KNN Regression Simulation

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1 Project: K-Nearest-Neighbor Regression Simulation

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1.1 Motivation

Consider a single dimension (variable) X. Obtain N = 100 iid samples x1, x2, · · · of X uniformly randomly between 1 and 10, and then obtain the corresponding y values as the natural logarithm of x plus a Gaussian noise (mean 0, standard deviation 0.1), with different points having different amounts of noise.

Now use K-NN regression to obtain ˆy values (= estimates of y) at x-values of 1, 3, 5, 7 and 9 for each of the following three schemes:

- the K neighbors contribute equally (separately for K = 1, 3, 50)
- each of the K neighbors has an influence that is inversely proportional to the distance from the point (separately for K = 1, 3, 50)
- all the N points contribute, with each contribution proportional to $e^{-\frac{1}{2}d^2}$, where d represents distance.

Print the numerical values of the $(x, \hat{y})$ pairs for each of the above cases (there should be a total of $3 + 3 + 1 = 7$ cases and 5 $(x, \hat{y})$ pairs for each case). Also, plot the (x', y') and $(x, \hat{y})$ points for each of these seven cases, where x is the point (out of the 100 sample points) closest to x and y is the y-value of x (there should be a total of 7 plots for this, each plot showing the (x', y') and $(x, \hat{y})$ points). It is possible but unlikely that x and x coincide.

[2]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.neighbors import KNeighborsRegressor

np.random.seed(seed=42)
```

1.2 Data Creation

[3]:
```
# N=100 independently and identically distributed samples drawn from the␣
↪uniform distribution on the interval [1, 10]
train_X = np.random.uniform(1, 10, 100)
print(type(train_X))
```

```
print(train_X[:5])
```

```
<class 'numpy.ndarray'>
[4.37086107 9.55642876 7.58794548 6.38792636 2.40416776]
```

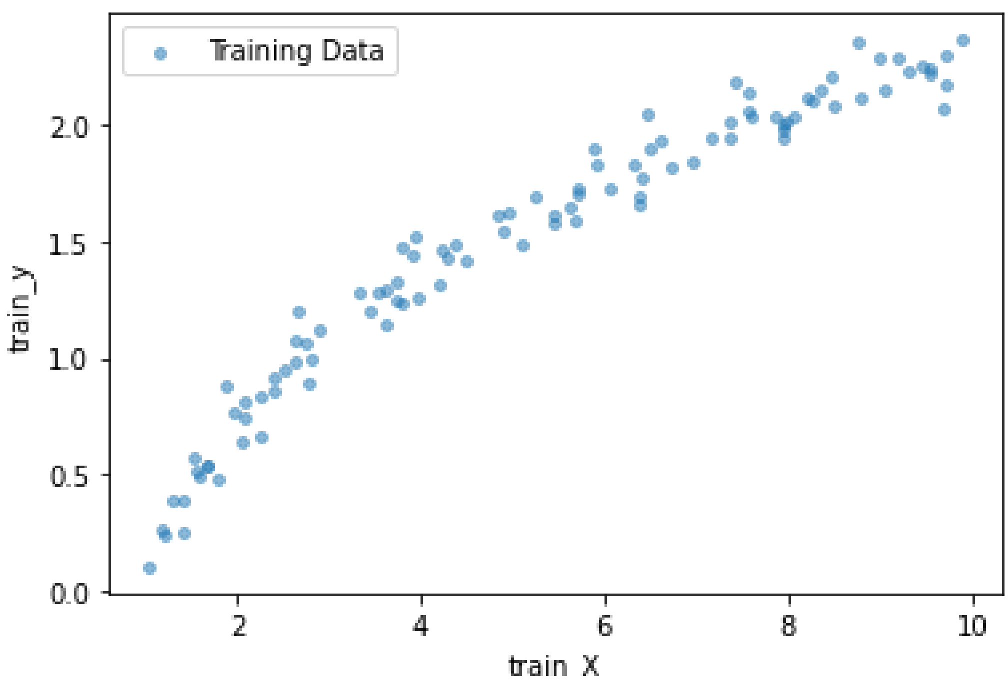
[4]:
```
# y values generated for the train_X values as ln(X)+epsilon, where epsilon is␣
↪Normal w/ mean 0 and sd 0.1
train_y = [np.log(i) + np.random.normal(0, 0.1) for i in train_X]
train_y = np.asarray(train_y)
print(type(train_y))
print(train_y[:5])
```

