1 Universal Constructions

Construction	Definition	Computation	Equality	Commutative Diagram
Initial Object	$\dfrac{}{0 \xrightarrow{[]} A}$		$\dfrac{f:0\to A}{f=[]}$	$0 \overset{[]}{\dashrightarrow} A$
Terminal Object	$\dfrac{A}{A \xrightarrow{\langle\rangle} 1}$		$\dfrac{f:A\to 1}{f=\langle\rangle}$	$A \overset{\langle\rangle}{\dashrightarrow} 1$
Products	$\dfrac{A \xrightarrow{f} X \quad A \xrightarrow{g} Y}{A \xrightarrow{\langle f,g\rangle} X\times Y}$	$\pi_1 \circ \langle f, g\rangle = f, \pi_2 \circ \langle f, g\rangle = g$ $\langle \pi_1, \pi_2\rangle \circ h = h : A \to X \times Y$	$\dfrac{\pi_1\circ h=f:A\to X \quad \pi_2\circ h=g:A\to Y}{h=\langle f,g\rangle:A\to X\times Y}$	$\begin{tikzcd} & A \arrow[dl, "f"'] \arrow[d, dotted, "\langle f,g\rangle"] \arrow[dr, "g"] & \\ X & X \times Y \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] & Y \end{tikzcd}$
Coproducts	$\dfrac{A \xrightarrow{f} X \quad B \xrightarrow{g} X}{A+B \xrightarrow{[f,g]} X}$	$[f, g] \circ \iota_1 = f, [f, g] \circ \iota_2 = g$ $[h \circ \iota_1, h \circ \iota_2] = h : A + B \to X$	$\dfrac{h\circ\iota_1=f:A\to X \quad h\circ\iota_2=g:B\to X}{h=[f,g]:A+B\to X}$	$\begin{tikzcd} A \arrow[r, "\iota_1"] \arrow[dr, "f"'] & A + B \arrow[d, dotted, "{[f,g]}"] & B \arrow[l, "\iota_2"'] \arrow[dl, "g"] \\ & X & \end{tikzcd}$
Exponentials	$\dfrac{A\times X \xrightarrow{f} Y}{A \xrightarrow{\mathrm{cur}f} X\Rightarrow Y}$	$\mathrm{app} \circ (\mathrm{cur} f \times \mathrm{id}_X) = f : A \times X \to Y$ $\mathrm{cur}(\mathrm{app} \circ (h \times \mathrm{id}_X)) = h : A \to X \Rightarrow Y$	$\dfrac{\mathrm{app}\circ(h\times \mathrm{id}_X)=f:A\times X\to Y}{h=\mathrm{cur}f:A\to X\Rightarrow Y}$	$\begin{tikzcd} A \times X \arrow[r, dotted, "\mathrm{cur}f\times\mathrm{id}"] \arrow[dr, "f"'] & (X \Rightarrow Y) \times X \arrow[d, "\mathrm{app}"] \\ & Y \end{tikzcd}$
Free monoids	$\dfrac{X \xrightarrow{f} M \in \underline{\mathrm{Set}}}{\underline{L}X \xrightarrow{f^\sharp} \underline{M} \in \underline{\mathrm{Mon}}}$	$f^\sharp \circ s_X = f : X \to M$ $(g \circ s_X)^\sharp = g : \underline{L}X \to \underline{M}$	$\dfrac{h\circ s_X=f:X\to M}{h=f^\sharp:\underline{L}X\to \underline{M}}$	$\begin{tikzcd} X \arrow[r, "s_X"] \arrow[dr, "f"'] & \underline{L}X \arrow[d, dotted, "f^\sharp"] \\ & \underline{M} \end{tikzcd}$

2 Equational Reasoning Laws

$$f = \langle\rangle : X \to 1_C$$

$$k = \langle f, g\rangle \iff (\pi_1 \circ k = f \wedge \pi_2 \circ k = g) : X \to Y \times Z$$

$$\pi_1 \circ (f \times g) = f \circ \pi_1 : X \times Y \to X_2$$

$$\pi_2 \circ (f \times g) = g \circ \pi_2 : X \times Y \to Y_2$$

$$\langle \pi_1, \pi_2, \rangle = id_{X\times Y} : X \times Y \to X \times Y$$

$$id_X \times id_Y = id_{X\times Y} : X \times Y \to X \times Y$$

$$(h \times k) \circ (f \times g) = (h \circ f) \times (k \circ g) : X_1 \times Y_1 \to X_3 \times Y_3$$

$$f = g \iff cur(f) = cur(g)$$

$$(g = cur(f)) \iff f = app \circ (g \times id)$$

$$cur(app) = id_{X \implies Y} : X \implies Y \to X \implies Y$$

$$\langle g_1, g_2\rangle \circ f = \langle g_1 \circ f, g_2 \circ f\rangle$$

$$(f_1 \times f_2) \circ \langle g_1, g_2\rangle = \langle f_1 \circ g_1, f_2 \circ g_2\rangle$$

$$u : X \to Y, g : F(Y) \to Z, v : Z \to W$$

$$\theta_{X,W}(v \circ g \circ F(u)) = G(v) \circ \theta_{Y,Z}(g) \circ u$$

$$\theta_{X,W}(g \circ F(u)) = \theta_{Y,Z}(g) \circ u$$

$$\theta_{X,W}(v \circ g) = G(v) \circ \theta_{Y,Z}(g)$$

$$u : X \to Y, g : Y \to G(Z), v : Z \to W$$

$$\theta^{-1}_{X,W}(G(v) \circ g \circ u) = v \circ \theta^{-1}_{Y,Z}(g) \circ F(u)$$

$$\theta^{-1}_{X,W}(g \circ u) = \theta^{-1}_{Y,Z}(g) \circ F(u)$$

$$\theta^{-1}_{X,W}(G(v) \circ g) = v \circ \theta^{-1}_{Y,Z}(g)$$

3 Categories

4 Functors

5 Natural Transformations

$$\eta : Id_C \to GF$$
$$\eta_X = \theta_{X,FX}(id_{FX})$$

$$\varepsilon : FG \to Id_D$$
$$\varepsilon_X = \theta^{-1}_{GX,X}(id_{GX})$$

$$\theta(f : A \to G(B)) = \varepsilon_B \circ F(f) : F(A) \to B$$
$$\theta^{-1}(g : F(A) \to B) = G(g) \circ \eta_A : A \to G(B)$$

6 Adjunctions

7 cartesian closed structure

8 left and right adjoints

9 dependent products and functions

10 exponentials in presheaf categories