K. Jawed to calculate elastic forces. Initially, the implicit solver is to use a timestep of Δt = .01 seconds, while the explicit solver is to use a timestep of Δt = .00001 seconds.

A. Problem 1

The following plots show the shape of the structure over time, as well as the y-position and y-velocity of the center node, the largest sphere. These plots are from the implicit solver:

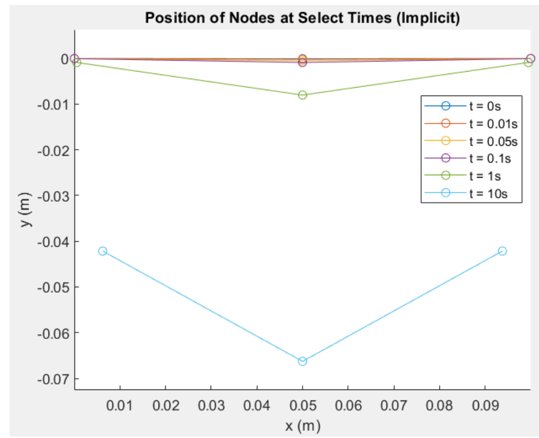

Figure 2: Structure shape (implicit solver).

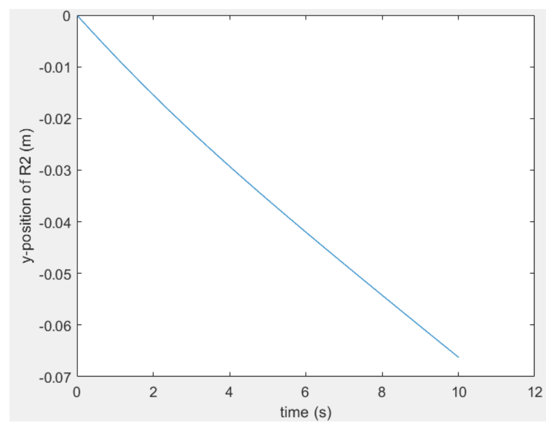

Figure 3: Position of middle sphere over time.

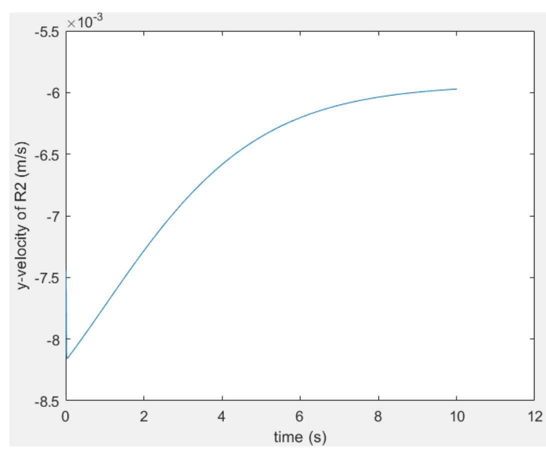

Figure 4: Velocity of middle sphere over time.

The following plots are from the explicit solver:

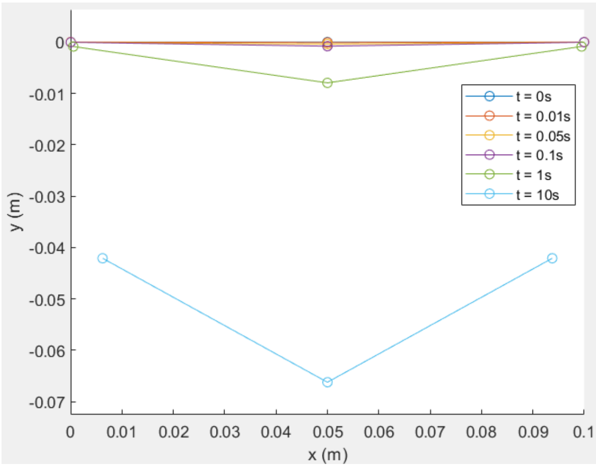

Figure 5: Structure shape (explicit solver).

