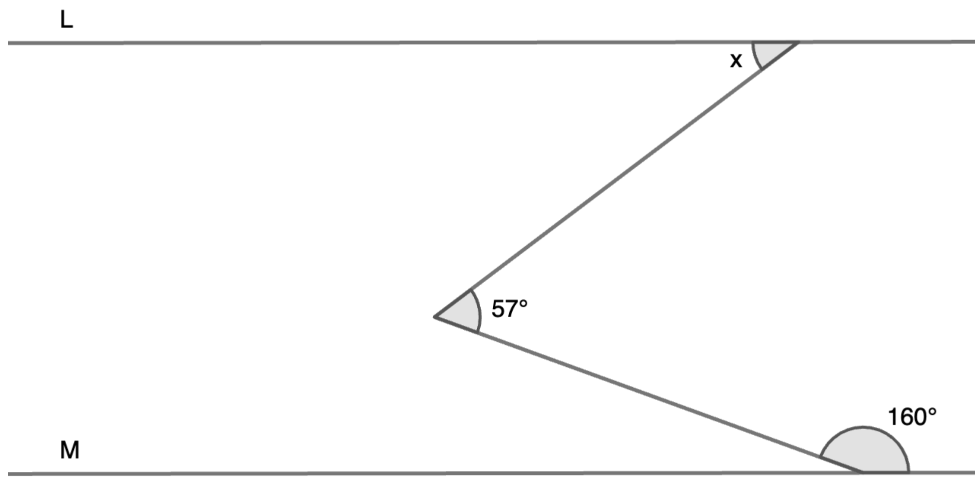
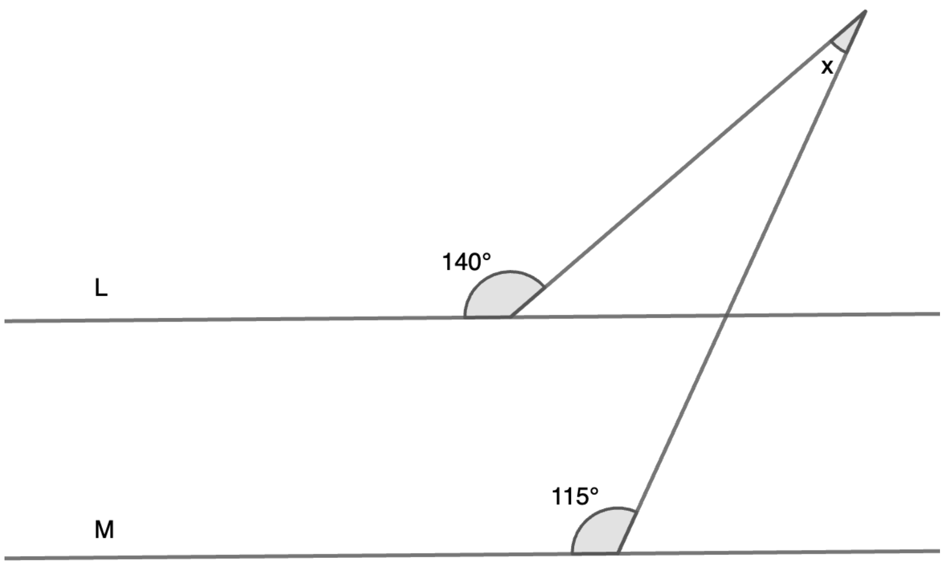
Find the size of angle x.

Note: For each diagram, line L and line M are parallel.
Hint: For (1) and (4), it may be useful to draw auxiliary lines between line L and line M. [1]

(1)

