Fill in the blanks below.[1]

As shown in Figure 1, triangle *A'B'C'* is made when the bisectors of the exterior angles of A, B, and C are extended. Therefore, $\angle A' =$ ________ $^\circ -$ ________ $\times A^\circ$. Now, the bisectors of the exterior angles of *A'*, *B'*, and *C'* are extended, giving us triangle *A"B"C"*, which is shown in Figure 2. Therefore, $\angle A'' =$ ________ $^\circ +$ ________ $\times A^\circ$.

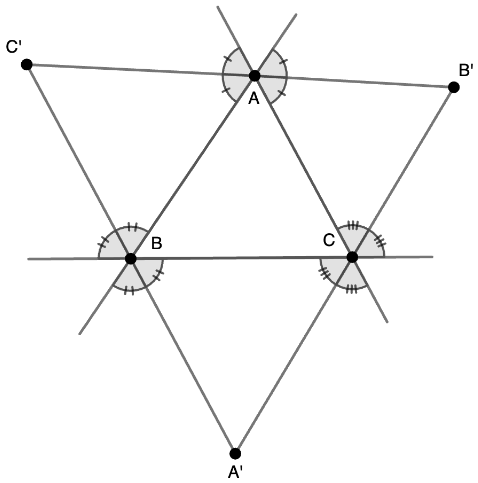

Figure 1

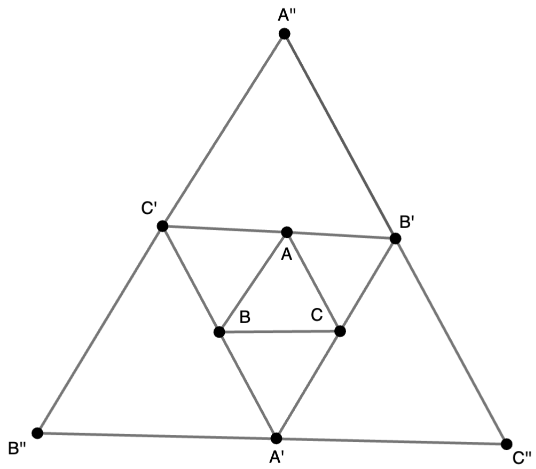

Figure 2

[1]Tsuchiura Nihon University High School, Ibaraki

Solution

$Answer: \angle A' = 90^\circ - \frac{1}{2} \times A^\circ, \angle A'' = 45^\circ + \frac{1}{4} \times A^\circ$

Proof: $\angle A' = 180^\circ - \frac{180^\circ - \angle ABC}{2} - \frac{180^\circ - \angle ACB}{2} = \frac{\angle ABC + \angle ACB}{2} = \frac{180^\circ - \angle A}{2} = \mathbf{90}^\circ - \mathbf{\frac{1}{2}} \times \mathbf{A}^\circ$.
The relationship between $\angle A''$ and $\angle A'$ is similar, with $\angle A'' = 90^\circ - \frac{1}{2} \times \angle A' = 90^\circ - \frac{1}{2}(90^\circ - \frac{1}{2} \times \angle A) = 90^\circ - 45^\circ + \frac{1}{4} \times \angle A = \mathbf{45}^\circ + \mathbf{\frac{1}{4}} \times \angle \mathbf{A}$.