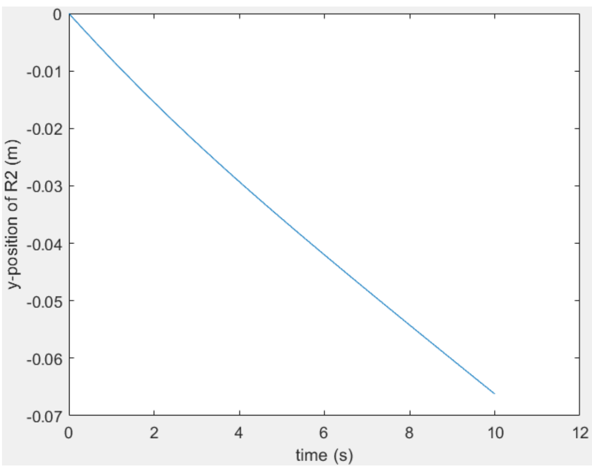

Figure 6: Position of middle sphere over time.

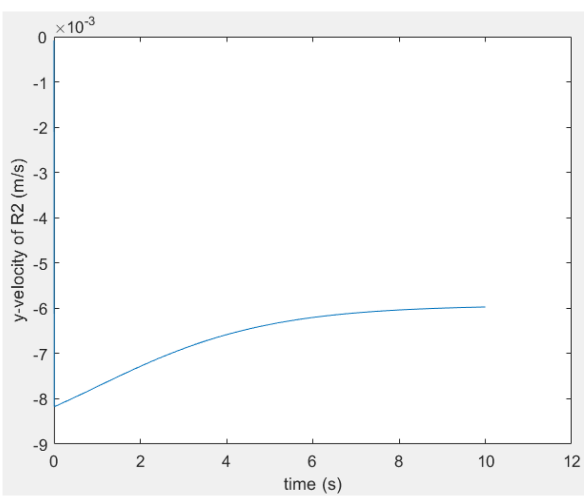

Figure 7: Velocity of middle sphere over time.

B. *Problem 2*

The terminal velocity along the y-axis of the system is about -0.0060 m/s, as shown by both the implicit and explicit solver.

C. *Problem 3*

If all three spheres are made to be the same radius, the turning angle stays at 0 over time, as shown by the implicit solver:

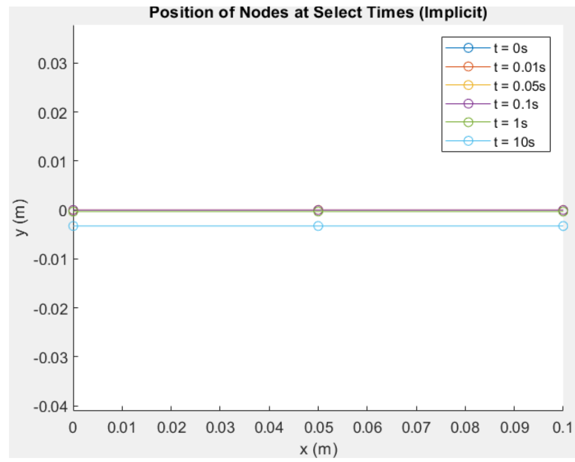

Figure 8: Structure shape if all spheres are the same radius.

This agrees with my intuition, as all spheres are the same weight, so the vertical force on all three spheres are the same. All spheres would fall at the same rate, so there would be no angle change between each sphere.

C. *Problem 4*

While changing the timestep sizes, it was observed that the implicit solver could still reach a relatively close result even if its timestep was increased from 0.01 seconds to 1 second. However, the explicit solver gave unreasonably large deformations after an increase in timestep from 1e-5 to 1e-4 seconds, with unrealistic oscillations in velocity and position towards the end of the simulation.

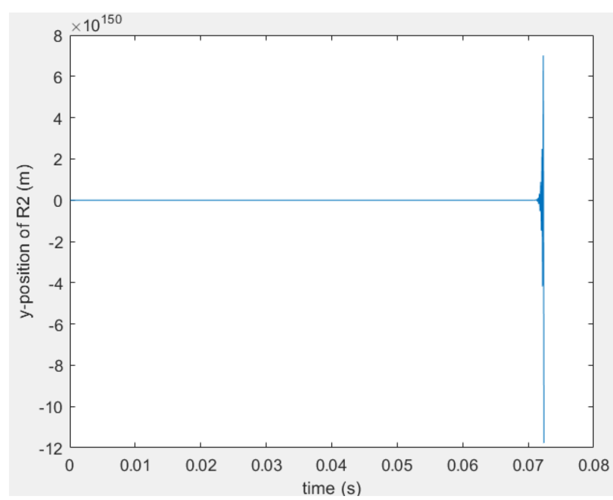

Figure 9: The y-position of the middle sphere if the explicit solver is set to a timestep of 1e-4 seconds.