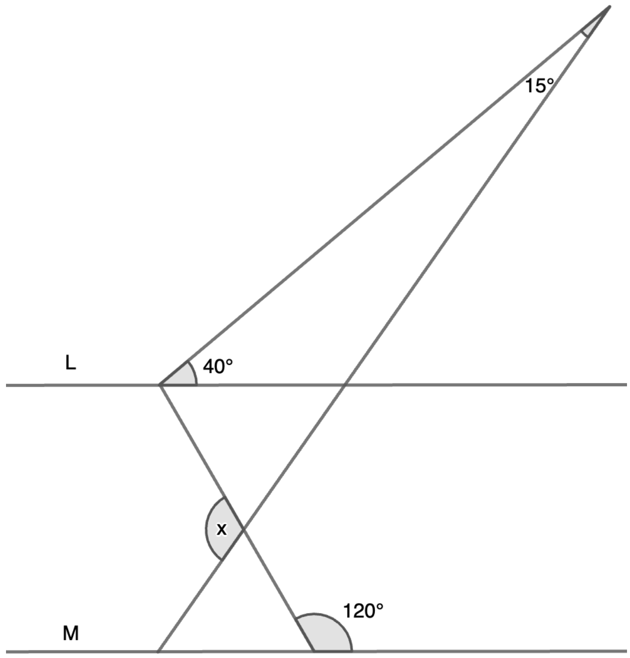


(2)

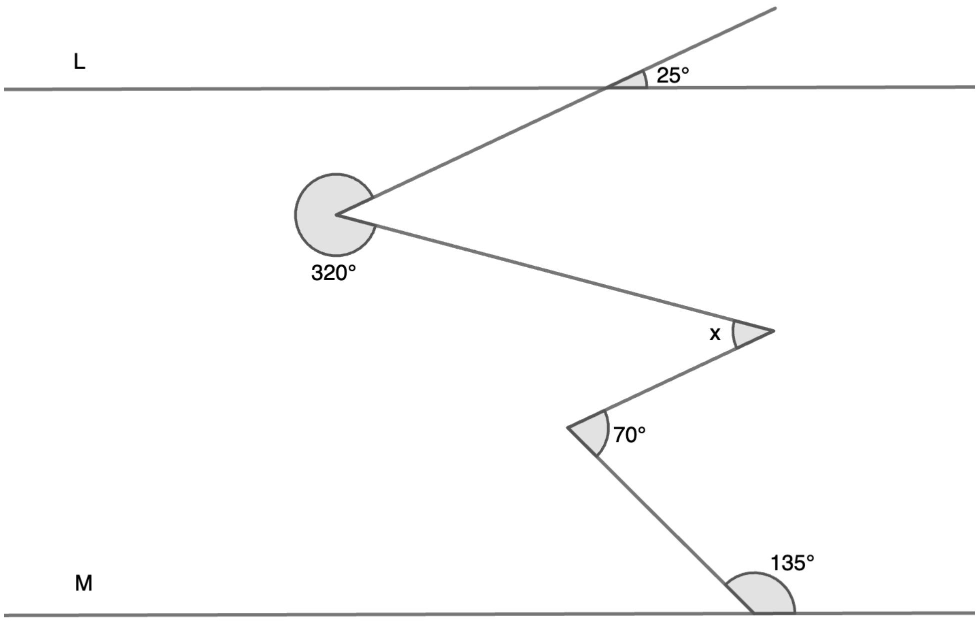


[1] (1) Tochigi Prefecture, (2) Institute of Science Tokyo High School, Tokyo, (3) Saga Prefecture, (4) Hosei University High School, Tokyo

(3)



(4)



Solution

Answer : (1) : 37°, (2) : 25°, (3) : 115°, (4) : 40°

Proof (1): As shown in Figure 1 below, you can draw an auxiliary line N, so that $L//M//N$, and line N goes through the vertex that contains the 57° angle. To find the measure of $\angle x$, we first must find the measure of $\angle a$, as well as the two smaller angles created by line N, which are denoted as b, and c. Since $\angle a$ and the 160° angle are supplementary angles, the sum of these two angles is 180°, allowing us to set up the equation:

$$\angle a + 160^\circ = 180^\circ \tag{1}$$

Solving this equation will give us $\angle a = 20^\circ$. Since $\angle a$ and $\angle b$ are alternate interior angles:

$$\angle a = \angle b = 20^\circ \tag{2}$$

As shown in the original figure, the sum of $\angle b$ and $\angle c$ is 57°, and from Equation (2) we know that $\angle b = 20^\circ$. Therefore, we can set up the equation:

$$20^\circ + \angle c = 57^\circ \tag{3}$$

giving us $\angle c = 37^\circ$. Finally, since $\angle x$ and $\angle c$ are alternate interior angles:

