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The Relation between Russia's Economic Growth and Oil Price

A VAR Approach

Time Series Analysis – Project

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1. Introduction

The current geopolitical destabilization which started with the war in Ukraine has turned the eyes of the world on Russia once again. In retaliation to the aggression, the so-called "Western" countries, in particular the European Union and the G7 countries, have imposed several economic sanctions on the Russian economy, ranging from its exclusion from the SWIFT payment system to the import of some strategic products for the functioning of its economy, inter-alia, on its oil. Gaidar (2007) addresses the perils of the Russian high reliance on commodities, especially oil, as it was one of the central motives of joy and disgrace of the defunct Soviet Union.

Rautava (2002), Katsuya (2010) as well as Benedictow, Fjærtoft and Løfsnæs, Fjærtoft and Løfsnæs (2010), find that the gross domestic product (GDP) of Russia and the oil price per barrel are correlated and that the proceeds from oil sales is a important factor in its monetary policy and in stabilizing the ruble. The time gap between the studies demonstrates the difficulty that the Russian economy has in transitioning to a solid economy based on value-added products.Kakanov, Blöchliger and Demmou (2018) study that economies with abundant resources, such as Russia, generally show lower growth than those economies where resources are scarce, this fact is called the "Resourse course”. This paper seeks to assess by means of a vector autoregressive model (VAR(p)) of order p of the type

$$\mathbf{y}_{t(kx1)} = \mathbf{\Phi}_{0(kx1)} + \sum_{i=1}^{p} \mathbf{\Phi}_{i(kxk)} \mathbf{y}_{t-i(kx1)} + \boldsymbol{\varepsilon}_{t(kx1)}, \text{ with } \boldsymbol{\varepsilon} \sim \mathbb{WN}\left(0, \mathbf{\Omega}\right), \quad (1.1)$$

whether and how deeply the direction of the Russian economy remains tied to oil price fluctuations and contributes to the literature insofar as it uses the most current data available for its analysis. In accordance to the literature, it is also expected that Russia’s economy remains closely tied to fluctuation in the oil price. For more in-depth information, the reader is encouraged to look at the works cited.

2. Data Description and Unit Root Test

As the indicator that measures the market value of all goods and services produced by a country within a period of time, it is natural that the GDP of the Russian Federation to be used as a proxy for its macroeconomic development. Quarterly Data about the GDP of the Russian Federation in current quotation were obtained from the Russian Federal Statistical Office (Rosstat, 2022) for the period $1995Q_1 - 2021Q_4$. Following Ruatava (2002) and Katsuya (2010), the North Sea Brent oil spot price per barrel, retrived from EIA (2022), was chosen for the analysis of the development in the oil price, once the Russian's Urals crude oil is expected to be closely related to it, as the major buyers of Russia's oil are located in Europe. Further, as there is a lag of about 4 months between the spot oil price and the actual contract price, a four quarter moving average for the oil price series is used. In order to maintain consistency in the monetary value of measurement, the price of crude oil has been converted to rubles using the current exchange rate for each quarter, retrived from OECD-FRED (2022). For analysis purposes, both the GDP and the ruble oil price were corrected for inflation using the GDP-deflator obtained from the Rosstat with 2017 as the base year. The series have been used in their logarithmic forms and will be referred to $log\,(RGDP)$ and $log\,(OIL)$. An illustration of the series is given in Fig. 1.

Time series demands a certain amount of care. Statistical inference is usually practiced on asymptotic theory, the basis of which is the convergence in probability and distribution of the sampling distribution of random variables to their true population values. In the case of time series the idea of convergence is subsumed by the concept of stationarity (Johnston and DiNardo, 2007: 55-57). Stationarity in strict sense requires that the joint distribution of a stochastic process $\{y_t\}$ to be the same at any point in time. For the purposes of time-series analysis the so called covariance stationarity suffices, meaning that $\{y_t\}$ has constant mean, constant variance and that the covariance between two observations y_t and y_{t+h} depends only on the distance between these observations but not on time (Wooldridge, 2009: 381). Given an autoregressive process AR(p) $\Phi\,(\mathbb{L})\,y_t = \varepsilon_t$, stationarity requires that the roots of the polynomial $\Phi\,(\mathbb{L}) = [1 - \phi_1 z - \cdots - \phi_p z^p]$ to lie outside the unit circle. In the simplest case of an AR(1) process stationarity condition requires that $\left|\phi\right| < 1$. Dickey and Fuller (1979) derived a test statistic which carries their names for testing the unit root of the polynomial, whose null hypothesis (H_0) is the of non-stationarity of the process.. Kwiatkowski, Phillips and Schmidt, Phillips and Schmidt (1991: 1-8) criticize the low performance of the (augmented)-Dickey-Fuller test (ADF) for coefficients close to unity, leading to type II error: an inherent problem of how statistical tests are conducted, in which the zero hypothesis is only rejected when there is strong evidence for the alternative hypothesis. To address the problem, the authors proposed a test consisting in decomposing the series as a combination of a random walk – $r_t = r_{t-1} + \upsilon_t$, with $\upsilon_t \overset{i.i.d.}{\sim} \mathbb{WN}\left(0, \sigma_\upsilon^2\right)$ – a time trend (ξt) and a white noise ε_t and whose $H_0 : \sigma_\upsilon^2 = 0$ (stationarity of the process) is tested by means of a Lagrange-multiplier test (L_M).

Tab. 1 summarizes the results of the two tests applied to the series used in this project, as well as the critical values under $\alpha = 5\%$ significance level. Both tests were conducted with the inclusion of 9 lags in the model, chosen based on Bayesian Information criterion (BIC). The ADF test fails to reject non-stationarity for both series, while the KPSS test rejects stationarity for $log\,(RGDP)$ but not $log\,(OIL)$.

3. Testing for Cointegration: Engle-Granger 2 Steps Procedure and Johansen Test

In the case of non-stationary series, one of the methods to be used is the induction of stationarity by taking first differences. In the literature the order of integration of a series alludes to the number of differences (d) required to make a series stationary and is denoted $\mathbf{I}\,(d)$. Generally, a linear combination of two series (y_t, x_t) of different integration order (d, b) produces a new series $z_t\,(y_t, x_t) \sim \mathbf{I}\,(\max\{d, b\})$. However, in some specific cases, there exists a linear combination of the variables $z_t = \alpha y_t - \beta x_t$, which issues a series with the order of integration $\mathbf{I}\,(d-b)$, $b > 0$. In such a case, the series are said to be *cointegrated*, and the economic interpretation of the cointegrating relation is of the a long-run equilibrium between the variables, such that $\mathbb{E}\,[z_t] = 0$ and under the null hypothesis inferences based on the standard statistic tests apply (Engle and Granger, 1987: 251-259).[1] Engle and Granger (1987) proposed that since the residuals are a linear combination of the variables in the model, if a cointegrating relation exists, it will be reflected in the residuals and these must be stationary. The procedure consists in estimating the long-term equilibrium (z_t) by means of OLS and testing the stationarity of its residuals. The problem with the Engle-Granger procedure is its inability to check for the number of cointegrating relationships as it only

[1]This is only the case when the variables are integrated of the same order. Otherwise the variables are still cointegrated but $\mathbb{E}\,[z_t] \neq 0$.

tests for a single cointegration relation and, in the case of more than two variables, multiple cointegration relations might exist. As shown by Engle-Granger's Theorem, for each cointegrating relation there exists an error correction representation. Its vectorized version is called vector autoregressive error correction model VECM$(p-1)$: $\Delta \mathbf{y}_t = \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{y}_{t-i} + \mathbf{\Pi} \mathbf{y}_{t-1}$.

