LISTA 5 - PROBABILIDADE 1

(1)

$3x + x = 1 \rightarrow 4x = 1 \rightarrow x = \frac{1}{4}$

$P(\text{SAIR CARA}) = 3x \rightarrow 3 \cdot \frac{1}{4} = \frac{3}{4}$

$P(\text{SAIR COROA}) = x \rightarrow \frac{1}{4}$

$S_x = \{0, 1, 2\} \rightarrow \{(c,c); (c,k); (k,c); (k;k)\}$

$P(x=0) = \frac{1}{4} \cdot \frac{1}{4} = \boxed{\frac{1}{16}}$

$P(x=1) = \frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16} + \frac{3}{16} = \boxed{\frac{6}{16}}$

$P(x=2) = \frac{3}{4} \cdot \frac{3}{4} = \boxed{\frac{9}{16}}$

(2)

$S_x = \{0, 1, 2, 3\} \rightarrow \{(P,P,P); (P,P,D); (P,D,P); (D,P,P); (P,D,D); (D,P,D); (D,D,P); (D,D,D)\}$

a) COM REPOSIÇÃO

$P(x=0) = \frac{15}{20} \cdot \frac{15}{20} \cdot \frac{15}{20} = \boxed{\frac{3.375}{8.000}}$

$P(x=1) = \frac{15}{20} \cdot \frac{15}{20} \cdot \frac{5}{20} + \frac{15}{20} \cdot \frac{5}{20} \cdot \frac{15}{20} + \frac{5}{20} \cdot \frac{15}{20} \cdot \frac{15}{20}$

$= 3\left(\frac{1.125}{8000}\right) = \boxed{\frac{3.375}{8.000}}$

$P(x=2) = \frac{15}{20}\cdot\frac{5}{20}\cdot\frac{5}{20} + \frac{5}{20}\cdot\frac{15}{20}\cdot\frac{5}{20} + \frac{5}{20}\cdot\frac{5}{20}\cdot\frac{15}{20}$

$= 3\left(\frac{375}{8000}\right) = \boxed{\frac{1.125}{8000}}$

$P(x=3) = \frac{5}{20}\cdot\frac{5}{20}\cdot\frac{5}{20} = \boxed{\frac{125}{8000}}$

b) SEM REPOSIÇÃO

$P(x=0) = \frac{15}{20}\cdot\frac{14}{19}\cdot\frac{13}{18} = \boxed{\frac{2.730}{6.840}}$

$P(x=1) = \frac{15}{20}\cdot\frac{14}{19}\cdot\frac{5}{18} + \frac{15}{20}\cdot\frac{5}{19}\cdot\frac{14}{18} + \frac{5}{20}\cdot\frac{15}{19}\cdot\frac{14}{18}$

$= 3\left(\frac{1.050}{6.840}\right) = \boxed{\frac{3.150}{6.840}}$

$P(x=2) = \frac{15}{20}\cdot\frac{5}{19}\cdot\frac{4}{18} + \frac{5}{20}\cdot\frac{15}{19}\cdot\frac{4}{18} + \frac{5}{20}\cdot\frac{4}{19}\cdot\frac{15}{18}$

$= 3\left(\frac{300}{6.840}\right) = \boxed{\frac{900}{6.840}}$

(3)

$x = a + b$

$S_x = \{2, 3, 4, 5\}$

$(1,1) = 2$

$(2,1); (1,2) = 3$

$(3,1); (1,3); (2,2) = 4$

$(3,2); (2,3) = 5$

$P(x=2) = \frac{2}{5}\cdot\frac{1}{4} = \boxed{\frac{2}{20}}$

$P(x=3) = \frac{2}{5}\cdot\frac{2}{4} + \frac{2}{5}\cdot\frac{2}{4} = \frac{4}{20} + \frac{4}{20} = \boxed{\frac{8}{20}}$

$$P(x=4) = \frac{1}{5}\cdot\frac{2}{4} + \frac{2}{5}\cdot\frac{1}{4} \cdot \frac{2}{5}\cdot\frac{1}{4} = \frac{2}{20} + \frac{2}{20} + \frac{2}{20} = \boxed{\frac{6}{20}}$$

$$P(x=5) = \frac{2}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{2}{4} = \frac{2}{20} + \frac{2}{20} = \boxed{\frac{4}{20}}$$

(4)

