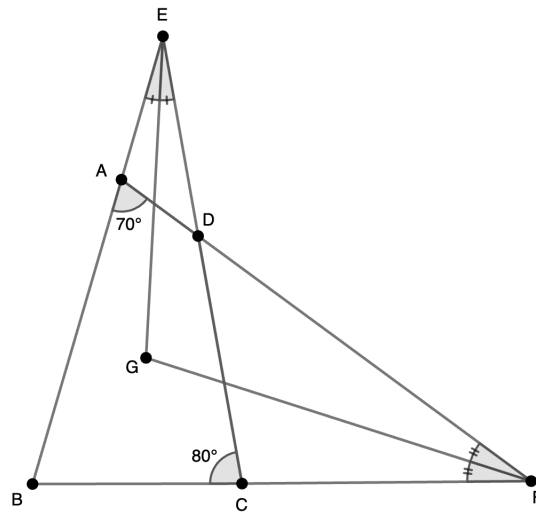From quadrilateral $ABCD$, sides BA and CD are extended, with the intersection of those extensions being point E. Sides AD and BC are also extended, with the intersection of those extensions being point F. Point G marks the intersection of the angle bisectors of $\angle E$ and $\angle F$. If $\angle DAB = 70°$, and $\angle BCD = 80°$, find the measure of $\angle EGF$.[1]

[1]Sugamo High School, Tokyo

Solution

Answer : 103°

Proof: Since EG and FG are bisectors of $\angle BEC$ and $\angle AFB$, we can say that $\angle BEG = \angle GED$ (1), $\angle BFG = \angle GFD$ (2). Looking at quadrilateral $GEDF$, $\angle EGF + \angle GED + \angle GFD = \angle EDF$ (3), and looking at quadrilateral $BEGF$, $\angle B + \angle BEG + \angle BFG = \angle EGF$ (4). Using equalities (1) and (2), and calulcating (3) - (4) as a system of equations, we get $\angle EGF - \angle B = \angle EDF - \angle EGF$, which simplifies to $2\angle EGF = \angle B + \angle ADC = 360° - (70° + 80°) = 210°$, giving us $\angle \textbf{EGF} = \textbf{105°}$.