In the case of a VAR as in equation 1.1, the assumption of a static equilibrium vector $\bar{\mathbf{y}}$ gives $\mathbf{\Pi}\bar{\boldsymbol{y}} = \mathbf{\Phi}_0$, where $\mathbf{\Pi} = [\mathbf{I} - \sum_{i=1}^{p} \mathbf{\Phi}_p]$ and the uniqueness of a solution to $\bar{\mathbf{y}}$ will depend on the invertibility of $\mathbf{\Pi}$. If $\mathbf{\Pi}$ has full rank, it is non-singular and invertible, hence a static equilibrium for the system exists (the VAR process is stationary) and the model can be estimated in level (Johnston and DiNardo, 2007: 288-289). Given that $\mathbf{\Pi}$ is a $(k\text{x}k)$– matrix, its null space $N(\mathbf{\Pi})$ will contain $k - r_\pi$ basis $\bar{\mathbf{y}}$, whose linear combinations span the whole $N(\mathbf{\Pi})$ (Strang 2009: 132-139), indicating the number of cointegration relations . In this case, the best model is a VECM$(p-1)$, but when $r_\pi = 0$, there is no linear independence between the columns of $\mathbf{\Pi}$ and a solution to $\bar{\mathbf{y}}$ does not exist, the system is not stationary and stationarity must be induced . Johansen $(1987, 1991)$ created two methods to test for the rank of $\mathbf{\Pi}$ called maxeigenvalues and trace-statistics and has been applied along with the Engel-Granger-2-Steps method to variables in the present project.

Tab.2 summarizes the results for the ADF test on the stationarity of the residuals of the regression of $log\,(RGDP)$ on $log\,(OIL)$. The hypothesis of non-stationarity could not be rejected and becomes clear by graphic inspection of Fig. 2; such a result is corroborated by the Johansen maxeigenvalue and trace-statistic tests, which do not reject that $r_\pi = 0$. Accordingly, under $\alpha = 5\%$ a cointegrating relation between $log\,(RGDP)$ and $log\,(OIL)$ could not be found and their stationarity has been induced by first differencing and proofed by means of an ADF test (see Tab. 1). As the variables were in logs, their first differences interpretation is of their growth rates (see Fig. 1), which will be referred to $dRGDP$ and $dOIL$ respectively.

4. Model Estimation and Diagnosis Test

One of the biggest challenges when building a VAR model is determining its order, that is, the number of lags to include in the model. Three approaches are proposed in the literature:1) start with a a-priori chosen number of lags and, by means of a Likelihood

Ratio-test (L_R), discard those that do not add value to the model's explanatory power ; 2) Find the model that minimizes a given information criterion; 3) resort to economic theory. The business cycle theory postulates that the economy has non-periodic phases of contraction and expansion, so that the state of the economy in the past helps to explain its present and future developments. Kilian and Zhou (2020: 36-37), however, writes about the difficulty of finding a correct index that represents the cycle in the oil market. By the very atheoretical nature of the VAR model, the method chosen was to let the data speak for itself and to rely on the L_R test and on the BIC, since both yield similar results. By applying a L_R-test to the VAR system starting with a maximum number of 10 lags, the model could be reduced to 4, this also being the number of lags that minimizes the BIC. Further, Brooks (2019: 315) suggests as Rule of thumbs base the number of lags on the data frequency. The conclusion was the estimation of a VAR(4) by ordinary least squares (OLS) as given in Eq. (4.1)

$$\begin{bmatrix} dOIL \\ dRGDP \end{bmatrix}_t = \begin{bmatrix} \phi_{1,0} \\ \phi_{2,0} \end{bmatrix} + \sum_{i=1}^{4} \begin{bmatrix} \phi_i^{11} & \phi_i^{12} \\ \phi_i^{21} & \phi_i^{22} \end{bmatrix} \begin{bmatrix} dOIL \\ dRGDP \end{bmatrix}_{t-i} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}_t . \tag{4.1}$$

The OLS-estimator is said to be the best unbiased linear estimator when the so-called Markov Assumptions are fulfilled. To check whether the assumptions are met, diagnostic and specification tests regarding the stability of parameters, the distribution and independency of innovations, as well as the use of the correct functional form were applied to the data. The assumption of the correct functional form, tested by means of a Ramsey Reset Test, as well as of the normality of the residuals, tested by means of Jarque-Bera test, of the homoscedasticity assessed by means of ARCH-Effects - LM and White Test and of the stability of the parameters, tested by recursive regression CU-SUM were violeted (see Fig. 3 and Tab. 4 for further information)

In the case of violations of the Markov's assumptions, in special the independence of $\varepsilon \overset{i.i.d.}{\sim} \mathbb{N}(0, \sigma^2\mathbf{I})$, statistic tests (e.g. t-test, F-test, W-test), do not follow their standad distributions invalidating inferences made based on them. The solution to the problem was to estimate the confidence intervals for the estimators $\phi \forall i$ by bootstrapping. Accoding to the confidence intervals none of the lags concerning the $dRGDP$ helps to explain at 5% significance level, the current $dOIL$; on the orther hand, the confidence intervals for ϕ_i^{21} for $i = 1, 3$ do not include zero, such those coefficient helps in explaining current $dOIL$. Corcerning $dRGDP$, all of its own 4 lags show explanatory power on its current state, whereas only the second lag of $dOIL$ contributes to explaining today's $dRGDP$. An in-sample prediction is given by Fig. 4.[2]

5. Interpretation of the Estimation

Due to the interdependence of the equations in the VAR system and the large number of estimated coefficients, an interpretation of each coefficient is infeasible.In order to help interpret the results three approaches are used: Granger Causality test, (cumulative) impulse response function and variance decomposition. Granger (1969: 428) defines causality as: "[...] we say that y_t is causing x_t if we are better able to predict x_t using all available information than if the information apart from y_t had been used."[3] Specifically, given Eq. (4.1) $dOIL$ will be said to granger-cause $dRGDP$ if, jointly, $\phi_i^{11} \neq 0 \, \forall i$. In the case of only two variables, the significance of the parameters can be tested by a Wald-Test (W) applied separately to each equation in the system (Lütkepohl and Krätzig, 2004:145-147). When the Markov-assumptions are met, the $W \sim \chi^2\,(4)$. Unfortunately the some assumptions were violated and the empirical distribution of the test needed to be derived by bootstrapping, and inferece was made based on the computed Monte-Carlo p-value (p_{mc}). The p_{mc} computed for each equation were 0.032 and 7.5x10^{-5} respectively, resulting in the rejection of the null-hypothesis of common insignificance of the coefficients in both cases, so that under 95% probability both variables are endogenous and granger-cause each other.