$$\angle \mathbf{x} = \angle \mathbf{c} = \mathbf{37^\circ}$$

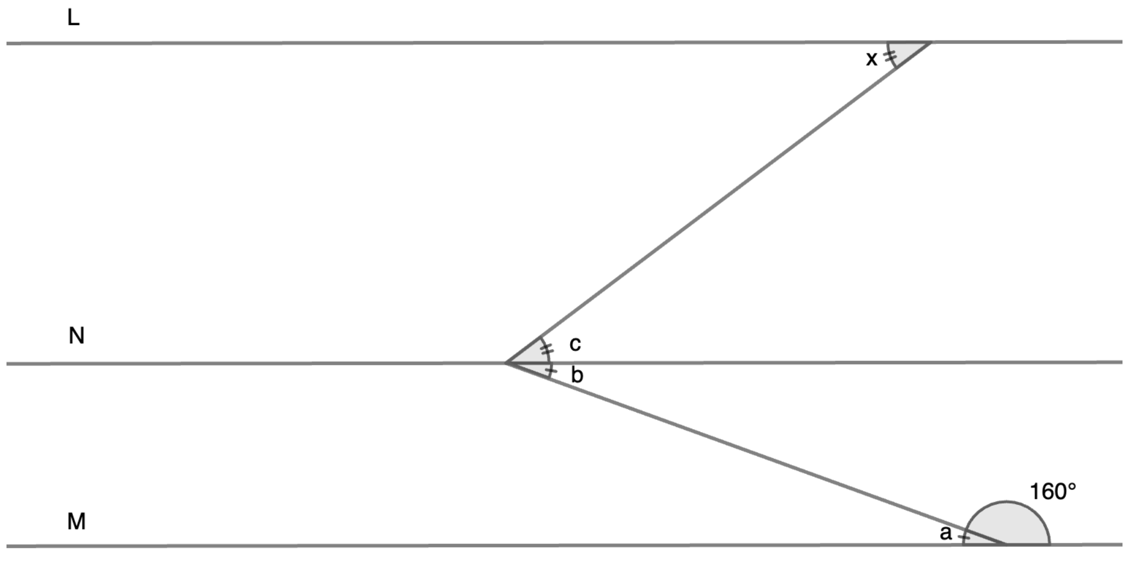


Figure 1

Proof (2): To find the measure of $\angle x$, we first must find the values of $\angle d$ and $\angle e$, as shown in Figure 2 below. Since $\angle d$ and the 115° angle on line M are corresponding angles:

$$\angle d = 115^\circ \tag{4}$$

Next, we know that $\angle e$ and the 140° angle are supplementary angles, the sum of these two angles is 180°, allowing us to set up the equation:

$$\angle e + 140^\circ = 180^\circ \tag{5}$$

giving us $\angle e = 40^\circ$. We can also see that $\angle d$, $\angle c$, and $\angle x$ make up the angles of a triangle, so

$$\angle d + \angle e + \angle x = 180^\circ \rightarrow x = 180^\circ - 115^\circ - 40^\circ \tag{6}$$

Simplifying to:

$$\angle \mathbf{x} = \mathbf{25}^\circ$$

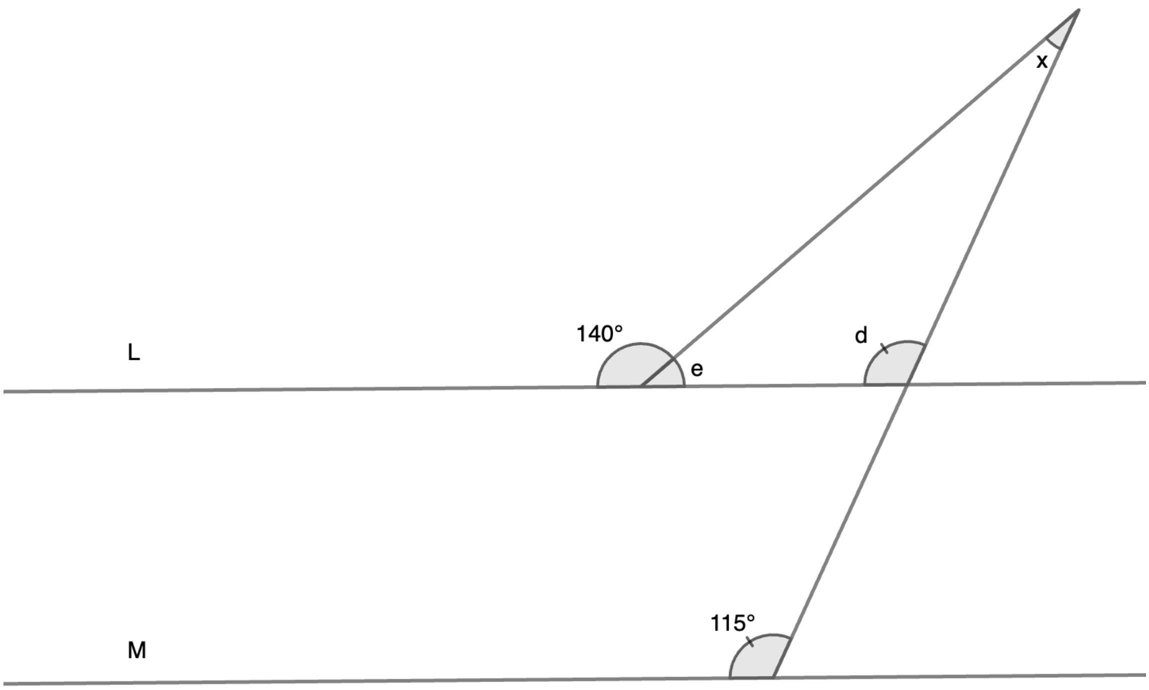


Figure 2

Proof (3): To find the measure of $\angle x$, we first must find the measures of $\angle f$, $\angle g$, and $\angle h$, as shown in Figure 3. Since $\angle f$ is the exterior angle of the triangle containing the 15° angle and the 40° angle, we can sum the two opposite interior angles to find the measure of $\angle f$:

$$\angle f = 15^\circ + 40^\circ = 55^\circ \tag{7}$$

Since $\angle f$ and $\angle g$ are corresponding angles:

$$\angle f = \angle g = 55^\circ \tag{8}$$

To find the measure of $\angle h$, we can again apply the theorem that an exterior angle of a triangle is the sum of the two opposite interior angles, giving us:

$$\angle g + \angle h = 120^\circ \rightarrow \angle h = 120^\circ - 55^\circ = 65^\circ \tag{9}$$

Finally, $\angle x$ and $\angle h$ are supplementary angles, so the sum of these two angles is 180°, leading to the equation:

$$\angle x + \angle h = 180^\circ \rightarrow \angle \mathbf{x} = \mathbf{180^\circ} - \mathbf{65^\circ} = \mathbf{115^\circ}$$

