Example 1: Effect of Downtime on Queueing

Consider a single machine that processes a wide variety of parts that arrive in random mixes at random times. Data analysis has shown that an exponentially distributed processing time with a mean of 7.5 minutes provides a fairly accurate representation. Parts arrive at random, time between arrivals being exponentially distributed with mean 10 minutes. The machine fails at random times. Downtime studies have shown that time-to-failure can be reasonably approximated by an exponential distribution with mean time 1000 minutes. The time to repair the resource is also exponentially distributed, with mean time 50 minutes. When a failure occurs, the current part in the machine is removed from the machine; when the repair has been completed, the part resumes its processing.

When a part arrives, it queues and waits its turn at the machine. It is desired to estimate the size of this queue. An experiment was designed to estimate the average number of parts in the queue. To illustrate the effect of an accurate treatment of downtimes, the model was run under a number of different assumptions. For each case and replication, the simulation run length was 100,000 minutes.

Table 1 shows the average number of parts in the queue for six different treatments of the time between breakdowns. For each treatment that involves randomness, five replications of those treatments and the average for those five replications are shown.

Case A ignores the breakdowns. The average number in the queue is 2.31 parts. Across the 5 independent replications, the averages range from 2.05 to 2.70 parts. This treatment of breakdowns is not recommended.

Case B increases the average service time from 7.5 minutes to 8.0 minutes in an attempt to approximate the effect of downtimes. On average, each downtime and repair cycle is 1050 minutes, with the machine down for 50 minutes. Thus the machine is down, on the average in the long run, $50/1050 = 4.8\%$ of total time. Thus, some have argued that downtime has approximately the same effect as increasing the processing time of each part by 4.8%, which is about 7.86 minutes. Therefore, an assumed constant 8 minutes per part should be (it is argued) a conservative approach. For this treatment of downtimes, the average number of parts in the queue, over the five replications, is about 3.26 parts. Across the 5 replications, the range is from 2.81 to 4.03 parts. (Note that the variability as shown in the range of values is very small compared to the other cases.) The treatment in Case B

Table 1 Average Number of Parts in Queue for Machines with Breakdowns

Case	*1st Rep*	*2nd Rep*	*3rd Rep*	*4th Rep*	*5th Rep*	*Avg Rep*
A. Ignore the breakdowns	2.36	2.05	2.38	2.05	2.70	2.31
B. Increase service time to 8.0	3.32	2.82	3.32	2.81	4.03	3.26
C. All random	4.05	3.77	4.36	3.95	4.43	4.11
D. Random processing, deterministic breakdowns	3.24	2.85	3.28	3.05	3.79	3.24
E. All deterministic						0.52
F. Deterministic processing, random breakdowns	1.06	1.04	1.10	1.32	1.16	1.13

might be appropriate under some limited circumstances, but, as was discussed in a previous section, it is not appropriate under the assumptions of this example.

The proper treatment, shown as Case C, treats the randomness in processing and breakdowns properly, with the assumed correct exponential distributions. The average value is about 4.11 parts waiting for the machine. Across the 5 replications, the average queue length ranges from 3.77 to 4.43 parts. The average number waiting differs from that of Case B by almost one part.

Case D is a simplification that treats the processing randomly, but treats the breakdowns as deterministic. The results average about 3.24 parts in the queue. The range of averages is from 2.85 to 3.79 parts, quite a reduction in variability from Case C.

Case E treats all of the times as deterministic. Only one replication is needed, because additional replications (using the same seed) will reproduce the result. The average value in the queue is 0.52 parts, well below the value in Case C, or any other case for that matter. The conclusion: Ignoring randomness is dangerous and leads to totally unrealistic results.

Case F treats arrivals and processing as deterministic, but breakdowns are random. The average number of parts in the queue at the machine is about 1.13. The range is from 1.04 to 1.32 parts. For some machines and processing in manufacturing environments, Case F is the realistic situation: Processing times are constant, and arrivals are regulated—that is, are also constant. The reader is left to consider the inaccuracies that would result from making faulty assumptions regarding the nature of time to failure and time to repair.

In conclusion, there can be significant differences between the estimated average numbers in a queue, based on the treatment of randomness. The results using the correct treatment of randomness can be far different from those using alternatives. Often, one is tempted by the unavailability of detailed data and the availability of averages to want to use average time to failure as if it were a constant. Example 1 illustrates the dangers of inappropriate assumptions. Both the appropriate technique to use and the appropriate statistical distribution depend on the available data and on the situation at hand.