Fig. 5 plots the cumulative effect of a positive and orthogonalized shock of a standard deviation on $dOIL$, computed from a cholesky decomposition of the variance matrix in Eq. (C.2) . After a shock the oil price per barrel takes about 12 quarters to stabilize. Its effect on $dRGDP$ is mostly positive in the first four quarters when it begins to diminish. The long-run effects of the shock leads to 10.95% and 2.75% increase in the level of the oil price and real GDP. In case of a positive and orthogonalized shock of one standard deviation to $dRGDP$, the long-run effects lead to an increase of 2.45% to Russia's real GDP and to 2.34% decrease in the oil price per barrel, however, as

[2]The estimated confidence intervals, the values for the estimated coefficients as well as information about the bootstrapping method is found in Appendix C and D respectively.

[3]The definition proposed by Granger does not necessarily represent the causality of one variable over the other, and the proposed test is informative only in the sense of proving the existence of feedback between the variables.

illustrated in Fig. 5 the error interval does not exclude the possibility of the cumulative effect of a shock to $dRGDP$ on $dOIL$ to be zero. Such a statement is corroborated by the decomposition of the variance of the forecast error (see Fig. 6), where can be seen that a shock to $dRGDP$ contributes very little in explaining the variation in the $dOIL$ forecast error, whereas the variance of the forecast error of $dRGDP$, shows considerable participation coming from a shock to $dOIL$. Nevertheless, the statement that a shock to $dRGDP$ has no effect on $dOIL$ goes against the findings of the Granger Causality test.

6. Conclusion

In spite of its simplicity, the present analysis was able to show through a VAR model, in accordance with the literature, that the growth of the Russian economy remains closely related to movements in the price of oil, since a shock of one standard deviation to the growth rate of oil price leads to an accumulated increase of 2.97% in the Russian RGDP. Whether, however, the shock to the Russian economy has significant effects on the price of oil is uncertain. Theoretically, commodity prices should be formed in the market and not by movements in the Russian economy, which goes against the results of the grange-causality test. Again, it must be emphasized that the results must be seen with caution, due to the violation in the Markov Assumptions. Furthermore, as could be verified by means of appropriated tests, it was found that the series $log\left(RGDP\right)$ and $log\left(OIL\right)$ are integrated $\mathbf{I}\left(1\right)$ and that a common stochastic trend between the variables is non-existent. The VAR model is commonly used to predict future values for the variables, such forecast was not implemented in the project, since the coronavirus pandemic as well as the Russian invasion in Ukraine could not have been predicted by the model. For future research, I suggest analyzing whether such a disruption led to significant changes in the relationship between economic growth and oil prices on the Russian economy.

Appendices

A. Figures

Fig. 1: Russia's real GDP and oil price per barrel in Level and as Growth Rate. The picture on the upper-left illustrates Russia's $log\,(RGDP)$ and the one on the upper-right $log\,(Oil)$, both in Ruble with 2017 as the base year. The centered figure illustrates both series after first differencing.

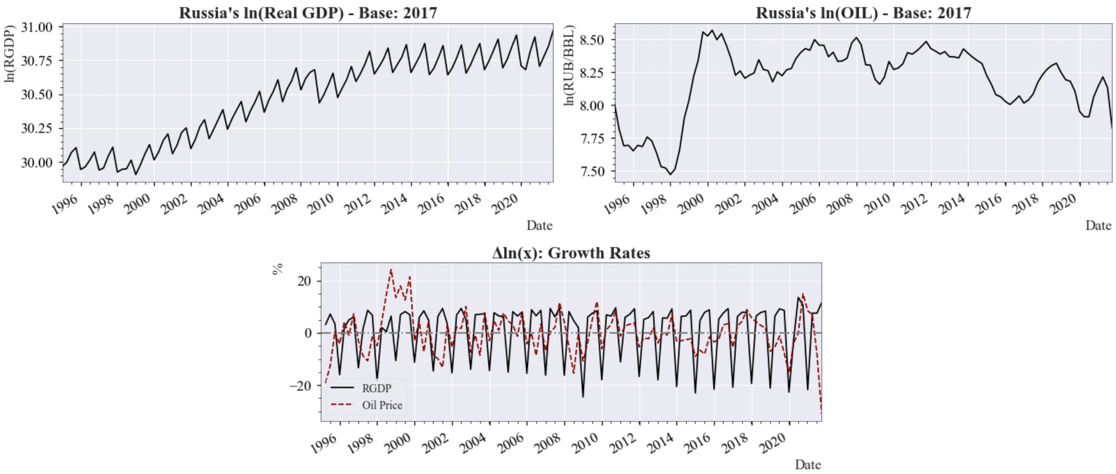


Fig. 2: EG-1-Step Residuals. The figure below illustrates the non-stationarity of the residuals of the long-term regression $dRGDP \sim dOIL$.

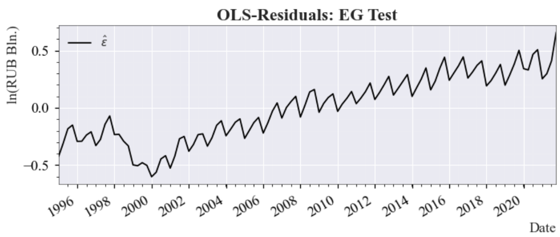


Fig. 3: Cumulative Sum. The Figure traces out the path of the cumulative sum W_t of the one-step ahead for cast error with $dRGDP$ on the left and $dOIL$ on the right. As shown by the left figure, $dRGDP$ has crossed the boundaries more than once.