Therefore, I observed that an implicit solver, while more complicated to write compared to the explicit solver, was much better at obtaining a reasonable solution at larger timesteps. The explicit solver, on the other hand, could diverge significantly if the timestep was not very fine. The explicit solver also took much longer to run due to the small timestep.

II. Assignment 2: Generalized Beam in Viscous Fluid

This problem calls for the implicit simulation of a beam in viscous flow with any arbitrary number of nodes or spheres. Most of the code was created following examples by M. Jawed. The parameters for the problem were similar to those in Assignment 1, with the exception of the number of nodes N and the radii of the sphere nodes. All nodes other than the one in the middle have a radius of $R = \Delta l/10$. The initial timestep for the solver was $\Delta t = 10^{-2}$s, and the simulation was run for 50 seconds. The initial number of nodes was set to be N = 21.

A. *Problem 1*

The following plots show the y-position and y-velocity of the center node, the largest sphere:

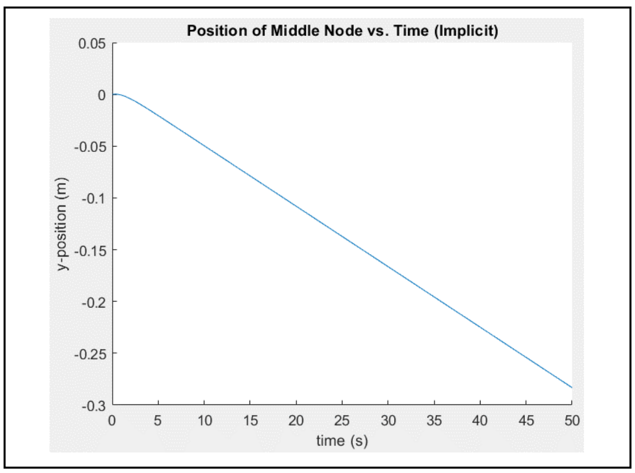

Figure 10: Vertical position of the center node over time.

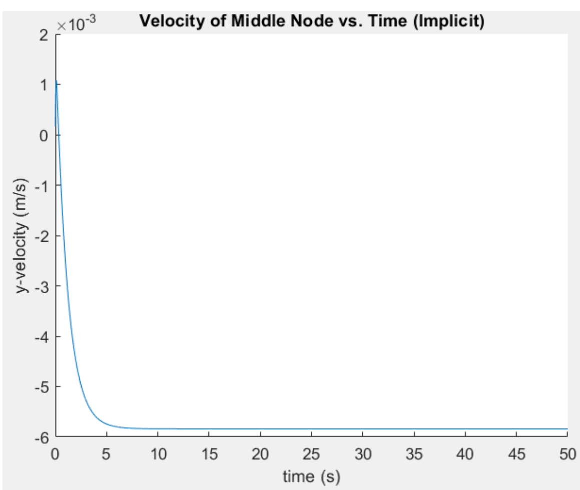

Figure 11: Vertical velocity of the center node over time.

The terminal velocity of the middle node is -0.0058 m/s.

B. *Problem 2*

The final shape of the beam after 50 seconds is:

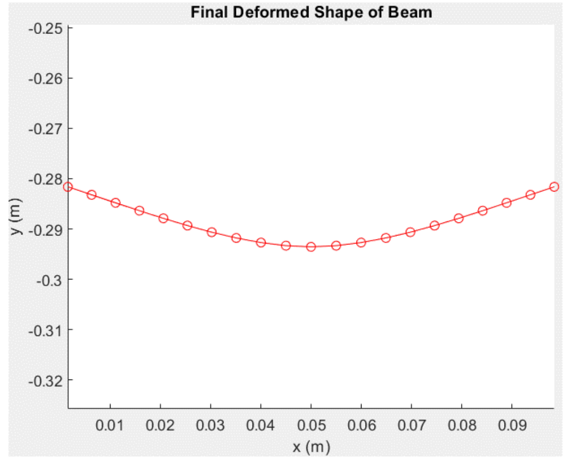

Figure 11: Final shape of beam.

