

Bài 1:

3;

$$\begin{cases} x_1 - x_2 + x_3 + x_4 = 1 \\ 2x_1 + x_2 + x_3 + 3x_4 = 8 \\ -3x_1 + 2x_2 - x_3 = b \\ 4x_1 + 4x_2 + 3x_3 + ax_4 = 14 \end{cases}$$

Với $a = 4$; $b = -5$

$$\Rightarrow \begin{cases} x_1 - x_2 + x_3 + x_4 = 1 \\ 2x_1 + x_2 + x_3 + 3x_4 = 8 \\ -3x_1 + 2x_2 - x_3 = -5 \\ 4x_1 + 4x_2 + 3x_3 + 4x_4 = 14 \end{cases}$$

$$(=) \left| \begin{array}{ccccc} 1 & -1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 3 & 8 \\ -3 & 2 & -1 & 0 & -5 \\ 4 & 4 & 3 & 4 & 14 \end{array} \right|$$

$$\Leftrightarrow \left| \begin{array}{ccccc} 1 & -1 & 1 & 1 & 1 \\ 0 & 3 & -1 & 1 & 6 \\ 0 & -1 & 2 & 3 & -2 \\ 0 & -8 & 1 & 0 & -10 \end{array} \right|$$

$$(=) \left| \begin{array}{ccccc} 1 & -1 & 1 & 1 & 1 \\ 0 & 3 & -1 & 1 & 6 \\ 0 & 0 & 5 & 10 & 0 \\ 0 & 0 & -5 & 8 & 18 \end{array} \right|$$

$$(=) \left| \begin{array}{ccccc} 1 & -1 & 1 & 1 & 1 \\ 0 & 3 & -1 & 1 & 6 \\ 0 & 0 & 5 & 10 & 0 \\ 0 & 0 & 0 & 18 & 18 \end{array} \right|$$

$r(A^B)$ = $r(A)$ \Rightarrow có 1 nghiệm duy

nhất

$$x_1, x_2, x_3, x_4 = \left\{ \begin{array}{l} 3 \\ 1 \\ -2 \\ 1 \end{array} \right.$$

$$b) \begin{cases} x_1 - x_2 + x_3 + x_4 = 1 \\ 2x_1 + x_2 + x_3 + 3x_4 = 8 \\ -3x_1 + 2x_2 - x_3 = b \\ 4x_1 + 4x_2 + 3x_3 + ax_4 = 14 \end{cases}$$

$$\Leftrightarrow \left| \begin{array}{ccccc} 1 & -1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 3 & 8 \\ -3 & 2 & -1 & 0 & b \\ 4 & 4 & 3 & a & 14 \end{array} \right|$$

$$\Leftrightarrow \left| \begin{array}{ccccc} 1 & -1 & 1 & 1 & 1 \\ 0 & 3 & -1 & 1 & 6 \\ 0 & -1 & 2 & 3 & 3+b \\ 0 & 8 & -1 & a-4 & 10 \end{array} \right|$$

$$\Rightarrow \left| \begin{array}{ccccc} 1 & -1 & 1 & 1 & 1 \\ 0 & 3 & -1 & 1 & 6 \\ 0 & 0 & 5 & 10 & 15+3b \\ 0 & 0 & 5 & 20-3a & 18 \end{array} \right|$$

$$\Leftrightarrow \left| \begin{array}{ccccc} 1 & -1 & 1 & 1 & 1 \\ 0 & 3 & -1 & 1 & 6 \\ 0 & 0 & 5 & 10 & 15+3b \\ 0 & 0 & 30-3a & 33+3b & \end{array} \right|$$

A vô nghiệm (2) $\lambda(A^{bs}) \neq \lambda(A)$.

$$(=) \quad \left\{ \begin{array}{l} 3x_0 - 3a = 0 \\ a = 30/3. \end{array} \right.$$

$$3b + 3b \neq 0$$

$$(=) \quad \left\{ \begin{array}{l} b \neq -11. \\ b \neq 11. \end{array} \right.$$