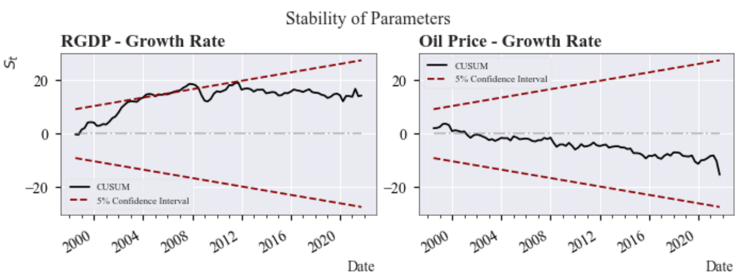


Fig. 4: In-Sample Prediction. The figure on the left (right) plots the in-sample prediction for the $dOIL$ ($dRGDP$) against the observed values.

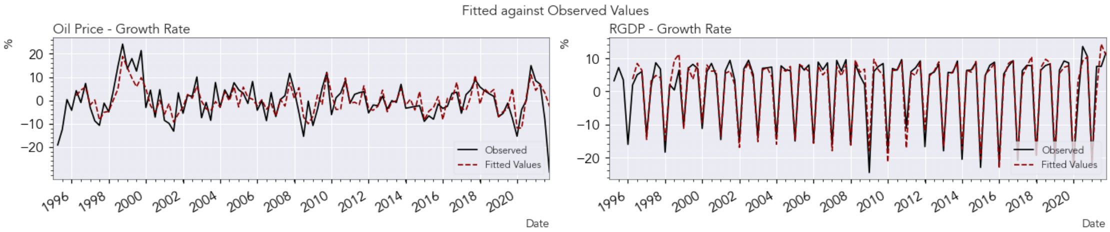
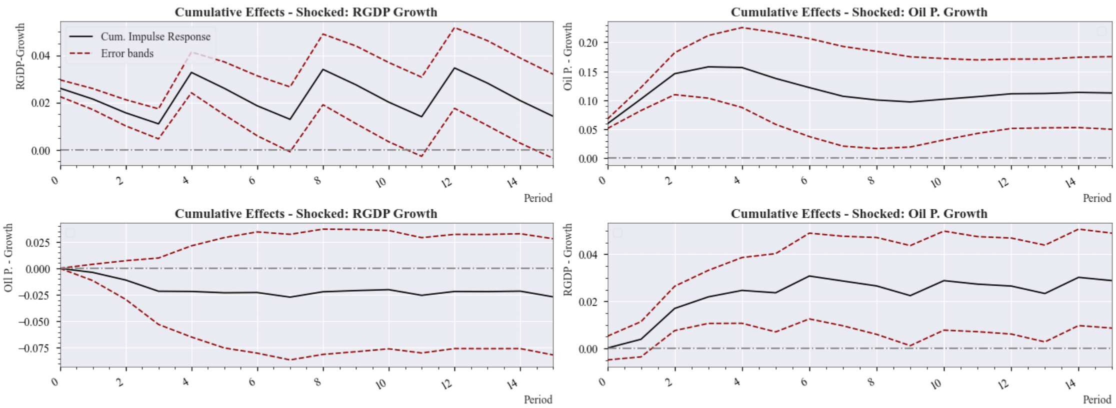
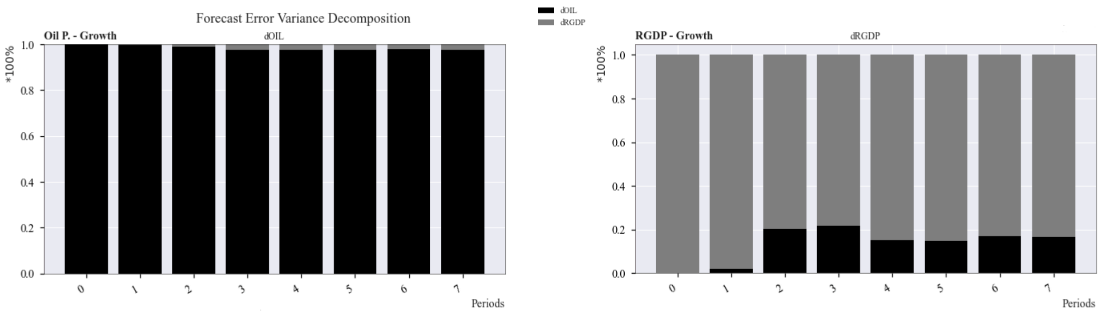


Fig. 5: Cumulative Impulse Response Function. The pictures on the left (right) traces out the cumulative effects of a orthogonalized shock of one standard deviation to $dRGDP$ ($dOIL$) on both endogenous variables. The accompanying table contains the plotted values and the long-run effects on its last row.



Impulse Response – Cumulative Effects				
Schoked:	$dRGDP$		$dOIL$	
Periods/ Response	$dOIL$	$dRGDP$	$dOIL$	$dRGDP$
0	0	0.0260	0.0595	0.0002
1	−0.0039	0.0214	0.1023	0.0039
2	−0.0110	0.0156	0.1459	0.0170
3	−0.0216	0.0109	0.1577	0.0219
4	−0.0218	0.0327	0.1565	0.0246
⋮	⋮	⋮	⋮	⋮
14	−0.0215	0.0208	0.1136	0.0302
15	−0.0269	0.0141	0.1127	0.0288
Long-Run effects:	−0.0234	0.0245	0.1095	0.0275

Fig. 6: Variance decomposition of forecast errors. The left (right) figure plots the results of the variance decomposition for $dOIL$ ($dRGDP$) up to 7 periods-ahead prediction. The table accompanying the figures contain the values plotted.



Forecast Errors - Variance Decomposition				
for:	$dRGDP$		$dOIL$	
Periods	$dOIL$	$dRGDP$	$dOIL$	$dRGDP$
0	$37 * 10^{-5}$	0.9999	1.0000	0.0000
1	0.0196	0.9803	0.9972	0.0028
2	0.2018	0.7982	0.9910	0.0090
3	0.2171	0.7828	0.9766	0.0234
4	0.1500	0.8500	0.9766	0.0234
5	0.1460	0.8540	0.9774	0.0226
6	0.1680	0.8319	0.9781	0.0219
7	0.1669	0.8331	0.9765	0.0235

B. Tests: Results and Description

B.1. Tests for Stationarity: ADF and KPSS

Tab. 1: ADF and KPSS – stationarity tests results. The table contains the results of the ADF and KPSS on the series in log-level and on the series in first-differences.

ADF-Test H_0: Non-Stationarity $\sim \mathbb{DF}(\mu, \sigma^2)$				
	$\log(RGDP)$	$\log(OIL)$	$dRGDP$	$dOIL$
Lags:	9	9	4	4
Test Statistic (t_{stat}):	-1.5086	-2.0302	-4.034	-5.216
Crit. Values (DF) $\alpha = 0.05$:	-2.89			
Reject H_0:	False	False	True	True
KPSS-Test H_0: Stationarity $\sim \chi^2(q)$ / Brownian Bridge				
	$\log(RGDP)$	$\log(OIL)$	$dRGDP$	$dOIL$
Lags:	9	9	N.A.	N.A.
Test Statistic (L_M):	1.1231	0.272	N.A.	N.A.
Crit. Values (DF) $\alpha = 0.05$:	0.463			
Reject H_0:	True	False	N.A.	N.A.