```
<class 'numpy.ndarray'>
[1.48366474 2.22731336 2.03573694 1.65565281 0.85523661]
```

[50]:
```
# Create an array of test values
test_X = np.array([1, 3, 5, 7, 9])
test_X
```

[50]:
```
array([1, 3, 5, 7, 9])
```

[49]:
```
plt.scatter(train_X, train_y, alpha=0.5, s=15, label='Training Data')
plt.xlabel('train_X')
plt.ylabel('train_y')
_ = plt.legend()
```



1.3 Creation of Helper Functions

[7]:
```python
def nearest_neighbor(test_x, x_train):
    '''
    Find the closest x value in a training set to a test_x numeric value.
    Returns the closest training x value and its associated y value.
    '''
    dist = 10000000000
    x_value = 100000000
    for i in x_train:
        t_dist = np.absolute(test_x-i)
        if t_dist < dist:
            dist = t_dist
            x_value = i
    y_index = np.where(train_X == x_value)[0][0]
    y_value = train_y[y_index]
    return x_value, y_value
```

[47]:
```python
# A quick test for validation
nearest_neighbor(7, train_X)
```

[47]: (6.962700559185838, 1.8496286649871818)

[9]:
```python
def KNN_Regression_homework(wts='uniform', K_sizes=[1, 3, 50]):
    '''
    Pass in the wts argument for the KNeighborsRegressor and a list of K␣
↪integer values (K_sizes).
    Function returns values and plots using the test_X variable and the model's␣
↪predictions on those values.
    '''
    x_prime = []
    y_prime = []
    for i in test_X:
            u, v = nearest_neighbor(i, train_X)
            x_prime.append(u)
            y_prime.append(v)
    plt.scatter(x_prime, y_prime, s=60, label='Nearest Neighbors')
    estimates = {}
    size = 20
    alpha = 1
    for K in K_sizes:
        model = KNeighborsRegressor(n_neighbors = K, weights=wts)
        model.fit(train_X.reshape(-1,1), train_y)
        predicted = model.predict(test_X.reshape(-1,1))
        predicted_short = [round(i, 3) for i in predicted]
        estimates[K] = predicted_short
```

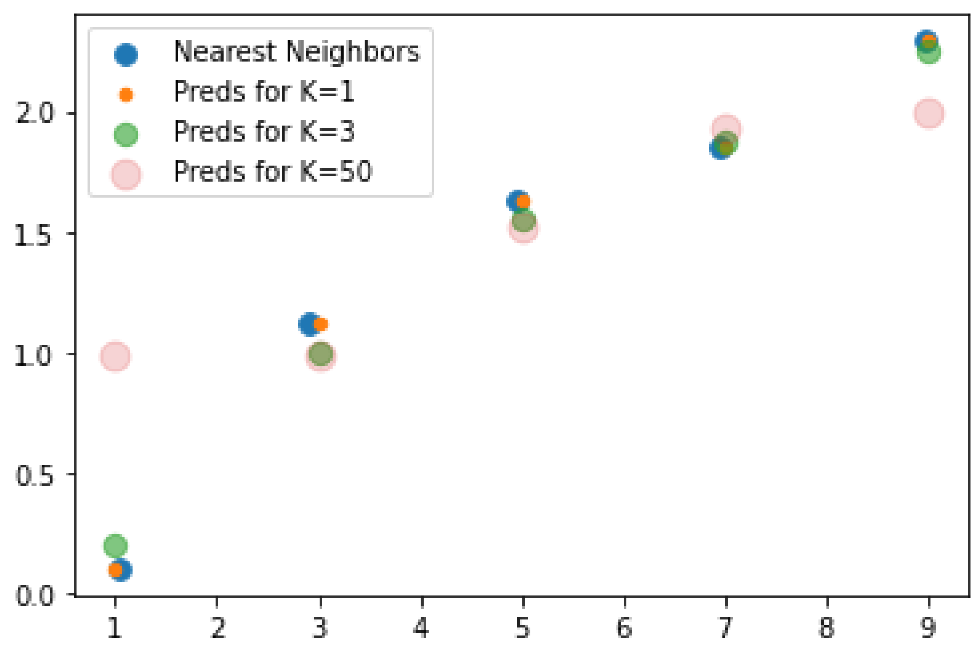
```
        plt.scatter(test_X, predicted, s=size, alpha=alpha, label=f'Preds for␣
↪K={K}')
        _ = plt.legend()
        size += 50
        alpha -= .40

    for key in estimates:
        output = [(test_X[i], estimates[key][i]) for i in range(len(test_X))]
        print(f"K = {key}, Predictions: {output}")
```

1.4 K Neighbors Contribute Equally

The default value for the weights argument of the KNeighborsRegressor object is "uniform" which will yield the above outcome.

[10]:
```
KNN_Regression_homework()
```