Bài 2:

$$\left\{ \begin{array}{l} x_1 + x_2 + x_3 + ax_4 = 5. \\ \text{c)} \end{array} \right.$$

$$\left\{ \begin{array}{l} 2x_1 + 2x_2 - x_3 + 3x_4 = 10 \\ -2x_1 - x_2 + x_3 + x_4 = b \\ 2x_1 + 3x_2 + 4x_3 + 2x_4 = 11. \end{array} \right.$$

$$(1) \quad \left\{ \begin{array}{l} x_1 + x_2 + x_3 + x_4 = 5 \\ 2x_1 + 2x_2 - x_3 + 3x_4 = 10 \end{array} \right.$$

$$\left\{ \begin{array}{l} -2x_1 - x_2 + x_3 + x_4 = b \\ 2x_1 + 3x_2 + 4x_3 + 2x_4 = 11. \end{array} \right.$$

$$(2) \quad \left\{ \begin{array}{l} 1. \quad 1. \quad 1. \quad 1. \quad 5. \\ 2. \quad 2. \quad -1. \quad 3. \quad 10. \\ -2. \quad -1. \quad 1. \quad 1. \quad 3. \\ 2. \quad 3. \quad 4. \quad 2. \quad 11 \end{array} \right.$$

$$(2) \quad \left| \begin{array}{ccccc} 1 & 1 & 1 & 1 & 5 \\ 0 & 0 & -3 & 1 & 0 \\ 0 & 1 & 3 & 3 & 13 \\ 0 & 1 & 2 & 0 & 1 \end{array} \right|.$$

$$\left| \begin{array}{ccccc} 1 & 1 & 1 & 1 & 5 \\ 0 & 0 & -3 & 1 & 0 \\ 0 & 1 & 3 & 3 & 13 \\ 0 & 1 & 2 & 0 & 1 \end{array} \right|.$$

$$\left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 3 & 3 & 3 & 13 \\ 0 & 0 & -3 & 1 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & -1 & -3 & -12 \\ 0 & 0 & -3 & 1 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & -1 & -3 & -12 \\ 0 & 0 & -3 & 1 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & -1 & -3 & -12 \\ 0 & 0 & -3 & 1 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & -1 & -3 & -12 \\ 0 & 0 & -3 & 1 & 0 \end{array} \right)$$

$$\Rightarrow \begin{cases} x_1 = 1,6 \\ x_2 = -1,4 \\ x_3 = 1,2 \\ x_4 = 3,6 \end{cases}$$

$$b_j \left| \begin{array}{ccccc} 1 & 1 & 1 & a & 5 \\ 2 & 2 & -1 & 3 & 10 \\ -2 & -1 & 1 & 1 & b \\ 2 & 3 & 4 & 2 & 11 \end{array} \right|$$

$$(=) \left| \begin{array}{ccccc} 1 & 1 & 1 & a & 5 \\ 2 & 2 & -1 & 3 & 10 \\ -2 & -1 & 1 & 1 & b \\ 2 & 3 & 4 & 2 & 11 \end{array} \right|$$

$$(+) \left| \begin{array}{ccccc} 2 & 3 & 4 & 2 & 11 \\ 2 & 2 & -1 & 3 & 10 \\ -2 & -1 & 1 & 1 & b \\ 1 & 1 & 1 & a & 5 \end{array} \right|$$

$$(=) \left| \begin{array}{ccccc} 2 & 3 & 4 & 2 & 11 \\ 0 & 1 & 5 & -1 & 1 \\ 0 & 2 & 5 & 3 & 11+b \\ 0 & 1 & 2 & 2-2a & 1 \end{array} \right|$$

$$(=) \left| \begin{array}{ccccc} 2 & 3 & 4 & 2 & 1 \\ 0 & 1 & 5 & -1 & 1 \\ 0 & 0 & -5 & 5 & 9+b \\ 0 & 0 & 3 & 2a-3 & 0 \end{array} \right|$$