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

$x = a + b$

$Y = \max(a, b)$

$S_x = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$(1,1) = 2$

$(2,1); (1,2) = 3$

$(3,1); (2,2), (1,3) = 4$

$(4,1); (3,2); (2,3); (1,4) = 5$

$(5,1); (4,2); (3,3); (2,4); (1,5) = 6$

$(6,1); (5,2); (4,3); (3,4), (2,5); (1,6) = 7$

$(6,2); (5,3); (4,4); (3,5); (2,6) = 8$

$(6,3); (5,4); (4,5); (3,6) = 9$

$(6,4); (5,5); (4,6) = 10$

$(6,5); (5,6) = 11$

$(6,6) = 12$

$S_Y = \{1, 2, 3, 4, 5, 6\}$

$$\begin{cases} (1,1)=1 \\ (1,2);(2,2);(2,1)=2 \\ (1,3);(2,3);(3,3);(3,2);(3,1)=3 \\ (1,4);(2,4);(3,4),(4,4);(4,3);(4,2);(4,1)=4 \\ (1,5);(2,5);(3,5),(4,5);(5,5);(5,4);(5,3);(5,2);(5,1)=5 \\ (1,6),(2,6);(3,6);(4,6);(5,6);(6,6);(6,5);(6,4);(6,3);(6,2);(6,1)=6 \end{cases}$$

$$P(Y=2)=\frac{1}{6}\cdot\frac{1}{6}=\boxed{\frac{1}{36}}$$

$$P(Y=3)=\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}+\frac{1}{36}=\boxed{\frac{2}{36}}$$

$$P(Y=4)=\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}=\boxed{\frac{3}{36}}$$

$$P(Y=5)=\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}=\boxed{\frac{4}{36}}$$

$$P(Y=6)=\frac{1}{6}\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}=\boxed{\frac{5}{36}}$$

$$P(Y=7)=\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}$$

$$=\boxed{\frac{6}{36}}$$

$$P(Y=8)=\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}=\boxed{\frac{5}{36}}$$

$$P(Y=9)=\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\frac{1}{6}=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}=\boxed{\frac{4}{36}}$$

$$P(Y=10)=\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}=\boxed{\frac{3}{36}}$$

$$P(Y=11)=\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}+\frac{1}{36}=\boxed{\frac{2}{36}}$$

$$P(Y=12)=\frac{1}{6}\cdot\frac{1}{6}=\boxed{\frac{1}{36}}$$

$P(x=1) = \frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$

$P(x=2) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{3}{36}}$

$P(x=3) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{5}{36}}$

$P(x=4) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{7}{36}}$

$P(x=5) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{9}{36}}$

$P(x=6) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{11}{36}}$

(5)

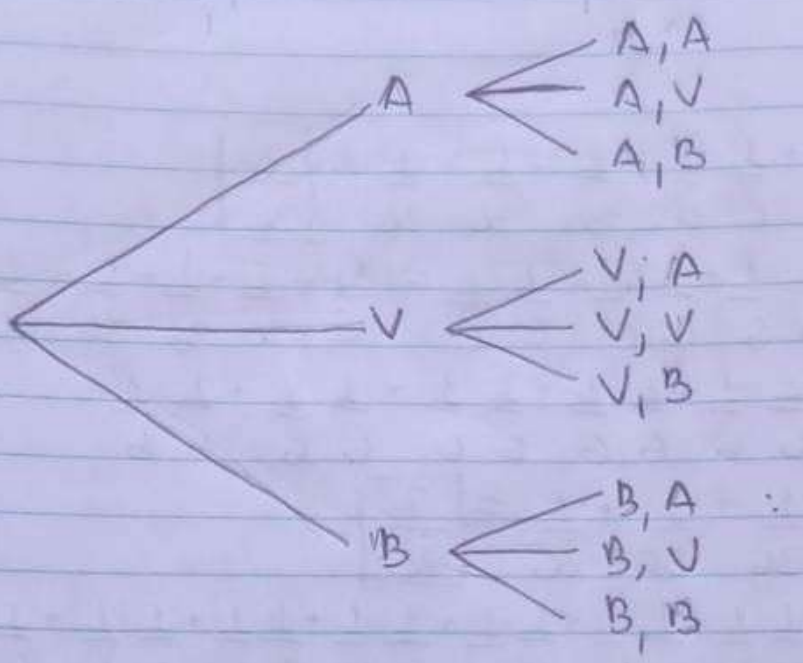
a) $2a + a + 4a = 1 \rightarrow 7a = 1 \rightarrow a = \frac{1}{7}$

b) $P(0 \leq X \leq 3) = P(x=1) + P(x=2) + P(x=3)$

$2a + a + 4a = 7a \rightarrow 7 \cdot \frac{1}{4} = \boxed{\frac{7}{7}}$

$P(0 < x < 2) = P(x=1) = 2a \rightarrow 2 \cdot \frac{1}{7} = \boxed{\frac{2}{7}}$