C. *Problem 3*

This assignment examines how the number of nodes and timestep affect the solution by the solver. The plots for the terminal velocity vs. the number of nodes and the timestep are as follows.

The terminal velocity vs. number of nodes were plotted for a fixed timestep of dt = 0.01 s.

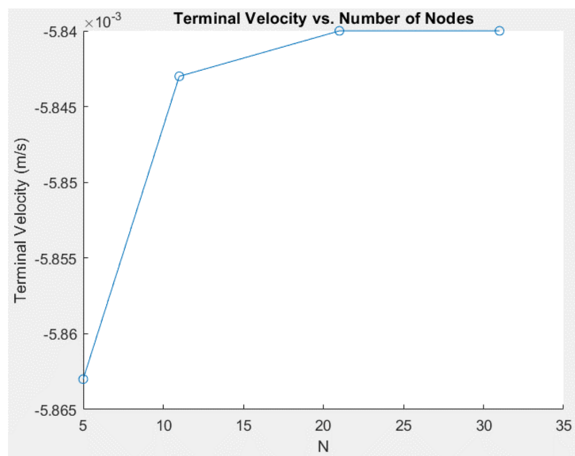

Figure 12: Calculated terminal velocity vs. the number of nodes defined.

The terminal velocity vs. timestep were also plotted for a fixed number of nodes of N = 21.

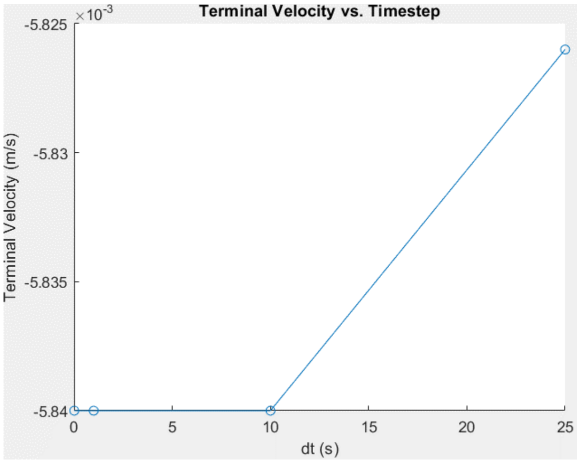

Figure 13: Calculated terminal velocity vs. size of timestep used in the solver.

The spatial discretization splits the beam into multiple nodes. The deformation of the beam is smoother with a finer discretization of the beam; i.e., more nodes. However, the computation time also increases with the nodes of the beam. Additionally, below an N of about 20, the terminal velocity calculated by the solver begins to deviate significantly.

The temporal discretization of the solver greatly affected the computation time required. However, it seems that in this case, a sufficient number of nodes (N = 21) and a long enough time for the simulation (t = 50 seconds) allowed the solver to arrive at a fairly accurate terminal velocity up to timesteps as large as dt = 10 s.

III. Assignment 3: Elastic Beam Bending

This problem asked for the implicit simulation of an Euler-Bernoulli beam that is 1 meter in length. The left end is fixed, while the right end is left rolling. There is a point load of P = 2000 Newtons applied at a distance of d = 0.75 m away from the left end of the beam.

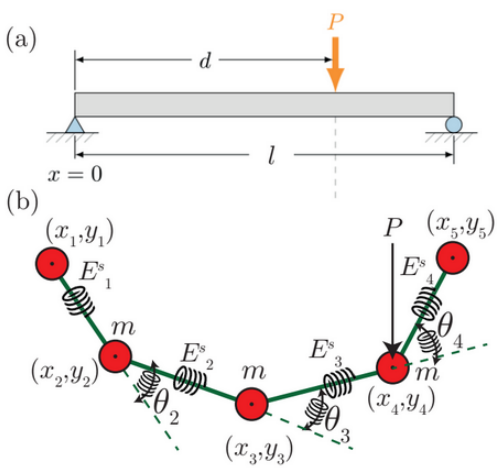

Figure 14: Euler-Bernoulli beam with point load (Jawed & Lim).

