Vizsga Optimális irányítások

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1. b.

$x(t) = x(t) + u(t)$ $\qquad$ $z(t) = x(t)$

$u(t) = -(k_1 \quad k_2) \cdot \begin{pmatrix} x(t) \\ z(t) \end{pmatrix}$

$J(u) = \int_0^\infty \left\{ (x(t) \quad z(t)) \begin{pmatrix} g & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x(t) \\ z(t) \end{pmatrix} + u^2(t) \right\} dt$

$g \geq 0$ $\qquad$ $y(t) = x(t)$

Megoldás

$\begin{matrix} x(t) = x(t) + u(t) \\ z(t) = x(t) \end{matrix} \Rightarrow A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$Q_c = (B \quad AB) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \Rightarrow \text{rang } Q_c = 2$

$J(u) = \int g x^2(t) + z^2(t) + u^2(t) \, dt$

$\Rightarrow Q = \begin{pmatrix} g & 0 \\ 0 & 1 \end{pmatrix} ; \quad R = 1$

$$\overset{P}{\begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix}} \cdot \overset{A}{\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}} - \overset{A^T}{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}} \cdot \overset{P}{\begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix}} + \overset{P}{\begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix}}$$

$$\cdot \overset{B}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} \cdot \overset{B^T}{\begin{pmatrix} 1 & 0 \end{pmatrix}} \cdot \overset{P}{\begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix}} - \overset{Q}{\begin{pmatrix} g & 0 \\ 0 & 1 \end{pmatrix}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} P_{11} + P_{12} & 0 \\ P_{12} + P_{22} & 0 \end{pmatrix} + \begin{pmatrix} P_{11} + P_{12} & P_{12} + P_{22} \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} P_{11} \\ P_{12} \end{pmatrix}$$

$$\cdot \begin{pmatrix} P_{11} & P_{12} \end{pmatrix} + \begin{pmatrix} g & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

- $2(P_{11} + P_{12}) - P_{11}^2 + g = 0$

  $P_{12} + P_{22} - P_{11} P_{12} = 0$

  $-P_{12}^2 + 1 = 0 \Rightarrow P_{12} = \pm 1$

$\underline{P_{12} = 1}$

$2(P_{11} + 1) - P_{11}^2 + g = 0$

$P_{11}^2 - 2P_{11} - g - 2 = 0$

$$P_{11} = \frac{2 \pm \sqrt{4 + 4g + 8}}{2} = 1 \pm \sqrt{g + 3}$$

$h_9 \quad 1 + P_{22} - P_{11} = 0 \Rightarrow P_{22} = 1 \pm \sqrt{g+3} - 1$

$$= \sqrt{g+3}$$

$\underline{p_{12} = -1}$

$$2(p_{11}-1) - p_{11}^2 + g = 0$$

$$p_{11}^2 - 2p_{11} - g + 2 = 0$$

$$p_{11} = \frac{2 \pm \sqrt{4+4g-8}}{2} \Rightarrow 1 \pm \sqrt{g-1}$$

ha $\quad -1 + p_{22} + p_{11} = 0 \Rightarrow p_{22} = -1\sqrt{g-1}+1$

$$= \sqrt{g+3}$$

$P = \begin{pmatrix} 1 \pm \sqrt{g+3} & 1 \\ 1 & \pm\sqrt{g+3} \end{pmatrix}$ illetve $P = \begin{pmatrix} 1 \pm \sqrt{g-1} & -1 \\ -1 & \sqrt{g-1} \end{pmatrix}$

Megvizsgálva a Riccati mátrix előjelét kapjuk, mint lehetséges megoldás.