```
K = 1, Predictions: [(1, 0.101), (3, 1.12), (5, 1.632), (7, 1.85), (9, 2.292)]
K = 3, Predictions: [(1, 0.199), (3, 1.003), (5, 1.556), (7, 1.875), (9, 2.246)]
K = 50, Predictions: [(1, 0.985), (3, 0.985), (5, 1.519), (7, 1.926), (9,
1.995)]
```

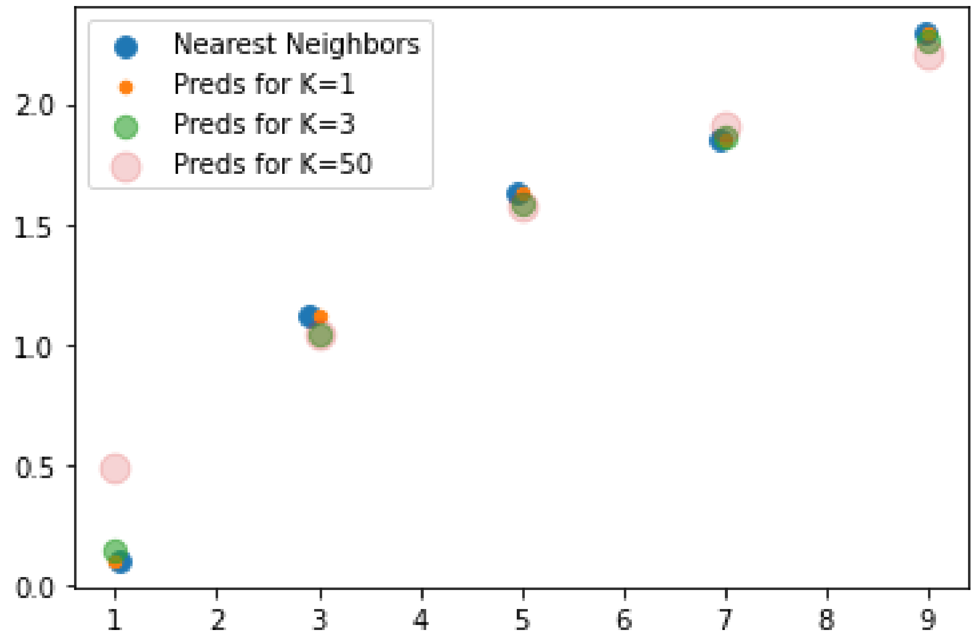


1.5 Influence of K Neighbors is Inversely Proportional to the Distance from x_0

The 'distance' value for the weights argument of KNeighborsRegressor does exactly this. Results below.

[11]:
```
KNN_Regression_homework('distance')
```

