HF_potter_modszer

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$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad R = 1 \quad Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} A & -B \cdot R^{-1} B^T \\ -Q & -A^T \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\det(\lambda \cdot I - H) = \det \begin{pmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & 0 & 1 \\ 1 & 0 & \lambda & 0 \\ 0 & 0 & 1 & \lambda \end{pmatrix} =$$

$$= \lambda \cdot \det \begin{pmatrix} \lambda & 0 & 1 \\ 0 & \lambda & 0 \\ 0 & 1 & \lambda \end{pmatrix} + 1 \cdot \det \begin{pmatrix} -1 & 0 & 0 \\ \lambda & 0 & 1 \\ 0 & 1 & \lambda \end{pmatrix} = 1 \cdot \lambda^3 + 1 \cdot 1 = \lambda^4 + 1$$

$\lambda^4 + 1 = 0$??

$$\lambda^4 + 1 + 2\lambda^2 - 2\lambda^2 = 0$$

$$(\lambda^2 + 1)^2 - 2\lambda^2 = 0 \Rightarrow (\lambda^2 + 1 - \sqrt{2}\lambda) \cdot (\lambda^2 + 1 + \sqrt{2} \cdot \lambda) = 0$$

$$\lambda^2 - \sqrt{2}\lambda + 1 = 0 \qquad \lambda_{1,2} = \frac{\sqrt{2} \pm \sqrt{2} i}{2} \qquad Re(\lambda) < 0$$

$$\lambda^2 + \sqrt{2}\lambda + 1 = 0 \qquad \boxed{\lambda_{3,4} = \frac{-\sqrt{2} \pm \sqrt{2} i}{2}}_{\text{stabil}} \Rightarrow$$

Saját vektorok:

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{pmatrix} = \lambda_3 \cdot \begin{pmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{pmatrix} \Rightarrow \begin{aligned} v_{12} &= \lambda_3 \cdot v_{11} \\ -v_{14} &= \lambda_3 \cdot v_{12} \\ -v_{11} &= \lambda_3 \cdot v_{13} \\ -v_{13} &= \lambda_3 \cdot v_{14} \end{aligned}$$

– hasonlóan felírható λ_4-re is

$$\Rightarrow \begin{aligned} v_{12} &= \lambda_3 \cdot v_{11} \\ v_{14} &= -\lambda_3 \cdot v_{12} \\ v_{11} &= -\lambda_3 \cdot v_{13} \\ v_{13} &= -\lambda_3 \cdot v_{14} \end{aligned} \longleftrightarrow \begin{aligned} v_{22} &= \lambda_4 \cdot v_{21} \\ v_{24} &= -\lambda_4 \cdot v_{22} \\ v_{21} &= -\lambda_4 \, v_{23} \\ v_{23} &= -\lambda_4 \cdot v_{24} \end{aligned} \qquad (*)$$

$$\begin{pmatrix} v_{13} & v_{23} \end{pmatrix} \cdot \begin{pmatrix} v_{11} & v_{21} \end{pmatrix}^{-1} = \begin{pmatrix} P_{11} & P_{12} \end{pmatrix}$$

$$P = \begin{pmatrix} v_{13} & v_{23} \\ v_{14} & v_{24} \end{pmatrix} \cdot \begin{pmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{pmatrix}^{-1} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

$$\downarrow$$

$$\frac{1}{\underbrace{v_{11} \cdot v_{22} - v_{12} \cdot v_{21}}_{d}} \cdot \begin{pmatrix} v_{22} & -v_{21} \\ -v_{12} & v_{11} \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{v_{13} \cdot v_{22} - v_{23} \cdot v_{12}}{d} & \dfrac{-v_{13} \cdot v_{21} + v_{23} \cdot v_{11}}{d} \\[2ex] \dfrac{v_{14} \cdot v_{22} - v_{24} \cdot v_{12}}{d} & \dfrac{-v_{14} \cdot v_{21} + v_{24} \cdot v_{11}}{d} \end{pmatrix}$$

