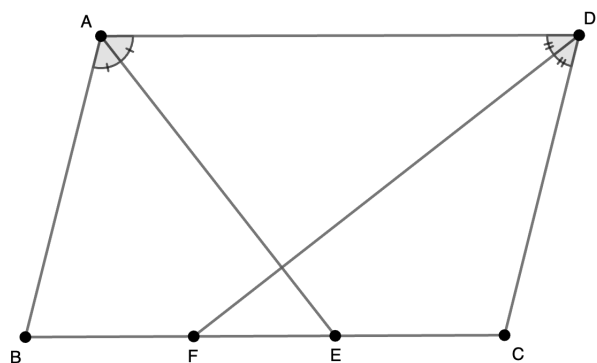$ABCD$ is a parallelogram. AE is an angle bisector of $\angle A$, and DF is an angle bisector of $\angle D$. Both E and F lie on segment BC. If $AB = 6.5$cm and $AD = 10$cm, find the length of EF.[1]

A

D

B

F

E

C

[1]Nagano Prefecture

Solution

Answer : $3cm$

Proof: $AD//BC$, and opposite angles are congruent so $\angle DAE = \angle AEB$. AE is a bisector of $\angle BAD$, so $\angle DAE = \angle EAB$. By transitive property, $\angle AEB = \angle EAB$, so triangle BAE is an isosceles triangle, with $AB = BE$. Since $AB = 6.5cm$, BE is also $6.5cm$. Similarly, $\angle ADF = \angle FDC = \angle DFC$, creating an isosceles triangle CDF, with $CD = CF$. Opposite sides of a parallelogram are congruent, so $AB = CD = CF = 6.5cm$. Similarly, $AD = BC = 10cm$. To find the length of EF, we must calculate $\mathbf{EF = BE - CF - BC = 6.5 + 6.5 - 10 = 3cm}$.