Labor_10

Thursday, April 29, 2021 9:39 AM

① AloH: $\begin{cases} x_{k+1} = 2\cdot x_k + u_k ; \\ y_k = x_k \end{cases}$ DLQR $J = \frac{1}{2}\cdot\sum_{k=0}^{N-1}(x_k^2 + u_k^2)$

a). $N \to \infty \to u_k = ? \to$ 2 módszerrel / Ric. e.)

b). $N = 3$ $\Gamma \to Bd$

$\phi \to Ad$

Megoldás:

$$u_k = -(R + \Gamma^T\cdot P_{k+1}\cdot\Gamma)^{-1}\cdot\Gamma^T\cdot P_{k+1}\cdot\phi\cdot x_k = -\mathcal{K}\cdot x_k$$

$Ad = \phi = 2$; $Bd = \Gamma = 1$; $Cd = 1$;

$R = 1$; $Q = 1$;

a).

$$u_k = -(R + Bd^T\cdot P\cdot Bd)^{-1}\cdot Bd^T\cdot P\cdot Ad\cdot x_k$$

a1) $P = Ad^T\cdot P\cdot A_d + Q - Ad^T\cdot P\cdot Bd\cdot(R + Bd^T\cdot P\cdot Bd)^{-1}\cdot Bd^T\cdot P\cdot Ad$ dare

$$P = 2\cdot P\cdot 2 + 1 - 2\cdot P\cdot 1\cdot(1 + 1\cdot P\cdot 1)^{-1}\cdot 1\cdot P\cdot 2$$

$$P = 4\cdot P + 1 - \frac{4\cdot P^2}{1+P}$$

$$3P + 1 - \frac{4P^2}{1+P} = 0$$

$$(3P+1)\cdot(1+P) - 4P^2 = 0$$

$$-4P^2 + 3P^2 + 3P + P + 1 = 0 \quad /\cdot(-1)$$

$$P^2 - 4P - 1 = 0$$

$$P_{1,2} = \frac{4 \pm \sqrt{16 - 4\cdot(-1)}}{2} = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$$

$\boxed{P_1 = 2 + \sqrt{5}} > 0$

$P_2 = 2 - \sqrt{5} < 0$

$$u_k = -(1 + 1\cdot(2+\sqrt{5})\cdot 1)^{-1}\cdot 1\cdot(2+\sqrt{5})\cdot 2\cdot x_k = -\frac{2(2+\sqrt{5})}{3+\sqrt{5}}x_k$$

a2 $\to$ Potter.

$$H = \begin{bmatrix} Ad + Bd\cdot R^{-1}\cdot Bd\cdot(Ad^T)^{-1}\cdot Q & -Bd\cdot R^{-1}\cdot Bd^T\cdot(Ad^T)^{-1} \\ (A^T)^{-1}Q & (Ad^T)^{-1} \end{bmatrix}$$

$$H = \begin{bmatrix} A_d + \dots & \dots \\ -(A_d^T)^{-1} \cdot Q & (A_d^T)^{-1} \end{bmatrix}$$

$$H = \begin{bmatrix} 2 + 1 \cdot 1^{-1} \cdot 1 \cdot \frac{1}{2} \cdot 1 & -1 \cdot 1^{-1} \cdot 1 \cdot \frac{1}{2} \\ -\frac{1}{2} \cdot 1 & \frac{1}{2} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{5}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\lambda \cdot I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$\det(\lambda I - H) = 0$$

$$\det \cdot \begin{bmatrix} \lambda - \frac{5}{2} & \frac{1}{2} \\ \frac{1}{2} & \lambda - \frac{1}{2} \end{bmatrix} = \left(\lambda - \frac{5}{2}\right) \cdot \left(\lambda - \frac{1}{2}\right) - \frac{1}{4}$$

$$= \lambda^2 - \frac{5}{2}\lambda - \frac{1}{2}\lambda + \frac{5}{4} - \frac{1}{4} =$$

$$= \lambda^2 - 3\lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9 - 4 \cdot 1}}{2} \quad \begin{cases} \lambda_1 = \dfrac{3 + \sqrt{5}}{2} > 1 \\ \lambda_2 = \dfrac{3 - \sqrt{5}}{2} < 1 \end{cases}$$

$$P = V_{21} \cdot V_{11}^{-1} \qquad V = \begin{array}{cc} V_1 & V_i \\ \downarrow & \downarrow \end{array} \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

