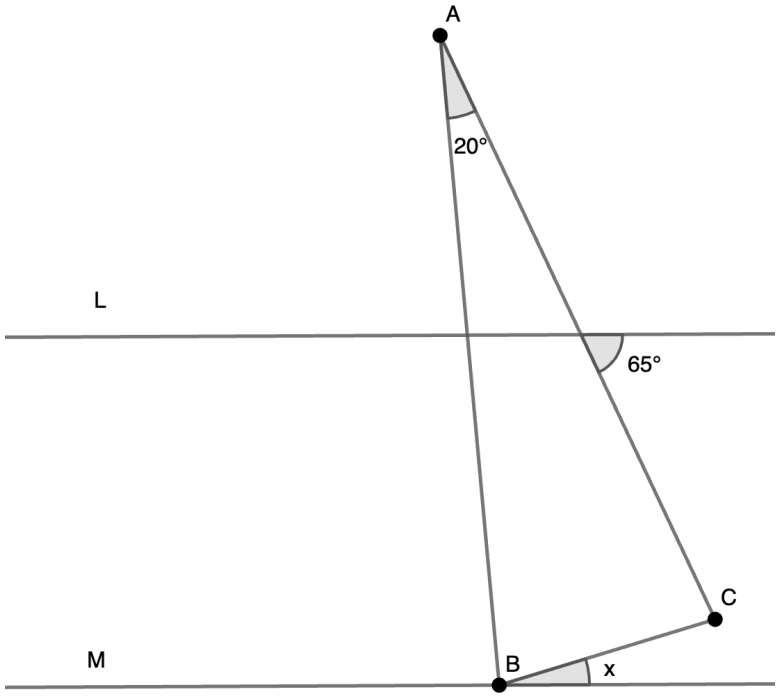
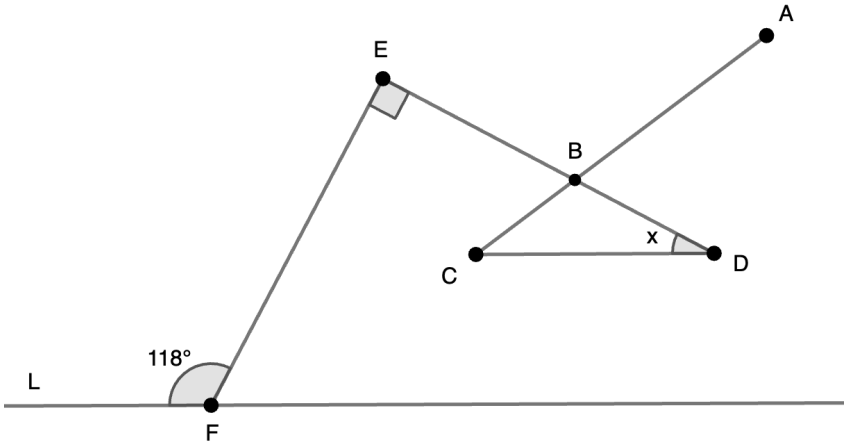
Find the measure of $\angle x$.[1]

(1): AB = AC, $L//M$

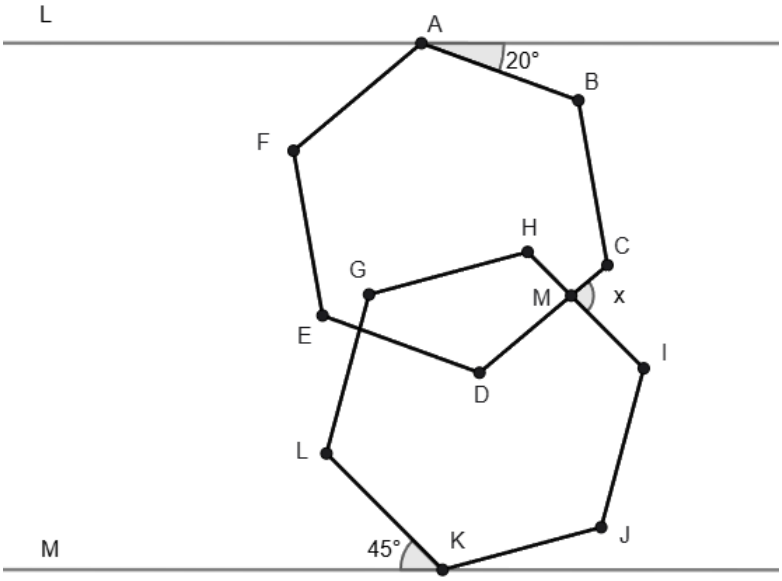


(2): $CD//L$, $\angle DEF = 90°$



[1](1) Hosei University High School, Tokyo, (2) Ikubunkan High School, Tokyo, (3) Rakunan High School, Kyoto

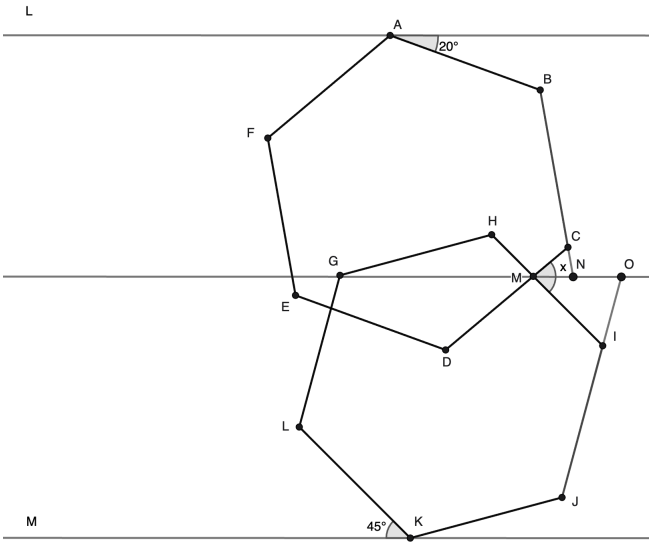
(3): $L//M$, both hexagons are regular hexagons.
Hint: Draw an auxiliary line and extend some of the sides of the hexagons.



Solution

Answer : (1) : $15°$, (2) : $28°$, (3) : $85°$

Proof (1): Triangle ABC is an isosceles triangle, with sides AB=AC, so $\angle C = (180° - 20°) \div 2 = 80°$. Drawing an auxiliary line through point C, we can derive that $\angle x + 65° = 80°$, giving us $\angle \mathbf{x} = \mathbf{15}°$.

Proof (2): Extend segment ED, creating point G, the intersection with line L. Since $CD//L$, $\angle x = EGF$. Also, we know that $\angle x + \angle EGF = 118°$, giving us $\angle \mathbf{x} = \mathbf{28}°$.

Proof(3): *Refer to diagram below.* Draw an auxiliary line through point M, and extend sides BC and IJ. Make the intersection of the extended sides BC and IJ with the auxiliary line points N and O, respectively. We know that an interior angle of a regular hexagon is $120°$, so each exterior angle of a regular hexagon is $60°$. $\angle CNO = 180° - 20° - 60° = 100°$. $\angle MCN = 60°$. Therefore, $\angle CMO = 100° - 60° = 40°$. Opposite angles of a parallelogram are congruent, so $\angle MOI = 75°$ and $\angle MIO = 60°$. Also, $\angle IMO = 180° - 75° - 60° = 45°$. Therefore, $\angle \mathbf{x} = \angle \mathbf{CMN} + \angle \mathbf{IMO} = \mathbf{40}° + \mathbf{45}° = \mathbf{85}°$.