$P = \begin{pmatrix} 1-\sqrt{g+3} & 1 \\ 1 & -\sqrt{g+3} \end{pmatrix}$ $\quad$ det $1-\sqrt{g+3} < 0$

tehát nem megoldás

$P = \begin{pmatrix} 1-\sqrt{g-1} & -1 \\ -1 & \sqrt{g-1} \end{pmatrix}$ $\quad$ det $1-\sqrt{g-1} > 0$ ha $g < 0$

$(1-\sqrt{g-1})\, g - 1 + 1 = \sqrt{g-1}\, g < 0$

tehát nem megoldás

$$P = \begin{pmatrix} 1+\sqrt{g+3} & 1 \\ 1 & \sqrt{g+3} \end{pmatrix}$$

det

$1+\sqrt{g+3} > 0$

$(1+\sqrt{g+3})\sqrt{g+3} - 1$

$= g+2+\sqrt{g+3} > 0$

$$P = \begin{pmatrix} 1+\sqrt{g-1} & -1 \\ -1 & -\sqrt{g-1} \end{pmatrix}$$

$1+\sqrt{g-1} > 0$

$(-1+\sqrt{g-1})\sqrt{g-1} - 1$

$= -g+1-\sqrt{g-1} - 1 < 0$

nem megoldás

tehát csak $P = \begin{pmatrix} 1+\sqrt{g+3} & 1 \\ 1 & \sqrt{g+3} \end{pmatrix}$ Riccati

mátrix lehet elfogadható, amivel

$$u(t) = -(1 \quad 0)\begin{pmatrix} 1+\sqrt{g+3} & 1 \\ 1 & \sqrt{g+3} \end{pmatrix}\begin{pmatrix} x(t) \\ z(t) \end{pmatrix}$$

$$= -(1+\sqrt{g+3})\begin{pmatrix} x(t) \\ z(t) \end{pmatrix}$$

$$\begin{pmatrix} x(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x(t) \\ z(t) \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}(1+\sqrt{g+3} \quad 1)\begin{pmatrix} x(t) \\ z(t) \end{pmatrix}$$

$$= \begin{pmatrix} -\sqrt{g+3} & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x(t) \\ z(t) \end{pmatrix}$$

$$\det\left(s\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -\sqrt{g+3} & -1 \\ 1 & 0 \end{pmatrix} \right)$$

$$\det\begin{vmatrix} s+\sqrt{g+3} & 1 \\ -1 & s \end{vmatrix} = s^2 + s\sqrt{g+3} + 1 = 0$$

$$s_{1,2} = \frac{-\sqrt{g+3} \pm \sqrt{g+3-4}}{2} = \frac{-\sqrt{g+3} \pm \sqrt{g-1}}{2}$$

1. a

$\dot{x}(t) = x(t) + u(t)$, $x_0 = 1$

$J(u) = \int_0^{\infty} gx^2 + r u^2 \, dt$

$A = 1$ $B = 1$ $Q = g$ $R = r$

$P \cdot A + A^T P - P \cdot B \cdot R^{-1} \cdot B^T \cdot P \cdot Q = 0$

$P + P - P \cdot 1 \cdot 1/r \cdot 1 \cdot P + g = 0$

$4P - P^2/r + g = 0$

$P_{1,2} = \frac{-2 \pm \sqrt{4(1 + g/r)}}{-2/r} = \frac{-2 \pm \sqrt{4 + 4g/r}}{-2/r}$

$= \frac{-1 \pm \sqrt{1 + g/r}}{-1/r}$

2.

$$x_{k+1} = 3x(k) + 2u(k)$$

$$J(u) = \frac{1}{2}\sum_{k=0}^{\infty} (x^2(k) + u^2(k))$$

$$u(k) = -(R + \Gamma^T \cdot P_{k+1} \cdot \Gamma)^{-1} \cdot \Gamma^T \cdot P_{k+1} \cdot \phi \cdot x_k$$

$Ad = \phi = 3$; $Bd = \Gamma = 1$; $Cd = 1$;