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$$\left| \begin{array}{ccccc} 2 & 3 & 4 & 2 & 11 \\ 0 & 1 & 5 & -1 & 1 \\ 0 & 0 & -5 & 5 & 9+b \\ 0 & 0 & 3 & 2a-3 & 0 \end{array} \right|$$

(=)

$$\left| \begin{array}{ccccc} 2 & 3 & 4 & 2 & 11 \\ 0 & 1 & 5 & -1 & 1 \\ 0 & 0 & -5 & 5 & 9+b \\ 0 & 0 & 0 & 10a & 3.(9+b) \end{array} \right|$$

Vô số nghiệm ($\Leftrightarrow \text{r}(A^{BS}) = \text{r}(A)$) và $a < n$

$$\Rightarrow \begin{cases} 10a = 0 \\ 3(9+b) = 0 \end{cases} \quad a = 0 \quad (=)$$

$$3(9+b) = 0 \quad b = -9$$

Bài 3.

i;

$$\begin{cases} x_1 + 2x_2 - 4x_3 + x_4 = 4 \\ 3x_1 + 5x_2 + x_3 - 2x_4 = 7 \\ 2x_1 + 3x_2 + ax_3 - 3x_4 = b \end{cases}$$

(*)

$$\left| \begin{array}{ccccc} 1 & 2 & -4 & 1 & 4 \\ 3 & 5 & 1 & -2 & 7 \\ 2 & 3 & a & -3 & b \end{array} \right|$$

$$(2) \left| \begin{array}{ccccc} 1 & 2 & -4 & 1 & 4 \\ 0 & -1 & 13 & -5 & 5 \\ 0 & -1 & a+8 & -5 & b-8 \end{array} \right|$$

$$(2) \left| \begin{array}{ccccc} 1 & 2 & -4 & 1 & 4 \\ 0 & -1 & 13 & -5 & 5 \\ 0 & 0 & 5-a & 0 & 13-b \end{array} \right|$$

Nếu $\begin{cases} 5-a = 0 \\ 13-b \neq 0 \end{cases} \Leftrightarrow \begin{cases} a=5 \\ b \neq 13 \end{cases} \Rightarrow$ pt vô nghiệm.

Nếu $\begin{cases} 5-a = 0 \\ 13-b = 0 \end{cases} \Leftrightarrow \begin{cases} a=5 \\ b=13 \end{cases} \Rightarrow$ pt vô số nghiệm.

$$\text{Nếu } (5-a)x_3 = 13-b$$

$$\Rightarrow x_3 = \frac{13-b}{5-a}$$

$$\Rightarrow -x_2 + 13x_3 - 5x_4 = 5$$

$$(2) x_2 = 13x_3 - 5x_4 - 5$$

$$(2) x_2 = 13 \cdot \frac{13-b}{5-a} - 5\alpha - 5$$

$$\Rightarrow x_1 = 4 - \alpha + 4 \cdot \frac{13-b}{5-a} - 2x_2$$

$$\Rightarrow \begin{cases} x_1 = 4d + 4 \frac{13-b}{5-a} - 2 \left(13 \cdot \frac{13-b}{5-a} - 5d \right) \\ x_2 = 13 \cdot \frac{13-b}{5-a} - 5d - 5 \end{cases}$$

$$\begin{cases} x_3 = \frac{13-b}{5-a} \\ x_4 = d \end{cases}$$

$$(2) \quad \begin{cases} x_1 = 14 + 9d - 22 \cdot \frac{13-b}{5-a} \\ x_2 = 13 \cdot \frac{13-b}{5-a} - 5d - 5 \end{cases}$$

