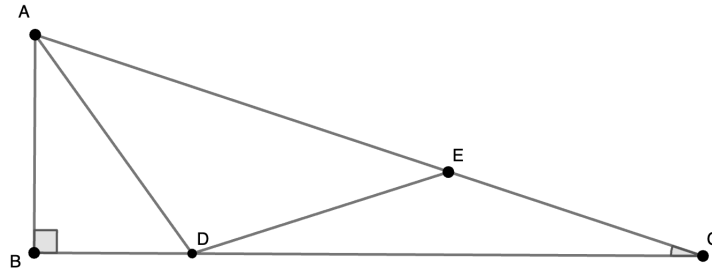


As shown in the figure below, triangle ABC is a right triangle, with $\angle ABC = 90^\circ$. Point D lies on segment BC , and point E lies on segment AC . If $\angle BAD = \angle CAD$, and segments $AD = DE = EC$, find the measure of $\angle ACB$.¹

Hint: Use properties of isosceles triangles to find the relationships of segments and angles.



¹Kunitachi High School, Tokyo

Solution

Answer : $\angle ACB = 18^\circ$

Proof: Since $EC = DE$, $\angle EDC = \angle C$. $\angle AED = \angle EDC + \angle C = 2\angle C$. Additionally, since $AD = DE$, $\angle EAD = \angle AED = 2\angle C$. Given this, we can say that $\angle BAC = \angle BAD + \angle CAD = 2\angle CAD = 2 \times 2\angle C = 4\angle C$. Now looking at triangle ABC , we know that the sum of interior angles in a triangle sum to 180° , so $\angle C + \angle BAC + \angle B = 180^\circ$. Substituting previous equalities and solving for $\angle C$, we get $\angle \mathbf{ACB} = \angle \mathbf{C} = \mathbf{18^\circ}$.