Augmented Dickey Fuller Test: The augmented Dickey Fuller test is derived by reparameterizing the autoregressive process. For example, given an AR(3) process

$$y_t = m + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \varepsilon_t \tag{B.1}$$

the reparametrization issues an error correction model of the type

$$\Delta y_t = m - [1 - \alpha_1 - \alpha_2 - \alpha_3]\, y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \varepsilon_t \tag{B.2}$$

$$\Delta y_t = m + \alpha y_{t-1} + \sum_{i=1}^{2} \Delta y_{t-2} + \varepsilon_t. \tag{B.3}$$

whose hypothesis is that $H_0 : \alpha = 0$ (the process is not stationary) and is tested by means of a t-test. The test-statistic does not follow a standard distribution since, under H_0, $\{y_t\}$ is not stationary and the t-statistic no longer follows the t-distribution, but the Dickey-Fuller (DF) distribution. If the t-statistic is larger in absolute value than the critical value of the $\mathbb{DF}$-distribution, H_0 is to be rejected.

B.2. Cointegration Tests: EG and Johansen (Maxeigen and Trace statistic)

Tab. 2: Results ADF-Test on EG-Residuals. The table contains the results of ADF test on the stationarity of the residuals of the long-term-relationship regression (1. Step: EG).

ADF-Test on Residuals $H_0 : \gamma = 0$	
Lags:	8
Test-statistic (ADF)	0.0504
Critical value $\alpha = 5\%$	-1.944
Reject H_0:	True

Engle-Granger 2 Steps Procedure The test consisted in estimating the long-term equilibrium $log\,(RGDP)_t = \alpha + \beta log\,(OIL)_t + \varepsilon_t$ and in assessing the stationarity of the residuals $\hat{\varepsilon}$ by means of an ADF test.

$$
\begin{aligned}
H_0 : \gamma &= 0 \text{ (residuals are non-stationary)} && \text{(B.4)}\\
\Delta\hat{\varepsilon}_t &= \sum_{i=1}^{7} \beta_i \Delta\hat{\varepsilon}_{t-i} + \gamma\hat{\varepsilon}_{t-i} + \eta_t && \text{(B.5)}\\
t &= \frac{\hat{\gamma} - \gamma_0}{SE\,(\hat{\gamma})} \sim \mathbb{DF}\left(\mu, \sigma^2\right) && \text{(B.6)}
\end{aligned}
$$

where $\gamma = -\left[1 - \sum_{i=1}^{8} \alpha_i\right]$.

Tab. 3: Johansen Maxeigen. and Trace Tests. The table shows the results of the trace and maxeigenvalues test statistics as well as the critical values at 5% significance level.

			Critical Values $\alpha = 5\%$	
Hypothesized No. CE	Maxeigen.	Trace-stat.	Maxeigen.	Trace-stat.
at most 0	5.9052	9.355	14.2649	15.4943
at most 1	3.4498	3.4498	3.8415	3.8415
$H_0 : r = 0$ cannot be rejected in both tests.				

The Johansen cointegration test: Maxeigenvalues and trace statistics The Johansen-cointegration test consists in assessing the rank of the $\mathbf{\Pi}$ matrix given the VECM($p-1$)

representation with k endogenous variables:

$$\Delta\mathbf{y}_{t(kx1)} = \mathbf{m}_{(kx1)} + \sum_{i=1}^{p-1} \beta_{i(kxk)} \Delta y_{t-i(kx1)} + \mathbf{\Pi}_{(kxk)} \mathbf{y}_{t-1} + \varepsilon_{t(kx1)} \tag{B.7}$$

Note that the eigenvalues of $\mathbf{\Pi}$ is the complement of $\mathbf{A}$ in the VAR(p) representation:

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + \varepsilon_t \tag{B.8}$$

such that, if the eigenvalue λ_i of $\mathbf{A}$ equals to 1, the eigenvalue λ_i^{π} of Π equals to zero[4]. Given that the VAR process could be decomposed in:

$$\mathbf{y}_t = \mathbf{S}\Lambda\mathbf{S}^{-1}\mathbf{y}_{t-1} + \varepsilon_t \tag{B.9}$$

where $\mathbf{S}$ is a matrix with linear independent eigenvectors of $\mathbf{A}$ in its columns and $\Lambda_{(kxk)}$ diagonal matrix with the eigenvalues of $\mathbf{A}$, it is natural that stationarity of the process requires (all) the eigenvalues of $\mathbf{A}$ to be less than one in absolute term, if $n < k$ eigenvalues $\left|\lambda_i\right| < 1$, there exists n reparametrization that issues the system stationary and $\mathbf{\Pi}$ can be decomposed in $\alpha\beta^T$.

The rank r of a matrix equals the number its eigenvalues that are different from zero, hence if the rank of $\mathbf{\Pi}$ is, say 1, there will be 1 $\left|\lambda_i\right| < 1$. The Maxeigenvalue and trace statistics are both tests based on the eigenvalues of $\mathbf{A}$ (inversely, on the eigenvalues of $\mathbf{\Pi}$) and they are respectively (Brooks: 364-367):

$$\lambda_{(tr.)} = -T \sum_{i=r+1}^{n} \ln\left(1 - \hat{\lambda}_i\right) \tag{B.10}$$

and

$$\lambda_{(maxeig.)} = -T \ln\left(1 - \hat{\lambda}_{r+1}\right) \tag{B.11}$$

with r being the number of cointegration relations. and their null hypothesis are:

[4] Note that all VAR(p) process has a VAR(1) representation. For further details check Johnston and DiNardo (2007: 321-322)

- For the trace-statistic

$$\begin{aligned} H_0 : r = 0 \quad vs. \quad & H_1 : 0 < r \leq 1 \\ \vdots \quad \vdots \quad & \vdots \\ H_0 : r = k-1 \quad vs. \quad & H_1 : r = k \end{aligned} \tag{B.12}$$

- For the Maxeigenvalue

$$H_0 : r = 0 \quad vs. \quad H_1 : r = r_0 + 1 \tag{B.13}$$

The critical value for the test statistics are non-standard.[5]

[5] For further reading see Johansen (1991, 1987).

B.3. Diagnostic and Stability Tests

Tab. 4: Results of Diagnostic and Specification Tests . The table presents the results for the diagnostic tests on the normality, homoscedasticity as well as the whiteness of the residuals. The two last rows contains the test on the correct functional form. Some tests were applied separately to each equation of the VAR system.