$$\underbrace{\underset{2\times 2}{H} \cdot \underset{2\times 1}{V_1}}_{2\times 1} = \underbrace{\underset{1\times 1}{\lambda_2} \underset{2\times 1}{V_1}}_{2\times 1}$$

$$\begin{bmatrix} \frac{5}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} = \lambda_2 \cdot \begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix}$$

$$\begin{cases} \frac{5}{2} V_{11} - \frac{1}{2} V_{21} = \lambda_2 \cdot V_{11} \quad /: V_{11} \\ -\frac{1}{2} \cdot V_{11} + \frac{1}{2} \cdot V_{21} = \lambda_2 \cdot V_{21} \end{cases}$$

$$\frac{5}{} \quad 1 \cdot \frac{V_{21}}{}$$

$$L \quad -\frac{?}{2}\cdot V_{11} + \frac{?}{2}\cdot V_{21} - \lambda_2 \cdot V_{21}$$

$$\frac{5}{2} - \frac{1}{2}\cdot\frac{V_{21}}{V_{11}} = \lambda_2$$

$$\frac{5}{2} - \frac{1}{2}\cdot P = \frac{3-\sqrt{5}}{2}$$

$$-P = 3-\sqrt{5} - 5 \;/\; (-1)$$

$$\boxed{P = 2+\sqrt{5}}$$

b).

$$u_k = -(R + \Gamma^T \cdot P_{k+1} \cdot \Gamma)^{-1} \cdot \Gamma^T \cdot P_{k+1} \cdot \phi \cdot x_k = -\mathcal{K} \cdot x_k$$

$$P_k = A_d^T \cdot P_{k+1} \cdot A_d + Q - A_d^T \cdot P_{k+1} \cdot B_d \cdot (R + B_d^T \cdot P_{k+1} \cdot B_d)^{-1} \cdot B_d^T \cdot P_{k+1} \cdot A_d$$

$u_0 = ?$ $\quad P_1 = ?$ $\qquad P_N = F$; $F = 0$

$u_1 = ?$ $\quad P_2 = ?$

$$P_3 = 0 \; ; \; P_2 = 2\cdot 0\cdot 2 + 1 - 2\cdot 0\cdot 1\cdot(1 + 1\cdot 0\cdot 1)^{-1}\cdot 1\cdot 0\cdot 2 = 1$$

$$P_1 = 2\cdot 1\cdot 2 + 1 - 2\cdot 1\cdot 1\cdot(1 + 1\cdot 1\cdot 1)^{-1}\cdot 1\cdot 1\cdot 2$$

$$P_1 = 4 + 1 - 4/2 = 5 - 2 = 3$$

$$u_0 = -(1 + 1\cdot 3\cdot 1)^{-1}\cdot 1\cdot 3\cdot 2\cdot x_0 = -\frac{1}{4}\cdot 6\cdot x_0 = -\frac{3}{2}\cdot x_0$$

$$u_1 = -(1 + 1\cdot 1\cdot 1)^{-1}\cdot 1\cdot 1\cdot 2\cdot x_1 = \ldots\ldots (\underbrace{2\cdot x_0 + u_0}_{x_1}) \qquad x_1 = 2\cdot x_0 + u_0$$

$$u_2 = -(1 + 1\cdot 0\cdot 1)\cdot 1 0\cdot 2\cdot x_2 = \ldots\ldots (2\cdot x_1 + u_1)$$

Thursday, April 29, 2021
12:31 PM

② $\dot{x}(t) = 5\cdot x(t) + u(t)$; $y(t) = 2\cdot x(t)$;

a). $J(u) = \int_0^{t_f} (x^2(\tau) + u^2(\tau))\,d\tau$ $\qquad u(t) = ?$

a1). Riccati e. → klasszikus köz. m. $\qquad t_f \to \infty$

a2). –II– → Potter m. $\qquad t_f \to \infty$

a3) magyarázat $t_f \not\to \infty$ $t_f = 5$

b). $J(u) = \frac{1}{2} \int_0^\infty (y(\tau) - \cos\omega\tau)^2 + 0.1 \cdot u^2(\tau))\, d\tau$

