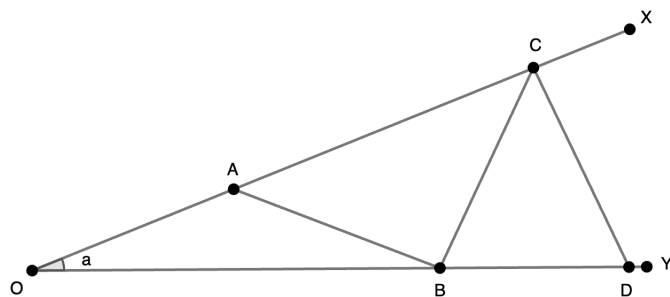


As shown in the figure below, points A and C lie on segment OX , and points B and D lie on segment OY . If segments $OA = AB = BC = CD$, and $\angle XOY = a^\circ$, find the measure of $\angle XCD$ in terms of a .¹

Hint: Use properties of isosceles triangles to find the relationships of segments and angles.



¹ Akita Prefecture

Solution

Answer : $\angle XCD = 4a^\circ$

Proof: Since $OA = AB$, we can say that $\angle OBA = \angle AOB = a^\circ$. Also, $\angle BAC = \angle AOB + \angle OBA = a^\circ + a^\circ = 2a^\circ$. Similarly, since $AB = BC$, $\angle BCA = \angle BAC = 2a^\circ$, so $\angle CBD = \angle BCO + \angle COB = 2a^\circ + a^\circ = 3a^\circ$. Finally, since $BC = CD$, $\angle ODC = \angle CBD = 3a^\circ$, so $\angle \mathbf{XCD} = \angle \mathbf{COD} + \angle \mathbf{ODC} = \mathbf{a^\circ} + \mathbf{3a^\circ} = \mathbf{4a^\circ}$.