$R = 1$; $Q = 1$

$$P = Ad^T \cdot P \cdot Ad + Q - Ad^T \cdot P \cdot Bd \cdot (R + Bd^T \cdot P \cdot Bd)^{-1}$$
$$\cdot Bd^T \cdot P \cdot Ad$$

$$P = \underbrace{3 \cdot P \cdot 3}_{6P} + \underbrace{1}_{1} - \underbrace{3 \cdot P \cdot 1}_{3P} \cdot (1 + \underbrace{1 \cdot P \cdot 1}_{P})^{-1} \cdot \underbrace{1 \cdot P \cdot 3}_{3P}$$

$$P = 9P + 1 - \frac{6P^2}{1+P}$$

$9P + 1 - 6P^2/1+P = 0$

$(9P+1)(1+P) - 6P^2 = 0$

$-6P^2 + 9P^2 + 6P + 1 = 0 \quad / \cdot (-1)$

~~$P^2 \; 6P + 1 = 2 \quad P = 1/6$~~

$A_d = 3$, $B_d = 0$, $C_d = 1$ $Q = 1, R = 1$

$P = 3 \cdot P \cdot 3 + 1 - 3 \cdot P \cdot 0 \cdot (1 + 0 \cdot P \cdot 0) \cdot 0 \cdot P \cdot 3$

$P = 9P + 1 - 1^{-1}$ $\Rightarrow P = \frac{1}{9}$

$9P^2 - 6P^2 + 6P + 1 = 0$

$3P^2 + 6P + 1 = 0$

$P_{1,2} = \frac{6 \pm \sqrt{36 - 4 \cdot 3 \cdot 1}}{2} = \frac{6 \pm \sqrt{24}}{2} =$

$= \begin{cases} 5,44 \\ 0,24 \end{cases}$

Potter

$$H = \begin{bmatrix} 6 + 1 \cdot 1^{-1} \cdot 1 \cdot 1/6 \cdot 1 & -1 \cdot 1^{-1} \cdot 1 \cdot 1/6 \\ -1/6 \cdot 1 & 1/6 \end{bmatrix}$$

$$= \begin{bmatrix} 6,16 & -0,16 \\ -1/6 & 1/6 \end{bmatrix}$$

$$\lambda \cdot I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$\det(\lambda \cdot I - H) = 0$

$$\det \begin{bmatrix} \lambda - 6,16 & \cancel{1/6} - 0,16 \\ -1/6 & \lambda - 1/6 \end{bmatrix}$$

$$= (\lambda - 6,16) \cdot (\lambda - 1,6) - 0,1 \cdot 0,2$$

$$= \lambda^2 - 6,16\lambda - 1,6\lambda - 0,02 = 0$$

$$= \lambda^2 - 4,56\lambda - 0,02 = 0$$

$$\lambda_{1,2} = \frac{4,56 \pm \sqrt{20,79 - 0,09}}{2}$$

$$\frac{4,56 + 9,02}{2} > 0$$

$$\frac{4,56 - 9,02}{2} < 0$$

3. $\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$ $\quad |x_1(0)| \leq 2$

$|x_2(0)| = 0$

$|u(t)| \leq 1$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$

$$H(x, u, p, t) = 1 + (p_1(t) \quad p_2(t)) \cdot \begin{pmatrix} x_2(t) \\ -x_1(t) + u(t) \end{pmatrix}$$

$$= 1 + p_1(t) \cdot x_2(t) - p_2(t)\, x_1(t) + p_2(t) \quad u(t)$$

$\frac{\partial H}{\partial u} = 0 \Rightarrow p_2(t) = 0 \rightarrow |u(t)| \rightarrow \{+1, -1\}$ $u^* = \{^{1}_{-1}$

$\frac{\partial H}{\partial x_1} = -p_1(t)$

$\Rightarrow \begin{cases} -p_1(t) = p_2(t) \\ -p_2(t) = p_1(t) \end{cases} \Rightarrow \begin{pmatrix} \dot{p}_1(t) \\ \dot{p}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

$\frac{\partial H}{\partial x_2} = -p_2(t)$

$$\left.\begin{matrix} y_1 = x_1 \\ y_2 = x_2 \end{matrix}\right\} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} u$$

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Aut III