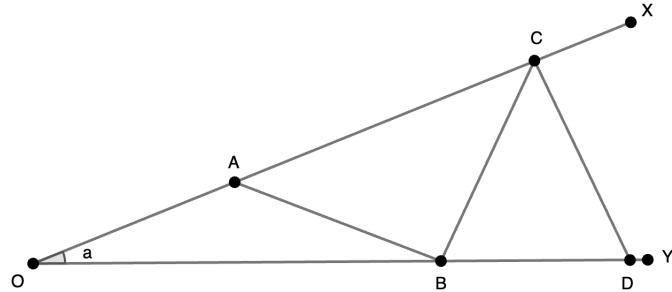


As shown in the figure below, points  $A$  and  $C$  lie on segment  $OX$ , and points  $B$  and  $D$  lie on segment  $OY$ . If segments  $OA = AB = BC = CD$ , and  $\angle XOY = a^\circ$ , find the measure of  $\angle XCD$  in terms of  $a$ .<sup>1</sup>

*Hint: Use properties of isosceles triangles to find the relationships of segments and angles.*



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<sup>1</sup>Akita Prefecture

## Solution

Answer :  $\angle XCD = 4a^\circ$

Proof: Since  $OA = AB$ , we can say that  $\angle OBA = \angle AOB = a^\circ$ . Also,  $\angle BAC = \angle AOB + \angle OBA = a^\circ + a^\circ = 2a^\circ$ . Similarly, since  $AB = BC$ ,  $\angle BCA = \angle BAC = 2a^\circ$ , so  $\angle CBD = \angle BCO + \angle COB = 2a^\circ + a^\circ = 3a^\circ$ . Finally, since  $BC = CD$ ,  $\angle ODC = \angle CBD = 3a^\circ$ , so  $\angle XCD = \angle COD + \angle ODC = a^\circ + 3a^\circ = 4a^\circ$ .