Bài 10

$$A = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -1 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 0 & -7 & -11 & 6 \\ 0 & -14 & -26 & 9 \\ 0 & -14 & -26 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 5 & -1 \\ 0 & -7 & -11 & 6 \\ 0 & 0 & -4 & -3 \\ 0 & 0 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 0 & -7 & -11 & 6 \\ 0 & 0 & -4 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rank = 4

$$B = \begin{bmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{bmatrix} = \begin{bmatrix} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 3 & 0 & -4 \\ 0 & 0 & -3 & 0 & 4 \\ 0 & 0 & 6 & 0 & -8 \\ 0 & 0 & 9 & 0 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & -5 & 2 & 3 \\ 0 & 0 & 3 & 0 & -4 \\ 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow r(B) = 3$$

Bài 11.

$$A = \begin{bmatrix} 1 & -1 & 1 & 2 \\ -1 & 2 & 2 & 1 \\ 1 & 0 & 4 & m \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 1 & 3 & m-2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1-m \end{bmatrix}$$

$$r(A) = 2 \Leftrightarrow 1 - m = 0 \Leftrightarrow m = 1$$

Bài 12. Tìm nghịch đảo

a; $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$; $A^{-1} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$

b; $B = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & 1 \end{bmatrix}$

$$B = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & 1 \end{bmatrix} \quad \det(B) = -3$$

$$P_B = \begin{bmatrix} 2 & -21 & -11 \\ 1 & -12 & 7 \\ -1 & 3 & -1 \end{bmatrix} \Rightarrow B^{-1} = \begin{bmatrix} -2/3 & 7 & -\frac{11}{3} \\ -1/3 & 4 & -\frac{7}{3} \\ 1/3 & -1 & \frac{1}{3} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -a & 0 & 0 \\ 0 & 1 & -a & 0 \\ 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \det(C) = 1$$

$$P(C) = \begin{bmatrix} 1 & a & a^2 & a^3 \\ 0 & 1 & a & -a \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bài 13. Tìm a để A khả nghịch

$$A = \begin{bmatrix} a+1 & -1 & a \\ 3 & a+1 & 3 \\ a-1 & 0 & a-1 \end{bmatrix}$$

Để A khả nghịch $\Leftrightarrow \det(A) \neq 0$

$$A = \begin{bmatrix} -1 & a+1 & a \\ a+1 & 3 & 3 \\ 0 & a-1 & a-1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & a+1 & a \\ 0 & 3+(a+1)^2 & 3+a(a+1) \\ 0 & 0 & (a-1)(3+(a+1)^2)-(3+a(a+1))(a-1) \end{bmatrix}$$

$\Rightarrow \det A \neq 0 \Leftrightarrow (a-1)(3+(a+1)^2)-(3+a(a+1))(a-1)$

$\Leftrightarrow (a-1)(3+a^2+2a+1)-(3+a^2+a)(a-1)$

$\Leftrightarrow \cancel{3a}+\cancel{a^3}+\cancel{2a^2}+\cancel{a}-3-\cancel{a^2}-2a-1-\cancel{3a}+3-\cancel{a^3}+a^2$
$-\cancel{a^2}-\cancel{a}$

$\Leftrightarrow a^2-2a-1 \neq 0$

$\Leftrightarrow a \neq 1 \pm \sqrt{2}$

Ma trận A khả nghịch $\forall a \neq 1 \pm \sqrt{2}$

Bài 14.

$$a_k A^k + a_{k-1} A^{k-1} + \dots a_1 A + a_0 E_0 = 0$$

$$a_k A^k + a_{k-1} A^{k-1} + \dots a_1 A = -a_0 E_0$$

$$-\frac{a_k}{a_0} A^k + \left(-\frac{a_{k-1}}{a_0}\right) A^{k-1} + \left(-\frac{a_1}{a_0}\right) A = E_0$$

$$\Rightarrow A \left(-\frac{a_k}{a_0} A^{k-1} - \frac{a_{k-1}}{a_0} A^{k-2} \dots - \frac{a_1}{a_0}\right) = E_0$$

$\forall$ ma trận vuông A thoả mãn $\Rightarrow$ A khả nghịch

Bài 15

$$AX + B = C \;;\; A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & 3 & 4 \\ 3 & 1 & -1 \end{bmatrix} \;;\; B = \begin{bmatrix} -1 & 2 \\ 3 & 4 \\ 0 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 18 & 10 \\ 6 & 16 & 7 \end{bmatrix}$$

$$\Rightarrow X = A^{-1}(C^T - B)$$

$$A^{-1} = \begin{pmatrix} -1/4 & \frac{3}{28} & 5/28 \\ \frac{1}{2} & \frac{-1}{14} & 3/14 \\ -1/4 & 1/4 & -1/4 \end{pmatrix}$$

$$C^T - B = \begin{bmatrix} 2 & 6 \\ 12 & 16 \\ 10 & 7 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 3 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 9 & 12 \\ 10 & 4 \end{bmatrix}$$

$$X = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix}$$