→ rajzoljuk fel a zárt szab. kört

Megoldás

$A = 5$; $B = 1$; $C = 2$

a) → állapot utáni szab. $R = 1$; $Q = 1$

$$u(t) = -R^{-1} \cdot B^T \cdot P \cdot x(t)$$

$$J = \frac{1}{2} x_{t_f}^T \cdot F \cdot x_{t_f} + \frac{1}{2} \int_{t_0}^{t_f} \Big(\underbrace{x^T(\tau) \cdot Q \cdot x(\tau)}_{x^2(\tau)} + u^T(\tau) \cdot R \cdot u(\tau) \Big)\, d\tau$$

a1. $t_f \to \infty$

$$P \cdot A + A^T \cdot P - P \cdot B \cdot R^{-1} \cdot B^T \cdot P + Q = 0 \quad \leftarrow \text{ARE}$$

$$5 \cdot P + 5P - P \cdot 1 \cdot 1 \cdot 1 \cdot P + 1 = 0$$

$$10P - P^2 + 1 = 0 \to P^2 - 10P - 1 = 0$$

$$P_{1,2} = \frac{10 \pm \sqrt{100 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{10 \pm \sqrt{104}}{2} = 5 \pm \sqrt{26}$$

$$P = 5 + \sqrt{26} > 0$$

$$u(t) = -1 \cdot 1 \cdot (5 + \sqrt{26}) \cdot x(t) = -(5 + \sqrt{26}) \cdot x(t)$$

a2). $t_f \to \infty$

$$H = \begin{bmatrix} A & -B \cdot R^{-1} \cdot B^T \\ -Q & -A^T \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -1 & -5 \end{bmatrix}$$

$$\det\left([\lambda \cdot I - H] \right) = 0$$

$$\det \begin{pmatrix} \lambda - 5 & 1 \\ 1 & \lambda + 5 \end{pmatrix} = 0 \Rightarrow (\lambda - 5) \cdot (\lambda + 5) - 1 = 0$$

$$\lambda^2 - 25 - 1 = 0$$

$$\lambda^2 = 26 \quad \lambda_1 = \sqrt{26}, \quad \lambda_2 = -\sqrt{26}$$

$$V = [V_s \ V_i] = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}; \quad \boxed{P = V_{21} \cdot V_{11}^{-1}}$$

$$V = [v_1 \quad v_2] \quad [v_{21} \quad \ldots]) \underbrace{[\ldots \quad v_{21} \quad v_{11}]}_{\to \frac{v_{21}}{v_{11}}}$$

$$H \cdot V = \lambda \cdot V$$

$$\begin{pmatrix} 5 & -1 \\ -1 & -5 \end{pmatrix} \cdot \begin{pmatrix} v_{11} \\ v_{21} \end{pmatrix} = -\sqrt{26} \cdot \begin{pmatrix} v_{11} \\ v_{21} \end{pmatrix}$$

$$\begin{cases} 5 \cdot v_{11} - v_{21} = -\sqrt{26} \cdot v_{11} & / \frac{1}{v_{11}} \\ -v_{11} - 5 v_{21} = -\sqrt{26} \cdot v_{21} \end{cases}$$

$$\to 5 - p = -\sqrt{26}$$

$$\boxed{p = 5 + \sqrt{26}}$$

a3)

$$t_f = 5$$

$$\dot{p}(t) = -p(t) \cdot A - A^T \cdot p(t) + p(t) \cdot B \cdot R^{-1} \cdot B^T \cdot p(t) - Q$$

$$\dot{p}(t) = -2 \cdot p(t) - 2 \cdot p(t) + p^2(t) - 1$$

$$\boxed{\dot{p}(t) = p^2(t) - 4 p(t) - 1} \leftarrow \text{ode45} \qquad t_f : -1 : 0$$

$$\dot{p}(t) \simeq \frac{p(t_k) - p(t_{k-1})}{h} \longrightarrow \frac{p(t_k) - p(t_k - h)}{h}$$

$$p(t_k) = p(t_{k+1}) + h \cdot (p^2(t_{k+1}) - 4 p(t_{k+1}) - 1)$$

$$p(t_{k+1}) = F = 0 \qquad p(t_f) = F$$

b).