$$x_3 = \frac{13-b}{5-a}$$

$$x_4 = d$$

$$(ii) \quad \begin{cases} x_1 + 3x_2 + x_3 - 4x_4 = 5 \\ 2x_1 + 9x_2 - 2x_3 + 2x_4 = 8 \\ x_1 + 4x_2 - 3x_3 + ax_4 = 6 \end{cases}$$

$$(=) \quad \begin{cases} 1 & 3 & 1 & -4 & 5 \\ 2 & 9 & -2 & 2 & 8 \\ 1 & 4 & -3 & a & b \end{cases}$$

$$(2) \left| \begin{array}{ccccc} 1 & 3 & 1 & -4 & 5 \\ 0 & 1 & -4 & 10 & -2 \\ 0 & 0 & 0 & 6-a & 3-b \end{array} \right|$$

Nếu $6-a=0$ và $3-b \neq 0 \Leftrightarrow a=6$

Và $b \neq 3$ thì pt vô nghiệm

Nếu $\begin{cases} 6-a=0 \\ 3-b=0 \end{cases} \Leftrightarrow \begin{cases} a=6 \\ b=3 \end{cases}$ pt vô số nghiệm

$$(3) \begin{cases} x_1 + 3x_2 + x_3 - 4x_4 = 5 \\ x_2 - 4x_3 + 10x_4 = -2. \end{cases}$$

$$(2) \begin{cases} x_1 = 11 - 13x_2 + 3x_3 \\ x_2 = -2 + 4x_3 - 10x_4 \end{cases}$$

$$x_3 = \alpha$$

$$x_4 = \beta$$

$$\text{Nếu } (b-a)x_4 = 3-b$$

$$\Rightarrow x_4 = \frac{3-b}{6-a}$$

$$\Rightarrow \begin{cases} x_1 + 3x_2 + x_3 - 4x_4 = 5 \\ x_2 - 4x_3 + 10x_4 = -2. \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = 5 - 3x_2 - x_3 + 4x_4 \\ x_2 = -2 - 10\left(\frac{3-b}{6-a}\right) + 4x_3 \end{cases}$$

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$$x_1 = 5 - 3 \left(-2 - 10 \left(\frac{3-b}{6-a} \right) + 4d \right) - d + 4 \left(\frac{3-b}{6-a} \right)$$

$$\Leftrightarrow x_1 = 11 + 34 \left(\frac{3-b}{6-a} \right) - 13d$$

$$\Rightarrow \begin{cases} x_1 = 11 + 34 \left(\frac{3-b}{6-a} \right) - 13d \\ x_2 = -2 - 10 \left(\frac{3-b}{6-a} \right) + 4d \end{cases}$$

$$\begin{cases} x_3 = d \\ x_4 = \frac{3-b}{6-a} \end{cases}$$

Bài 4

i) $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = -2 \\ 2x_1 + 2x_2 + 3x_3 + 5x_4 = -2 \\ 3x_1 + x_2 + 2x_3 + x_4 = 2 \\ 2x_1 + 6x_2 + 7x_3 + 13x_4 = -10 \end{cases}$

$$\begin{array}{c|ccccc} (\exists) & 1 & 1 & 2 & 3 & -2 \\ & 2 & 2 & 3 & 5 & -2 \\ & 3 & -1 & 2 & 1 & 2 \\ & 2 & 6 & 7 & 13 & -10 \end{array}$$

$$(1) \begin{vmatrix} 1 & 1 & 1 & 2 & 3 & -2 \\ 0 & 0 & -1 & -1 & -1 & 2 \\ 0 & -4 & -4 & -5 & 8 & 1 \end{vmatrix}$$

$$(2) \begin{vmatrix} 1 & 1 & 2 & 3 & -2 \\ 0 & 4 & 3 & 7 & -6 \\ 0 & -4 & -4 & -5 & 8 \\ 0 & 0 & -1 & -1 & 2 \end{vmatrix}$$

