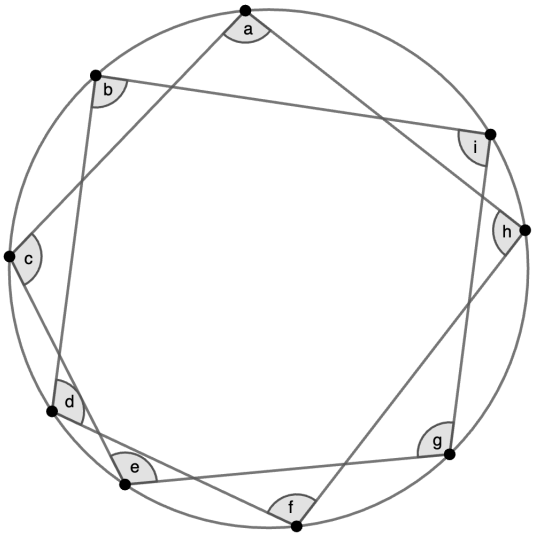
Nine points on the circumference of a circle are connected by segments, as shown in the figure below. Find the sum of $\angle a + \angle b + \angle c + ... + \angle i$.[1]
Hint: There are nine triangles, each containing $\angle a, \angle b, \angle c, ...,$ *or* $\angle i$.



[1]Nihon University Narashino High School, Chiba

Solution

Answer : 900°

Proof: As shown in the figure below, label the vertices from A to R. The sum of the exterior angles of the nonagon JKLMNOPQR can be considered in two different ways: The sum of the angles marked with a single hash-mark is 360°, and the sum of the angles marked with a double hash-mark is also 360°. Therefore, the (sum of the interior angles of the 9 triangles) $= \angle a + \angle b + \angle c + ... + \angle i + 360° + 360°$. From this equation, we can say that $\angle a + \angle b + \angle c + ... + \angle i = 180° \times 9 - 360° - 360° = \mathbf{900}°$.