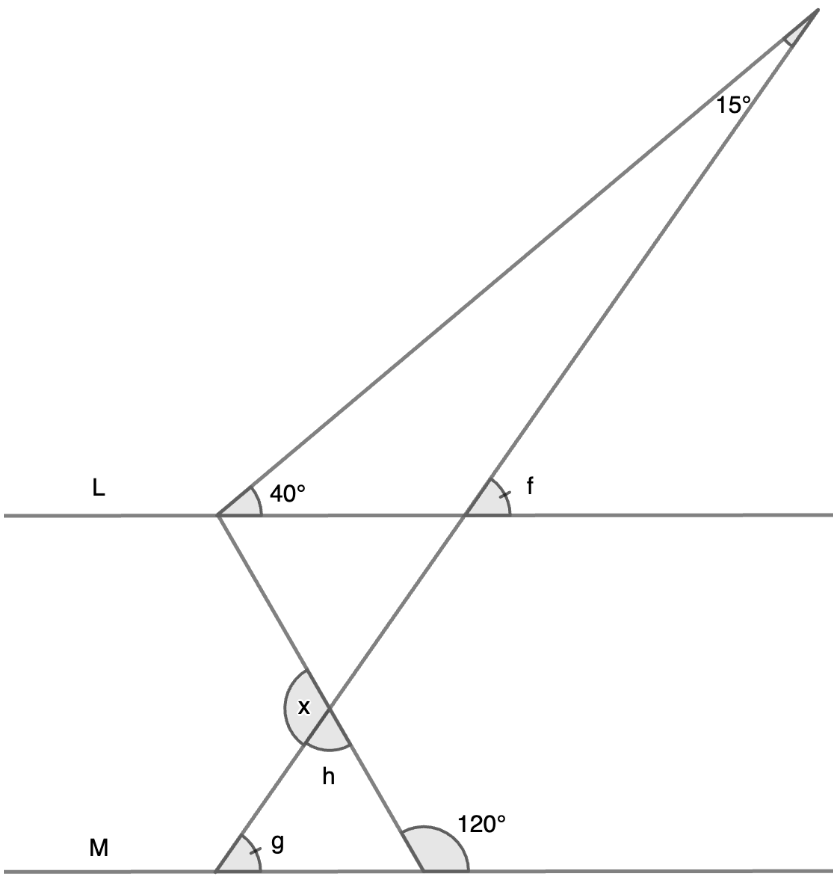


Figure 3

Proof (4): Similar to (1), we first must draw three auxiliary lines P, Q, R, each passing through one of vertices and so that $L//M//P//Q//R$. From these auxiliary lines, we have created $\angle i$, $\angle j$, $\angle k$, $\angle s$, $\angle t$, and $\angle u$, where

$$\angle k + \angle u = \angle x \tag{10}$$

which is shown in Figure 4. Starting with $\angle i$, we see that $\angle i$ and the 25° angle are corresponding angles, so $\angle i = 25°$. Also, we know that the sum of the angles around a single vertex is 360°, creating the equation:

$$\angle i + \angle j + 320° = 360° \rightarrow \angle j = 360° - 320° - 25° = 15° \tag{11}$$

Since $\angle j$ and $\angle k$ are alternate interior angles:

$$\angle j = \angle k = 15° \tag{12}$$

We can also see that $\angle s$ and the 135° angle are same-side interior angles, so we know that:

$$\angle s + 135° = 180° \rightarrow \angle s = 45° \tag{13}$$

As shown by the original figure, the sum of $\angle s + \angle t$ is 70°. Solving this equation will give us: $\angle t = 25°$ Since $\angle t$ and $\angle u$ are alternate interior angles,

$$\angle t = \angle u = 25° \tag{14}$$

Now we have the necessary values to calculate $\angle x$. From Equation (10), we know that $\angle k + \angle u = \angle x$, and we have the values of $\angle k$ and $\angle u$ from Equation (12) and Equation (14):

$$\angle \mathbf{x} = \angle \mathbf{k} + \angle \mathbf{u} = \mathbf{25° + 15° = 40°}$$

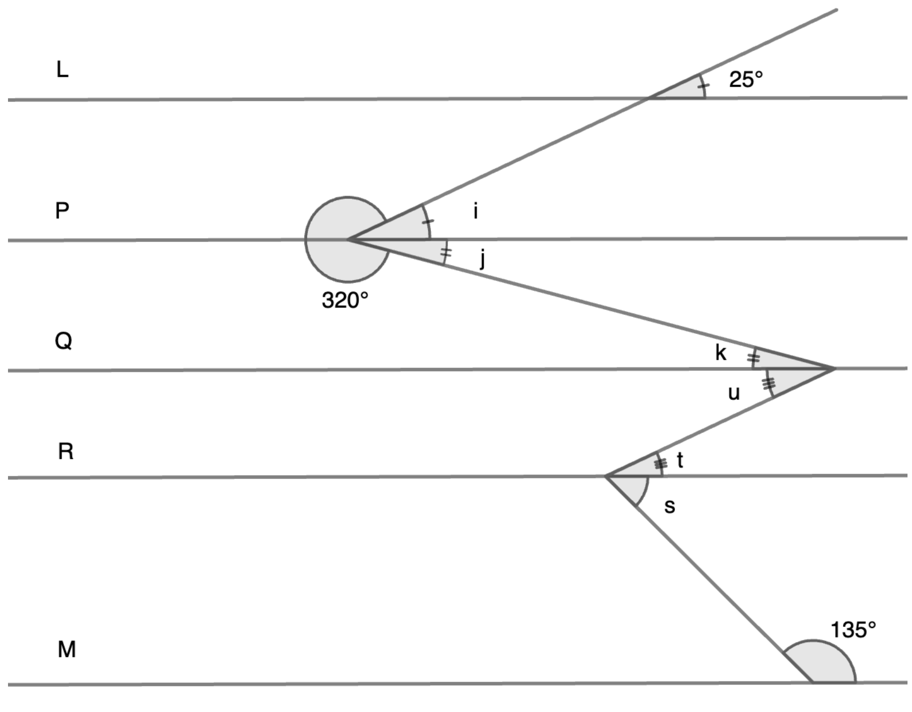


Figure 4