H_0: Hypothesis	Distribution	p–value	Reject H_0
Jarque Bera – Normality (Multivariate)			
$\varepsilon \sim \mathbb{N}(0, \Omega)$	$\chi^2(4)$	0.000	True
Arch - Effects – LM Test: $dOIL$			
$\sigma^2_{t\|\psi} = \sigma^2_\varepsilon$	$\chi^2(4)$	0.060	False
Arch - Effects – LM Test: $dRGDP$			
$\sigma^2_{t\|\psi} = \sigma^2_\varepsilon, \forall t$	$\chi^2(4)$	0.000	True
White Heteroscedasticity – LM Test: $dOIL$			
$\sigma^2_t = \sigma^2_\varepsilon, \forall t$	$\chi^2(44)$	0.008	True
White Heteroscedasticity – LM Test: $dRGDP$			
$\sigma^2_t = \sigma^2_\varepsilon, \forall t$	$\chi^2(44)$	0.09	False
Portmanteau test – Whiteness of Residuals (Multivariate)			
$corr(y_t, y_{t+s}) = 0$ $s = 1, \ldots, h.$	$\chi^2(20)$	0.058	False
Ramsey Reset Test – Functional Form Wald Test: $dOIL$			
$\beta_{y^x} = 0,$ $x = 2, 3$	$\chi^2(2)$	0.282	False
Ramsey Reset Test – Functional Form Wald Test: $dRGDP$			
$\beta_{y^x} = 0,$ $x = 2, 3$	$\chi^2(2)$	$4.992 * 10^{-6}$	True

Jarque-Bera (Normality Test) Jarque-Bera tests for the normality assumption of the innovations. The tests compares the skewness and the kurtosis of the data with that of the normal distribution, which are 0 and 3 respectively. If the test gets too large, then the hypothesis of a normal distribution can be rejected.

$$\begin{aligned} H_0 : \varepsilon &\sim \mathbb{N}\left(0, \sigma_\varepsilon^2\right) \\ JB &= \frac{T}{6}\left(S^2 + \left[\frac{K-3}{2}\right]^2\right) \sim \chi^2(2) \end{aligned} \tag{B.14}$$

where: $\begin{bmatrix} S & K \end{bmatrix} = \begin{bmatrix} \text{Skewness} & \text{Kurtosis} \end{bmatrix}$. H_0 hypothesizes that the innovations are normally distributed. In the multivariate case, which will not be explained here, the test ist $\chi^2_{2(p)}$ distributed. For the purpose of the variables in the present work, the test is $\chi^2_{(4)}$.

Arch-Effects (Autoregressive Condition Heteroscedasticity Test) The Arch-Effects test assesses the homoscedasticity of the conditional variance of the residuals. The test consists in regressing the squared residuals on its squared past values (Johnston and DiNardo, 2007: 195-196).

$$H_0 : \alpha_1 = \cdots = \alpha_4 = 0 \tag{B.15}$$

$$\varepsilon_t = \upsilon_t \left(\alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2\right)^{0.5}, \text{ with } \upsilon_t \sim \mathbb{N}(0,1) \tag{B.16}$$

$$\mathbb{E}\left[\varepsilon_t^2 \middle| \varepsilon_{t-i}\right] = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{B.17}$$

In the present work $p = 4$ in accordance with the number of lags included in the **VAR** model.. H_0 hypothesizes that $\left(\mathbb{E}\left[\varepsilon_t^2 \middle| \varepsilon_{t-i}\right] = \alpha_0\right)$ the conditional probability of the error term is constant (homoscedastic) and is assessed by a **LM-Test**: $L_M = nR^2 \sim \chi^2(4)$.

White Test (Test for Heteroscedasticity) The white test assesses the homoscedasticity of the residuals:

$$H_0 : \sigma_t^2 = \sigma^2, \forall t \tag{B.18}$$

$$\hat{\varepsilon}_t^2 = \alpha_0 + \sum_{i=1}^{2} \sum_{j=1}^{4} \beta_{ij} y_{it-i} + \text{cross products} \tag{B.19}$$

It consists in regressing the squared residuals on the original explanatory variables as well as on their cross products (including squared values) and testing for the joint significance of the parameters by means of a L_M-test: $L_M = nR^2$ (Johnston and DiNardo, 2007: 166). For our purpose each regression includes 9 variables (4 lagged $[y_1, y_2]$ and a constant). In total the auxiliary regression will have 45 parameters to be estimated, being 28 cross products, 8 squared regressors and 9 variables from the orignal regression; the test is $\chi^2 \sim (44)$. H_0 hypothesizes that the variance of the error term is constant, i.e., all coeffiencients but α_0 are not, jointly, statistically significant different than zero.

Portmanteau Test The Portmanteau test as described by Lütkepohl and Krätzig (2004: 169-170) checks for the whiteness of the residuals. Given a set of matrices of autocorrelation $\mathbf{R}_h = (R_1, R_2, \ldots, R_h)$, the null hypothesis states that:

$$\begin{aligned} H_0 : \mathbf{R}_h &= 0 \\ Q_h &= T \sum_{i=1}^{h} \text{tr}\left[\hat{R}_i^T \hat{R}_\varepsilon^{-1} \hat{R}_i \hat{R}_\varepsilon^{-1}\right] \sim \chi^2 (20) \end{aligned} \tag{B.20}$$

Kilian and Lütkepohl (2017) (2017: 53) write regarding the choice of h: "[...] The number h of autocovariance terms in the test statistic should be considerably larger than p for a good approximation to the null distribution. Choosing h too large, however, may undermine the power of the test. [...]. The Portmanteau test should only be applied to test for a large number of nonzero autocovariance. It is not suitable for testing the absence of low-order autocorrelation. For the latter purpose, the LM test is preferred." Thus, the number of h chosen is 10 (approximately 9% of the sample size). H_0 hypothesizes that there is no serial correlation in the innovation up to the h^{th} lag.

