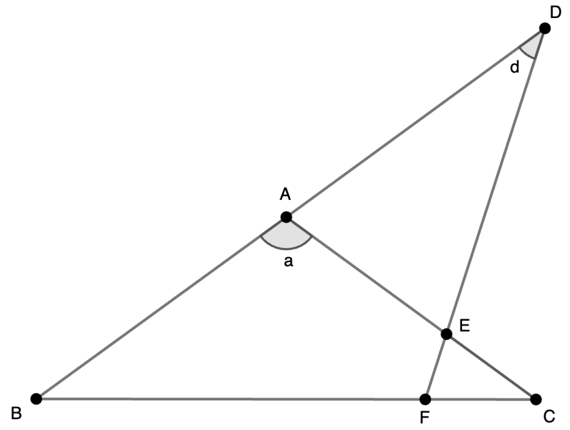
(1) In the diagram below, segments $AB = AC$, and segments $CE = CF$. Find the general measure of $\angle d$ in terms of $a$.
(2) In a specific case, segments $FB = FD$, in addition to the equalities listed in (1). Find the measure of $\angle a$ in this specfic case.[1]
Note: The answer to (1) should be written in terms of $a$, and (2) written as an integer value.
*Hint: Use properties of isosceles triangles to find the relationships of segments and angles.*




[1] Aiko High School, Ehime

# Solution

$Answer : (1) : \frac{3a-180}{4}, (2) : 108$

Proof (1): Since $AB = AC$, $\angle C = \frac{180^\circ - a^\circ}{2}$. Also, since $CE = CF$, $\angle CEF = (180^\circ - \angle C) \div 2 = (180^\circ - \frac{180^\circ - a^\circ}{2}) \div 2 = \frac{180^\circ + a^\circ}{4}$. $\angle DEA = \angle CEF$, so we can use substitution to solve for $\angle d$ in the following equation: $\angle d^\circ + \angle DEA = a^\circ$. Solving for $\angle d$, we get the general solution: $\angle \mathbf{d} = \mathbf{\frac{3a-180}{4}}$

Proof (2): Now, in the specific case of $FB = FD$, $\angle d^\circ = \angle B = \angle C = \frac{180^\circ - a^\circ}{2}$, allowing us to set up the equation: $\frac{3a-180}{4} = \frac{180-a}{2}$. Solving for $a$, we get $\mathbf{a = 108}$.