1 Universal Constructions

Construction	Definition	Computation	Equality	Commutative Diagram
Initial Object	$\frac{}{0 \xrightarrow{[]} A}$		$\frac{f : 0 \to A}{f = []}$	$0 \overset{[]}{\dashrightarrow} A$
Terminal Object	$\frac{A}{A \xrightarrow{\langle\rangle} 1}$		$\frac{f : A \to 1}{f = \langle\rangle}$	$A \overset{\langle\rangle}{\dashrightarrow} 1$
Products	$\frac{A \xrightarrow{f} X \quad A \xrightarrow{g} Y}{A \xrightarrow{\langle f,g\rangle} X \times Y}$	$\pi_1 \circ \langle f, g\rangle = f, \pi_2 \circ \langle f, g\rangle = g$ $\langle \pi_1, \pi_2\rangle \circ h = h : A \to X \times Y$	$\frac{\pi_1 \circ h = f : A \to X \quad \pi_2 \circ h = g : A \to Y}{h = \langle f,g\rangle : A \to X \times Y}$	$\begin{tikzcd} & A \arrow[dl, "f"'] \arrow[d, dotted, "\langle f,g\rangle"] \arrow[dr, "g"] & \\ X & X \times Y \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] & Y \end{tikzcd}$
Coproducts	$\frac{A \xrightarrow{f} X \quad B \xrightarrow{g} X}{A + B \xrightarrow{[f,g]} X}$	$[f, g] \circ \iota_1 = f, [f, g] \circ \iota_2 = g$ $[h \circ \iota_1, h \circ \iota_2] = h : A + B \to X$	$\frac{h \circ \iota_1 = f : A \to X \quad h \circ \iota_2 = g : B \to X}{h = [f,g] : A + B \to X}$	$\begin{tikzcd} A \arrow[r, "\iota_1"] \arrow[dr, "f"'] & A + B \arrow[d, dotted, "{[f,g]}"] & B \arrow[l, "\iota_2"'] \arrow[dl, "g"] \\ & X & \end{tikzcd}$
Exponentials	$\frac{A \times X \xrightarrow{f} Y}{A \xrightarrow{\mathrm{cur} f} X \Rightarrow Y}$	$\mathrm{app} \circ (\mathrm{cur} f \times \mathrm{id}_X) = f : A \times X \to Y$ $\mathrm{cur}(\mathrm{app} \circ (h \times \mathrm{id}_X)) = h : A \to X \Rightarrow Y$	$\frac{\mathrm{app} \circ (h \times \mathrm{id}_X) = f : A \times X \to Y}{h = \mathrm{cur} f : A \to X \Rightarrow Y}$	$\begin{tikzcd} A \times X \arrow[rr, dotted, "\mathrm{cur} f \times \mathrm{id}"] \arrow[dr, "f"'] & & (X \Rightarrow Y) \times X \arrow[dl, "\mathrm{app}"] \\ & Y & \end{tikzcd}$
Free monoids	$\frac{X \xrightarrow{f} M \in \underline{\mathrm{Set}}}{\underline{L}X \xrightarrow{f^\sharp} \underline{M} \in \underline{\mathrm{Mon}}}$	$f^\sharp \circ s_X = f : X \to M$ $(g \circ s_X)^\sharp = g : \underline{L}X \to \underline{M}$	$\frac{h \circ s_X = f : X \to M}{h = f^\sharp : \underline{L}X \to \underline{M}}$	$\begin{tikzcd} X \arrow[r, "s_X"] \arrow[dr, "f"'] & \underline{L}X \arrow[d, dotted, "f^\sharp"] \\ & \underline{M} \end{tikzcd}$

2 Categories

3 Functors

4 Natural Transformations

5 Adjunctions