CUSUM The CUSUM-Test (abb. for cumulative sum) is a test based on the residuals of recursive estimation.

$$\begin{aligned} H_0 : \mathbb{E}\left[W_t\right] &= 0 \\ W_t &= \sum_{j=k+1}^{t} w_j,\ t = 1 + k, \ldots, T \\ \mathbf{w} &\sim \mathbb{N}\left(0, \sigma_\varepsilon^2 \mathbf{I}\right) \end{aligned} \tag{B.21}$$

where w_j is the ratio between the one-step ahead prediction error and its standard deviation. $w_j = \frac{\mathbf{y}_t - \mathbf{X}\beta}{\hat{\sigma}\sqrt{1+\left(\mathbf{x}_t^T\left[\mathbf{X}_{t-1}^T\mathbf{X}_{t-1}\right]^{-1}\mathbf{x}_t\right)}}$, with $\hat{\sigma}^2 = \frac{RSS}{n-k}$, estimated by using all the observation in the data set. The idea of the test is that, if the parameters are stable (constant), then $\mathbb{E}\left[W_t\right] = 0$. The cumulative sum is plotted against time and the stability of the parameters is checked by graphic inspection, namely by analyzing how far the value of W_t diverges from zero. If W_t crosses the boundaries $k \pm a\sqrt{T-k}$ and $n \pm 3a\sqrt{T-k}$, one concludes that the parameters are not stable. a is a value dependent on the chosen significance level. In the present case $\begin{bmatrix}\alpha & a\end{bmatrix} = \begin{bmatrix}0.05 & 0.9408\end{bmatrix}$ (Johnston and DiNardo, 2007: 119).

Ramsey Reset Test The Ramsey-Reset test tests for the correct function form of the model, e.g., if a linear model is suitable to model the given data. It consists of three steps:

1. Estimate the model to obtain fitted values $\hat{y}$
2. Regress y on the original regressors as well as on polynomials of the fitted values

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \hat{\mathbf{Y}}\boldsymbol{\Psi} + \boldsymbol{\varepsilon}$$

   where $\hat{\mathbf{Y}} = \begin{bmatrix}\hat{\mathbf{y}}^2 & \dots & \hat{\mathbf{y}}^n\end{bmatrix}$
3. Test the joint significance of the parameters $\boldsymbol{\Psi}$

$$\begin{aligned} H_0 : \boldsymbol{\Psi} &= 0 \\ W &= \left[\mathbf{R}\boldsymbol{\Psi} - \boldsymbol{\Psi}_0\right]^T \left[\mathbf{R}\hat{\boldsymbol{\Sigma}}\mathbf{R}^T\right]^{-1} \left[\mathbf{R}\boldsymbol{\Psi} - \boldsymbol{\Psi}_0\right] \end{aligned}$$

   If H_0 cannot be rejected, the linear model is not the most appropriate to fit the data (Johnston and DiNardo 2007: 121).

C. The Model: Estimates and Confidence Intervals

The data set comprises 107 observations, as one observation was lost because of first-differencing. Tab. The estimates are given in 5as well as the bootstrapped confidence interval at 5% significance level. Further, the reader finds below the estimated covariance-matrix for the VAR residuals along with its cholesky decomposition, used

Tab. 5: Estimates: the table gives the estimated coefficients with the bootstrapped confidence intervals in parentheses. Estimates with $*$ are significant under 5% confidence level.

i	$\phi_{i,0}$	ϕ_i^{11}	ϕ_i^{12}	ϕ_i^{21}	ϕ_i^{22}
1	0.0014 $(-0.015;0.016)$	0.720^* $(0.394;1.066)$	-0.149 $(-0.408;0.13)$	0.063 $(-0.059;0.179)$	-0.177^* $(-0.334;-0.021)$
2	0.061 $(-0.01;0.013)$	0.224 $(-0.055,0.46)$	-0.194 $(-0.437;0.071)$	0.186^* $(0.051;0.317)$	-0.245^* $(-0.366;-0.107)$
3	–	-0.444^* $(-0.828;-0.124)$	-0.242 $(-0.563;0.13)$	-0.043 $(-0.161;0.081)$	-0.217^* $(-0.338;-0.093)$
4	–	0.0617 $(-0.159;0.299)$	0.167 $(-0.14;0.489)$	0.008 $(-0.114;0.138)$	0.782^* $(0.638;0.92)$

to initialize the orthogonal shock to the system, where the error is transformed such that $\boldsymbol{\upsilon} = \mathbf{P}\boldsymbol{\varepsilon}$.

- Endogenous variables: $\mathbf{y} = \begin{bmatrix} dOIL \\ dRGDP \end{bmatrix}$
- Model:

$$\mathbf{y}_t = \begin{bmatrix} \phi_{1,0} \\ \phi_{2,0} \end{bmatrix} + \sum_{i=1}^{4} \begin{bmatrix} \phi_i^{11} & \phi_i^{12} \\ \phi_i^{21} & \phi_i^{22} \end{bmatrix} \mathbf{y}_{t-i} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}_t \tag{C.1}$$

- Number of observations: 107
- VAR: ε-Variance covariance matrix

$$\boldsymbol{\Omega} = \begin{bmatrix} 3.55 * 10^{-3} & 9 * 10^{-6} \\ 9 * 10^{-6} & 6.78 * 10^{-4} \end{bmatrix} \tag{C.2}$$

- Cholesky decomposition of $\boldsymbol{\Omega} = \mathbf{P}^{-1}\left(\mathbf{P}^{-1}\right)^T$

$$\mathbf{P} = \begin{bmatrix} 5.954 * 10^{-2} & 0 \\ 1.578 * 10^{-4} & 2.604 * 10^{-2} \end{bmatrix} \tag{C.3}$$

D. Bootstraps: Description

The central idea of the bootstrap method is to consider the sample in question as an approximation of its population. Thus, through repeated resampling with replacements, an empirical distribution of the statistical variable of interest is derived allowing statistical inference to be made based on this distribution (Johnson, 2001: 49-50).

Since the diagnostic tests refuted the normality of the residuals as well as their homoscedasticity, a non-parametric bootstrap has been chosen, where the bootstrap-sampling is done by randomly resampling the original sample (pairwise), rather than creating the samples from a previously known distribution (using the error terms).

D.1. Bootstrap of Confidence Intervals ϕ_i

To create the confidence intervals for the ϕ_i estimators the following procedure (see Johnston and DiNardo (2007: 369)) was applied:

1. Given the original sample of size T, create another randomized sample with replacement of equal size and in pairs $(\mathbf{y}_t^*, \mathbf{x}_t^*)$;
2. Run the VAR regression as given in Eq. (4.1) and save the coefficient estimates ϕ_i^*.
3. Repeat the procedure 1 and 2 a large number of times (for the purpuse of this project $1,000,000$)
4. Create an ordered set (histogram) for each of the estimators.

The upper and the lower-bound of the intervals are given by:

$$CI_L^\alpha = Percentile\left(\frac{\alpha}{2}\right)100 \tag{D.1}$$

$$CI_U^\alpha = Percentile\left(1-\left[\frac{\alpha}{2}\right]\right)100 \tag{D.2}$$

where α is the confidence level (see Bittmann (2021: 33-34)).