$$(2) \begin{vmatrix} 1 & 1 & 2 & 3 & -2 \\ 0 & 4 & 3 & 7 & -6 \\ 0 & 0 & -1 & 2 & 2 \\ 0 & 0 & -1 & -1 & 2 \end{vmatrix}$$

$$(3) \begin{vmatrix} 1 & 1 & 2 & 3 & -2 \\ 0 & 4 & 3 & 7 & -6 \\ 0 & 0 & -1 & 2 & 2 \\ 0 & 0 & 0 & 3 & 0 \end{vmatrix}$$

$$\Rightarrow 3x_4 = 0 \Rightarrow x_4 = 0$$

$$\Rightarrow -x_3 = 2 \Rightarrow x_3 = -2$$

$$\Rightarrow x_2 = 0 \Rightarrow x_1 = 2$$

$$\Rightarrow x_1 = 1; x_2 = 0; x_3 = -2; x_4 = 0$$

Trung

Nguyễn

$$\text{ii) } \begin{cases} x_1 + 2x_2 + x_3 + x_4 = 4 \\ 2x_1 + x_2 + x_3 + 4x_4 = 3 \\ -x_1 + x_2 + 2x_3 + x_4 = -1 \\ 2x_1 + 4x_2 + 4x_3 + 6x_4 = 6 \end{cases}$$

$$\text{(=)} \left| \begin{array}{ccccc} 1 & 2 & 1 & 1 & 4 \\ 2 & 1 & 1 & 4 & 3 \\ -1 & 1 & 2 & 1 & -1 \\ 2 & 4 & 4 & 6 & 6 \end{array} \right|$$

$$\text{(=)} \left| \begin{array}{ccccc} 1 & 2 & 1 & 1 & 4 \\ 0 & -3 & -1 & 2 & -5 \\ 0 & 3 & 3 & 2 & 3 \\ 0 & -0 & 2 & 4 & -2 \end{array} \right|$$

$$\text{(=)} \left| \begin{array}{ccccc} 1 & 2 & 1 & 1 & 4 \\ 0 & -3 & -1 & 2 & -5 \\ 0 & 0 & 2 & 4 & -2 \end{array} \right|$$

$$\text{(=)} \left| \begin{array}{ccccc} 1 & 2 & 1 & 1 & 4 \\ 0 & -3 & -1 & 2 & -5 \\ 0 & 0 & 2 & 4 & -2 \end{array} \right|$$

$$\text{(=)} \left| \begin{array}{ccccc} 1 & 2 & 1 & 1 & 4 \\ 0 & -3 & -1 & 2 & -5 \\ 0 & 0 & 2 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right|$$

pt vô số nghiệm

$$(5) \quad \left\{ \begin{array}{l} x_1 + 2x_2 + x_3 + x_4 = 4 \\ -3x_2 - x_3 + 2x_4 = -5 \\ 2x_3 + 4x_4 = -2 \end{array} \right.$$

$$(6) \quad \left\{ \begin{array}{l} x_1 = 4 - 2x_2 - x_3 - x_4 \\ x_2 = \frac{-5 + x_3 - 2x_4}{-3} \\ x_3 = -1 - 2\lambda \end{array} \right.$$

$$\Rightarrow x_2 = -\frac{1}{3}(-5 + 1 - 2\lambda - 2\lambda).$$

$$x_2 = -\frac{1}{3}(-6 - 4\lambda).$$

$$\Rightarrow x_1 = 4 - \frac{2}{3}(-6 - 4\lambda) + (1 + 2\lambda) - \lambda$$

$$x_1 = 4 + 4 + \frac{8}{3}\lambda + 1 + 2\lambda - \lambda.$$

$$x_1 = 9 + \frac{11}{3}\lambda.$$

$$\Rightarrow \left\{ \begin{array}{l} x_1 = 9 + \frac{11}{3}\lambda \\ x_2 = -\frac{1}{3}(-6 - 4\lambda) \\ x_3 = -1 - 2\lambda \\ x_4 = \lambda \end{array} \right.$$