

Two lines pass through the origin O : $l : y = 3x$ and $m : y = \frac{1}{2}x$. Additionally, line n has slope $a < 0$, and passes through point $A(4, 2)$. There is also a binary variable P , with the following conditions:

- $P = 1$: A point (x, y) , where x and y are both integers, and lies in the region enclosed by the three lines l , m , and n (including points on the boundary lines).
- $P = 0$: Otherwise.

Given this information, answer the questions below.¹

- (1): How many unique points (x, y) on segment OA have x and y as integer values?
- (2): When $a = \frac{1}{2}$, how many unique points exist such that $P = 1$?
- (3): Let z be the number of unique points that make $P = 1$. Find the interval of a that makes $z = 11$.

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