D.2. Bootstrap: Granger Causality - Wald Test

The distribution of the Granger causality test using the Wald-Test is asymptotically $\chi^2(q)$ distributed, being q the number of restrictions. In this case, both the errors and the model coefficients are assumed to be normally distributed, which, as seen, is not the case. To increase the accuracy of the statement that $dOIL$ granger-causes $dRGDP$ and that $dRGDP$ granger-causes $dOIL$ the Wald-Test distribution imposing H_0 was derived and a Monte Carlo p-value (p_{mc}) computed.

Specifically, for each equation in the VAR model, and following the 2 Guidelines suggested by Hall and Wilson (1991: 758-759) and by Fox (2015: 661-662),

1. The model was estimated once using the original data to obtain estimates for the parameters $\mathbf{\Phi}$ and the (Wald) - test statistic W based on these parameters.
2. Given the original sample of size T, create another randomized sample with replacement of equal size and in pairs $(\mathbf{y}_t^*, \mathbf{x}_t^*)$ and obtain $\mathbf{\Phi}^*$ (estimates for the bootstrapped sample)
3. Calculate the Wald-test imposing H_0, in this case $H_0 : \mathbf{\Phi}^* = \mathbf{\Phi}$.

$$W^* = [\mathbf{R\Phi}^* - \mathbf{\Phi}]^T \left[\mathbf{R}\hat{\mathbf{\Sigma}}^* \mathbf{R}^T\right]^{-1} [\mathbf{R\Phi}^* - \mathbf{\Phi}] \tag{D.3}$$

   where $\mathbf{R} \equiv (4x9)$ restriction matrix, $\hat{\mathbf{\Sigma}}^* \equiv (9x9)$ estimated covariance matrix of the bootstrapped estimators.
4. Repita $2, 3$ a large number of times (here $200, 000$).
5. Compute Monte Carlo $p-$value as:

$$p_{mc} = \frac{1 + \#\{W^* \geq W\}}{T+1} \tag{D.4}$$

   In words: The number of bootstrapped Wald tests that are greater or equal to the Wald test calculated in 1) plus 1, divided by the number of repetition R plus 1, following Davison and Hinkely (1997: 140-141).

References

[1] Gaidar, Y. (2007), The Soviet collapse: grain and oil, *American Enterprise Institute for Public Policy Research No. 04-April*, AEI, N.W., Washington D.C..

[2] Benedictow, A., D., Fjærtoft and O., Løfsnæs (2010), Oil dependency of the Russian economy: an econometric analysis, *Discussion Papers No. 617*, Statistics Norway, Research Department, Kongsvinger.

[3] Bittmann, F. (2021), *Bootstrapping: An Integrated Approach with Python and Stata*, de Gruyter, Berlin.

[4] Brooks, C. (2019), *Introductory Econometrics for Finance*, Cambridge University Press, Cambridge.

[5] Davison, A.C. and D.V., Hinkely (1997), *Bootstrap Methods and their Applications*, Cambridge University Press, Cambridge.

[6] Dickey, D.A. and W.A., Fuller (1979), Distribution of the Estimators for Autoregressive Time Series With a Unit Root, *Journal of the American Statistical Association* 74, p. 427 - 431.

[7] Engle, R.F. and C.W.J., Granger (1987), Co-Integration and Error Correction: Representation, Estimation, and Testing, *Econometrica* 55, p. 251 - 276.

[8] Fox, J. (2016), *Applied Regression Analysis And Generalized Linear Models*, Sage, Los Angeles, London, New Delhi, Singapore, Washington DC, Boston.

[9] Granger, C.W.J. (1969), Investigating Causal Relations by Econometric Models and Cross-spectral Methods, *Econometrica* 37, p. 424 - 438.

[10] Hall, P. and S.R., Wilson (1991), Two Guidelines for Bootstraps Hypothesis Testing, *Biometrics* 47, p. 757 - 762.

[11] Johansen, S. (1987), *Statistical Analysis of Cointegration Vectors*, Institute of Mathematical Statistics No. Preprint 7, University of Copenhagen, Copenhagen.

[12] Johansen, S. (1991), Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica* 59, p. 1551 - 1580.

[13] Johnson, R.W. (2001), An Introduction to the Bootstrap, *The Statistician* 23, p. 49 - 54.

[14] Johnston, J. and J., DiNardo (2007), *Econometric Methods*, McGraw-Hill, New York.

[15] Kakanov, E., H., Blöchliger and L., Demmou (2018), Resource curse in oil exporting countries, *OECD Economics Department Working Papers No. 1511*, OECD Publishing, Paris.

[16] Katsuya, I. (2010), The impact of oil price volatility on macroeconomic activity of Russia, *Economic Analysis Working Papers No. 2010,5*, Colegio de Economistas de A Coruña, A Coruña.

[17] Kilian, L. and H., Lütkepohl (2017), *Structural Vector Autoregressive Analysis*, Cambridge University Press, Cambridge.

[18] Kilian, L. and X., Zhou (2020), The Econometric of oil market VAR models, *Working Paper - Research Department No. 2006*, Federal Reserve Bank of Dallas, Dallas, TX.

[19] Kwiatkowski, D., P.C.B., Phillips and P., Schmidt (1991), Testing the null hypothesis of stationarity against the alternative of a unit root: how sure are we that economic time series have a unit root?, *Cowles Foundation Discussion Paper No. 979*, Cowles Foundation for Research in Economics at Yale University, New Heaven, Connecticut.

[20] Lütkepohl, H. and M., Krätzig (2004), *Applied Time Series Econometrics, Cambridge University Press*, Cambridge, New York, Madrid, Cape Town, Singapore, São Paulo.

[21] Rautava, J. (2002), The role of oil prices and the real exchange rate in Russia's economy, *BOFIT - Discussion Papers No. 3*, Institute for Economies in Transition, Helsinki.

[22] Strang, G. (2009), *Introduction to Linear Algebra*, Wellesley-Cambridge Press, Wellesley, MA .

[23] Wooldridge, J.M. (2009), *Introductory Econometrics: A modern Approach*, Cengage, Australia, Brazil, Japan, Korea, Mexico, Singapore, United Kingdom, United States.

References: Online Sources - Datasets

[1] Federal Service Statistics - Rosstat (2022): National Accounts - GDP quarters (since 1995), URL:

https://rosstat.gov.ru/statistics/accounts, query-date: 30.06.2022, at: 17:33.

[2] OECD Main Economic Indicators - Complete Database; Retrived from Federal Reserve Bank of St. Louis - FRED (2022): National Currency to US Dollar Exchange Rate: Average of Daily Rates for the Russian Federation [CCUSMA02RUM618N], URL: https://fred.stlouisfed.org/series/CCUSMA02RUM618N, query-date: 30.06.2022, at: 18:05.

[3] U.S. Energy Information Administration (EIA) (2022): Europe Brent Spot Price FOB , URL: https://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=RBRTE&f=M, query-date: 30.06.2022, at: 17:54.