```
K = 1, Predictions: [(1, 0.101), (3, 1.12), (5, 1.632), (7, 1.85), (9, 2.292)]
K = 3, Predictions: [(1, 0.149), (3, 1.039), (5, 1.584), (7, 1.864), (9, 2.263)]
K = 50, Predictions: [(1, 0.49), (3, 1.039), (5, 1.569), (7, 1.908), (9, 2.21)]
```

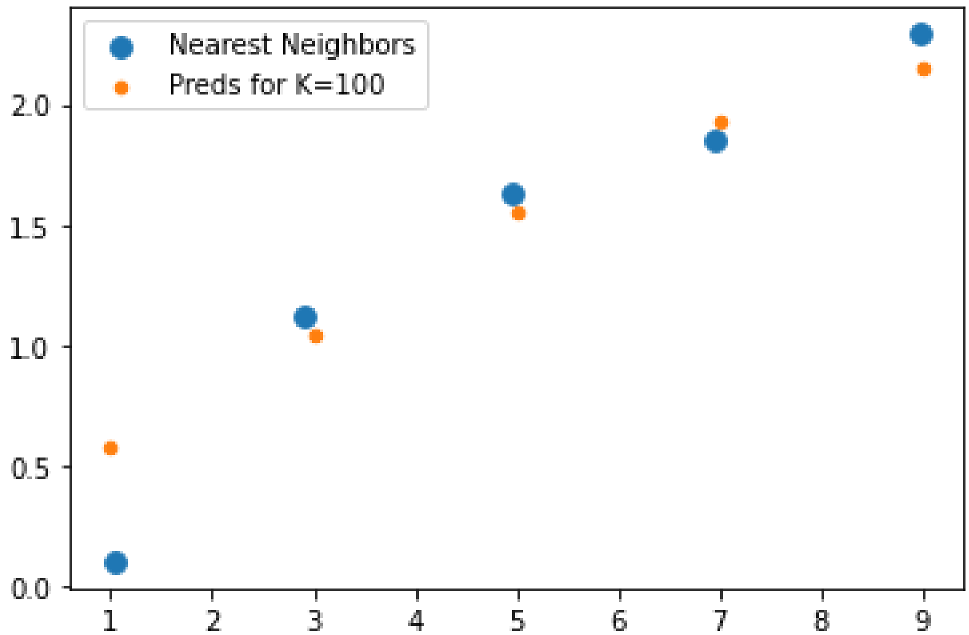


1.6 All N Points Contribute according to $e^{-\frac{1}{2}d^2}$

[28]:
```
def radial_basis(d):
    return (np.e)**(-0.5*(d)**2)
```

[29]:
```
KNN_Regression_homework(radial_basis, [100])
```

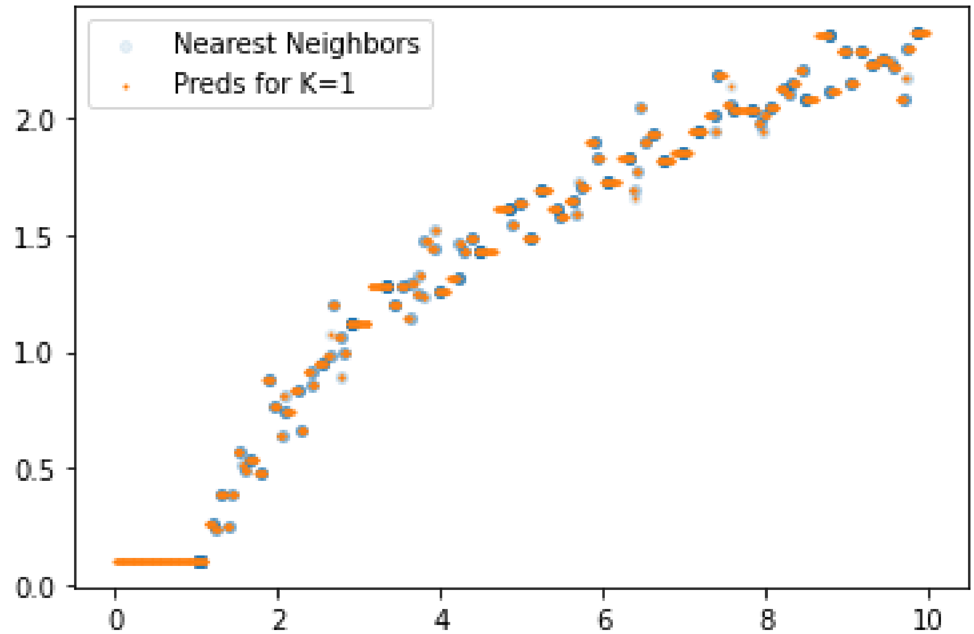
```
K = 100, Predictions: [(1, 0.578), (3, 1.037), (5, 1.554), (7, 1.925), (9,
2.156)]
```



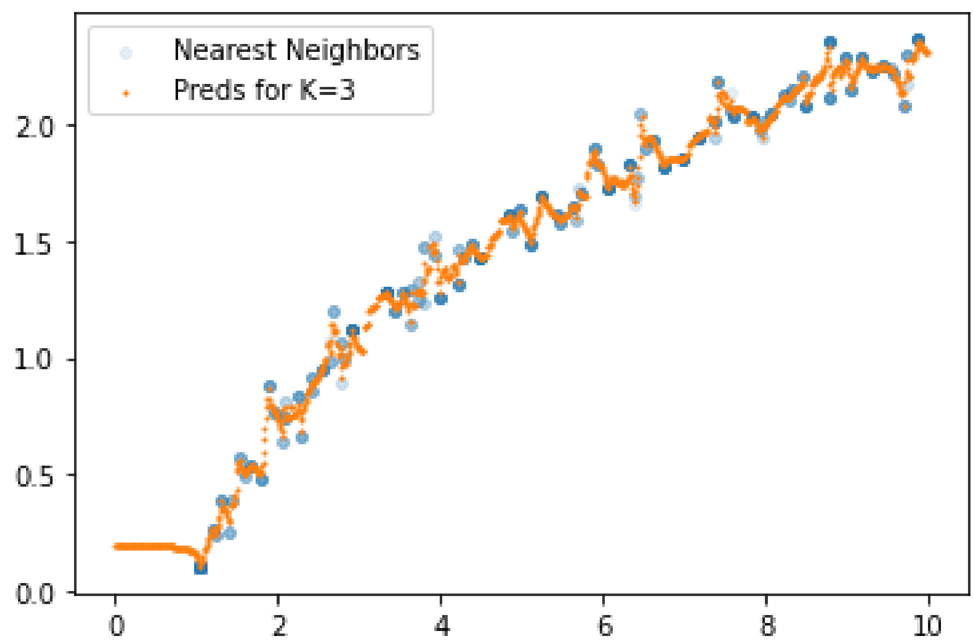
1.7 Extra Plots: 1000 points to Sketch Select Curves

[41]:
```
def KNN_Regression_extra(wts='uniform', K_sizes=[100]):
    test_X_2 = np.arange(0, 10, .01)
    x_prime = []
    y_prime = []
    for i in test_X_2:
            u, v = nearest_neighbor(i, train_X)