$$p_{11} = \frac{v_{13} \cdot v_{22} - v_{23} \cdot v_{12}}{v_{11} \cdot v_{22} - v_{12} \cdot v_{21}} \cdot \frac{v_{11} \cdot v_{21}}{v_{11} \cdot v_{21}} = \frac{\frac{v_{13}}{v_{11}} \cdot \frac{v_{22}}{v_{21}} - \frac{v_{23}}{v_{21}} \cdot \frac{v_{12}}{v_{11}}}{\frac{v_{22}}{v_{21}} - \frac{v_{12}}{v_{11}}}$$

$$p_{21} = \frac{v_{14} \cdot v_{22} - v_{24} \cdot v_{12}}{v_{11} \cdot v_{22} - v_{12} \cdot v_{21}} \cdot \frac{v_{22} \cdot v_{12}}{v_{22} \cdot v_{12}} = \frac{\frac{v_{14}}{v_{12}} - \frac{v_{24}}{v_{22}}}{\frac{v_{11}}{v_{12}} - \frac{v_{21}}{v_{22}}}$$

$$p_{22} = \frac{v_{24} \cdot v_{11} - v_{14} \cdot v_{21}}{v_{11} \cdot v_{22} - v_{12} \cdot v_{21}} \cdot \frac{v_{22} \cdot v_{12}}{v_{22} \cdot v_{12}} = \frac{\frac{v_{11}}{v_{12}} \cdot \frac{v_{24}}{v_{22}} - \frac{v_{14}}{v_{12}} \cdot \frac{v_{21}}{v_{22}}}{\frac{v_{11}}{v_{12}} - \frac{v_{21}}{v_{22}}}$$

$$p_{12} = \frac{v_{23} \cdot v_{11} - v_{13} \cdot v_{21}}{v_{11} \cdot v_{22} - v_{12} \cdot v_{21}} \cdot \frac{v_{11} \cdot v_{21}}{v_{11} \cdot v_{21}} = \frac{\frac{v_{23}}{v_{21}} - \frac{v_{13}}{v_{11}}}{\frac{v_{22}}{v_{21}} - \frac{v_{12}}{v_{11}}}$$

$(*) \Rightarrow$

$$\frac{v_{12}}{v_{11}} = -\frac{v_{14}}{v_{12}} = -\frac{v_{11}}{v_{13}} = -\frac{v_{13}}{v_{14}} = \lambda_3$$

$$\frac{v_{22}}{v_{21}} = -\frac{v_{24}}{v_{22}} = \frac{-v_{21}}{v_{23}} = \frac{-v_{23}}{v_{24}} = \lambda_4$$

3 2 $\lambda_3 + \lambda_4$

$$P_{11} = \frac{-\lambda_4/\lambda_3 - \left(\frac{-1}{\lambda_4}\right)\cdot\lambda_3}{\lambda_4 - \lambda_3} = \frac{\lambda_3^3 - \lambda_4^2}{\lambda_4\cdot\lambda_3(\lambda_4+\lambda_3)} = -\frac{\lambda_3+\lambda_4}{\lambda_3+\lambda_4}$$

$$P_{12} = \frac{-\frac{1}{\lambda_4}+\frac{1}{\lambda_3}}{\lambda_4-\lambda_3} = \frac{1}{\lambda_3\cdot\lambda_4} \quad ; P_{21} = \lambda_3\cdot\lambda_4$$

$$P_{22} = \frac{-\frac{\lambda_4}{\lambda_3} + \frac{\lambda_3}{\lambda_4}}{\frac{1}{\lambda_3} - \frac{1}{\lambda_4}} = -\lambda_4 - \lambda_3$$

$$P_{11} = \sqrt{2} \quad ; P_{22} = \sqrt{2} \quad ; \quad P_{12} = 1 \quad ; \quad P_{21} = 1$$

$$P = \begin{pmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{pmatrix}$$