MAE 263F Homework #2

Atti W. Lau

***Abstract*— This document contains the deliverable from chapter 7 for HW #2 of MAE 263F, other than the MATLAB code for the solvers.**

I. Assignment: Simulation of a Discrete Elastic Rod

This problem calls for the implicit simulation of an elastic rod. The rod is l = 20 cm in length, with a natural curvature of radius R_n = 2 cm. It has a circular cross-sectional area. Its initial position is defined as the following for its nodes:

$$\boldsymbol{x_k} = \left[R_n \cos\big((k-1)\Delta\theta\big), \qquad R_n \sin\,((k-1)\Delta\theta, \qquad 0\right]$$

where

$$\Delta\theta = \frac{l}{R_n}\frac{1}{N-1}$$

The twist angles between each node θ^k is initially 0. One end of the beam is clamped, represented by the first two nodes being fixed and the first twist angle between them staying 0 over time. The physical parameters of the beam are that the density is 1000 kg/m3, the cross-sectional radius is 1 mm, the Young's modulus is 10 MPa, the shear modulus is 1/3 the Young's modulus, and gravitational acceleration is -9.81 m/s2 "out of the page". The deformation of this rod under gravity is to be simulated over 5 seconds.

The governing equation for the motion of this rod is

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \frac{\partial \boldsymbol{E}^{elastic}}{\partial \boldsymbol{q}} - \boldsymbol{W} = 0$$

where **M** is the mass matrix containing the masses of the nodes, $\mathbf{E}^{\text{elastic}}$ is the elastic energy of the rod (including stretching, bending, and twisting), and **W** is the weight of the nodes.

The implicit solver was based on Newton-Raphson method and employed functions by M. K. Jawed for various intermediate calculations. The rod was discretized into 30 nodes, and the timestep of the solver was set to Δt = .01 seconds.

The code relies on various other functions, so global variables used in the main code and some of the functions are first defined. Then the physical parameters of the problem, as well as the timestep, number of nodes, and tolerance for the Newton-Raphson solver are written. The matrices containing data for the degrees of freedom, mass, and other parameters in the governing equation are added. The initial undeformed Voronoi lengths, edge lengths, reference frame, reference twist, and material frame are defined to be able to compute twist through the spatial parallel-transport theorem. The natural curvature of the rod and the indices of the fixed and free nodes are specified.

With the problem setup complete, the code then moves on to the Newton-Raphson solver. For the free indices, the code computes **F** and the gradients and hessians of the elastic energy terms to calculate the new positions of the rod nodes at each timestep. The z-position of the final node is also recorded at each timestep. Finally, the position of the rod nodes and the z-coordinate of the last node are plotted.

II. Result

The generated plot of the z-coordinate of the last node vs. time is:

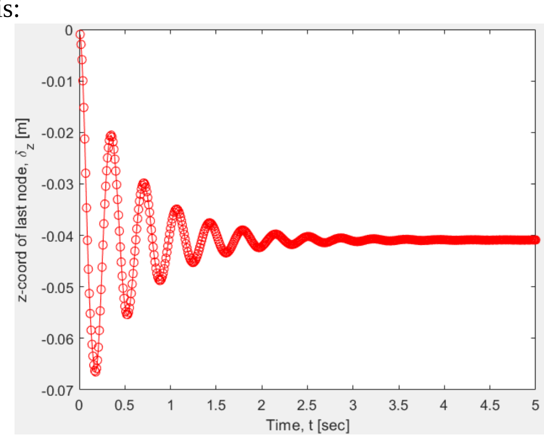

Figure 1: The z-coordinate of the last node vs. time over the rod simulation, with N = 30.

Over time, the z-coordinate of the last node oscillates in an underdamped manner and trends towards a resting position of around -0.04 m. The result generated by the code matches well with the provided answer plot, which simulated the rod with 50 nodes instead:

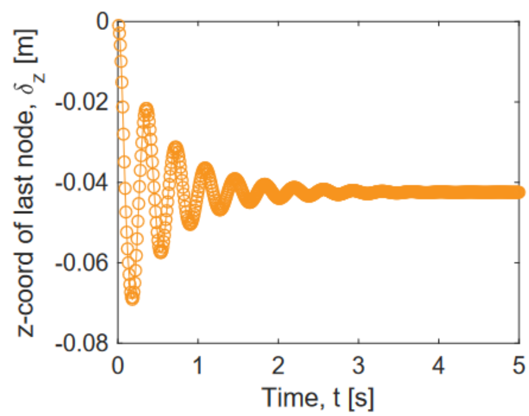

Figure 1: Provided answer to the rod simulation, with N = 50.

ACKNOWLEDGMENT

I thank Professor M. K. Jawed for his guidance and provided functions for completing this assignment.

REFERENCES

[1] M. K. Jawed and S. Lim, “Discrete simulation of slender structures” . UCLA, 2019, pp. 39–46.