            x_prime.append(u)
            y_prime.append(v)
    plt.scatter(x_prime, y_prime, s=15, alpha=0.1, label='Nearest Neighbors')
    estimates = {}
    size = 1
    alpha = 1
    for K in K_sizes:
        model = KNeighborsRegressor(n_neighbors = K, weights=wts)
        model.fit(train_X.reshape(-1,1), train_y)
        predicted = model.predict(test_X_2.reshape(-1,1))
        predicted_short = [round(i, 3) for i in predicted]
        estimates[K] = predicted_short
        plt.scatter(test_X_2, predicted, s=size, alpha=alpha, label=f'Preds for␣
↪K={K}')
        _ = plt.legend()
        size += 50
        alpha -= .40
```

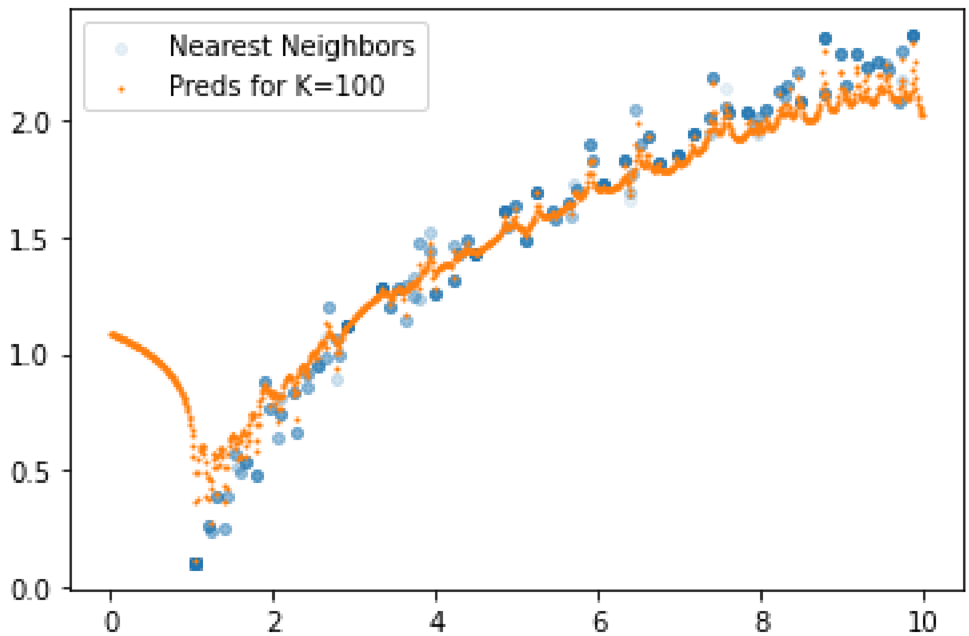
[46]:
```
# over fit
KNN_Regression_extra(wts="uniform", K_sizes=[1])
```



[39]:
```
KNN_Regression_extra(wts="distance", K_sizes=[3])
```



[51]:
```
KNN_Regression_extra(wts="distance", K_sizes=[100])
```



[42]:
```
KNN_Regression_extra(radial_basis)
```