The governing equation of the beam is as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \frac{\partial \boldsymbol{E}^{elastic}}{\partial \boldsymbol{q}} - \boldsymbol{P} = 0$$

In this case, the effect of the weight of the beam on its deformation is negligible compared to that of the point load, so its weight was ignored.

A. *Problem 1*

The absolute maximum vertical displacement of the beam over time as determined by the solver is:

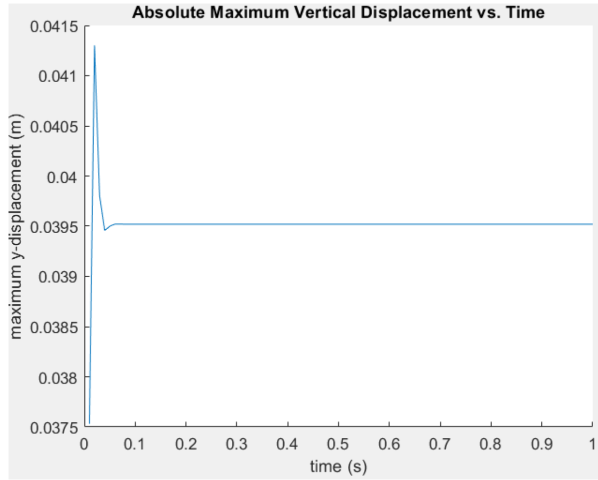

Figure 15: The absolute y-displacement of the beam over time.

The steady-state maximum vertical displacement is -0.039520 m (in the original coordinate system, where the beam displaces in the -y direction).

Using the theoretical prediction from the Euler beam theory:

$$y_{\max} = \frac{Pc\,(l^2 - c^2)^{1.5}}{9\sqrt{3}EI\,l} \qquad \text{where} \qquad c = \min(d, l-d)$$

The y_{max} predicted by the above equation is 0.03804 m, which is reasonably close to that predicted by the implicit solver.

B. *Problem 2*

When the point load is increased to P = 20000, the benefits of the implicit solver over a theoretical prediction according to Euler beam theory are shown. The implicit solver shows the following steady-state deformation:

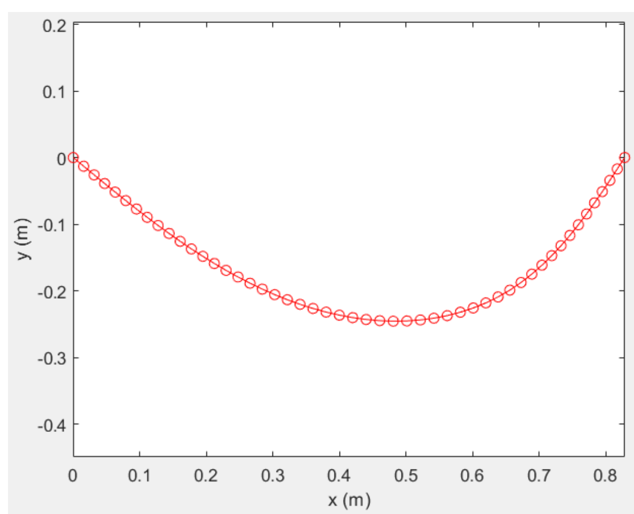

Figure 16: Final state of beam with P = 2000 N applied.

The solver determined the maximum vertical displacement to be 0.2456 m at steady state. In contrast, Euler beam theory predicts a y_{max} of 0.3804 m. The Euler beam theory's prediction is significantly larger than the one predicted by the solver. The solver should be more accurate and general than the Euler beam theory equation as the solver is based on the governing equation that is valid for any amount of deformation, while the Euler beam theory only addresses small deformation where linearity may be assumed.

Appendix

Appendixes should appear before the acknowledgment.

Acknowledgment

I thank Professor M. K. Jawed for his guidance and provided functions for completing this assignment.

References

[1] M. K. Jawed and S. Lim, "Discrete simulation of slender structures" . UCLA, 2019, pp. 16–22.