$$J(u) = \frac{1}{2} \int_0^\infty \{ (\underbrace{y(\tau) - \cos \ldots}_{e(\tau) = y(t) - y_{ref}(t)})^2 + 0.1 \cdot u^2(\tau) \} \, d\tau \qquad y_{ref}(t) = \cos \omega t$$

$$u(t) = -\underbrace{R^{-1} \cdot B^T \cdot P}_{K_{fb}} \cdot x(t) + \underbrace{(-R^{-1} \cdot B^T \cdot ((A - B \cdot R^{-1} \cdot B^T \cdot p)^T)^{-1} C^T Q)}_{K_{ff}} y_{ref}(t)$$

$$Q = 1 \quad ; \quad R = 0.1$$

$$p \cdot A + A^T \cdot p - p \cdot B \cdot R^{-1} \cdot B^T \cdot p + C^T \cdot Q \cdot C = 0 \quad \leftarrow \text{ARE} \quad \text{követő szab.}$$

$$10 p - p^2 \cdot 10 + 4 = 0$$

$$5 p^2 - 5 p - 2 = 0 \quad \begin{cases} p_1 \\ p_2 \end{cases}$$

$$p = \frac{5 \pm \sqrt{25 - 4 \cdot 10 \cdot (-2)}}{\ldots} = \frac{5 \pm \sqrt{105}}{\ldots} = \frac{5 + \sqrt{105}}{10} > 0$$

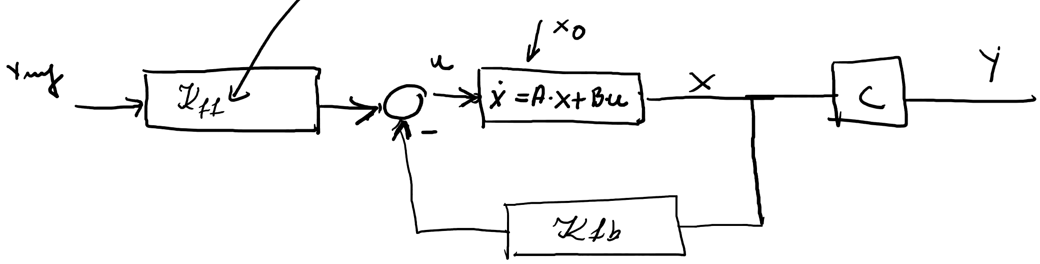
$$P = \frac{5 \pm \sqrt{25-4\cdot 10\cdot(-2)}}{10} = \frac{5 \pm \sqrt{105}}{10} \begin{cases} \frac{5+\sqrt{105}}{10} > 0 \\ \frac{5-\sqrt{105}}{10} < 0 \end{cases}$$

$$K_{fb} = +1 \cdot \frac{5+\sqrt{105}}{10}$$

$$K_{ff} = -\left(1\cdot 1\cdot\left(5 - 1\cdot 1\cdot 1\cdot \frac{5+\sqrt{105}}{10}\right)\right)^{-1} \cdot 2\cdot 1 =$$

$$= -\left(5 - \frac{5+\sqrt{105}}{10}\right)^{-1}\cdot 2 = -\left(\frac{50-5-\sqrt{105}}{10}\right)^{-1}\cdot 2 =$$

$$= -\frac{20}{45-\sqrt{105}} = \frac{20}{\sqrt{105}-45}$$



H1

③

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = u(t) \\ y(t) = x_1(t) + x_2(t) \end{cases}$$

állapot szab

a). $J(u) = \frac{1}{2}\int_0^{t_f} \left(x_1^2(t) + u^2(t)\right) dt$

a1) $t_f \to \infty$ $u(t) = ?$ Riccati e. < klasszikus / Potter

a2) $t_f = 4.$ magyarázat

követő

b). $J(u) = \int_0^{\infty} \left((y(t) - 3t)^2 + 2u^2(t)\right) dt$

Megoldás:

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 1 \end{pmatrix}; \quad D = 0$$

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \quad R = 1$$

$Q = \quad) \quad ; \quad R = 1$

$$J = \frac{1}{2} \underline{x}_{tf}^T F \underline{x}_{tf} + \frac{1}{2} \cdot \int_{t_0}^{t_f} \left(\underline{x}^T(\tau) \cdot Q \cdot \underline{x}(\tau) + \underline{u}^T(\tau) \cdot R \cdot \underline{u}(\tau) \right) \cdot d\tau$$

$$(x_1 \;\; x_2) \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

?

$$u(t) = -R^{-1} \cdot B^T \cdot P \cdot x(t)$$

$$\boxed{-P \cdot A - A^T \cdot P + P \cdot B \cdot R^{-1} \cdot B^T \cdot P - Q = 0} \rightarrow$$

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \qquad p_{12} = p_{21}$$

$$P = \begin{pmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{pmatrix}$$