(6)



X = BOLAS BRANCAS
Y = BOLAS VERDES

$S_x = \{0, 1, 2\}$

$S_y = \{0, 1, 2\}$

a) COM REPOSIÇÃO

$P(x=0) = \frac{9}{14} \cdot \frac{9}{14} = \boxed{\frac{81}{196}}$

$P(x=1) = \frac{5}{14} \cdot \frac{9}{14} + \frac{9}{14} \cdot \frac{5}{14} = \frac{45}{196} + \frac{45}{196} = \boxed{\frac{90}{196}}$

$P(x=2) = \frac{5}{14} \cdot \frac{5}{14} = \boxed{\frac{25}{196}}$

$P(y=0) = \frac{11}{14} \cdot \frac{11}{14} = \boxed{\frac{121}{196}}$

$P(y=1) = \frac{11}{14} \cdot \frac{3}{14} + \frac{3}{14} \cdot \frac{11}{14} = \frac{33}{196} + \frac{33}{196} = \boxed{\frac{66}{196}}$

$P(x=2) = \frac{3}{14} \cdot \frac{3}{14} = \boxed{\frac{9}{196}}$

b) SEM REPOSIÇÃO

$P(Y=0) = \frac{11}{14} \cdot \frac{10}{13} = \boxed{\frac{110}{182}}$

$P(Y=1) = \frac{3}{14} \cdot \frac{11}{13} + \frac{11}{14} \cdot \frac{3}{13} = \frac{33}{182} + \frac{33}{182} = \boxed{\frac{66}{182}}$

$P(Y=2) = \frac{3}{14} \cdot \frac{2}{13} = \frac{6}{182}$

$P(x=0) = \frac{9}{14} \cdot \frac{8}{13} = \boxed{\frac{72}{182}}$

$P(x=1) = \frac{5}{14} \cdot \frac{9}{13} + \frac{9}{14} \cdot \frac{5}{13} = \frac{45}{182} + \frac{45}{182} = \boxed{\frac{90}{182}}$

$P(x=2) = \frac{3}{14} \cdot \frac{2}{13} = \boxed{\frac{6}{182}}$

(7)

$X = a + b$

a) $P(x<5) = P(x=2) + P(x=3) + P(x=4)$

$= \left(\frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}\right)$

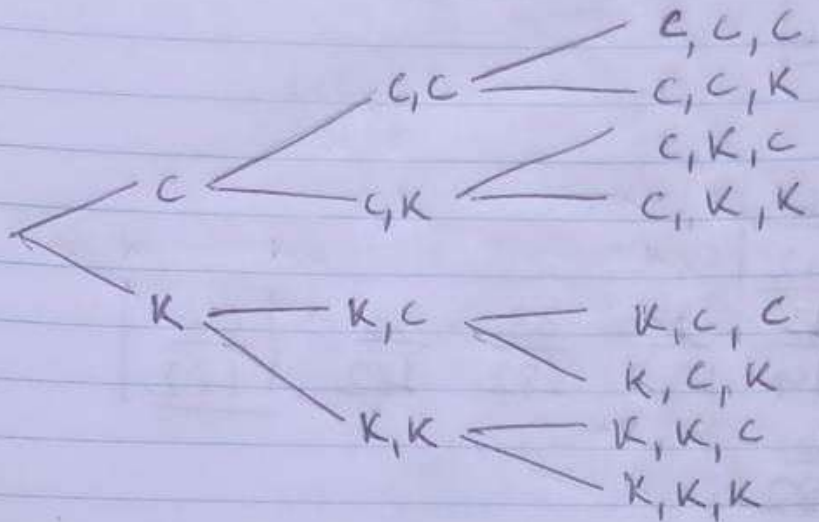
$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \boxed{\frac{6}{36}}$

b) $P(4<x<8) = P(x=5) + P(x=6) + P(x=7)$

$= \left(\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}\right) +$

$\left(\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}\right) = \frac{4}{36} + \frac{5}{36} + \frac{6}{36} = \boxed{\frac{15}{36}}$

(8)



X = Nº DE COROAS

$S_x = \{0, 1, 2, 3\}$

$P(x=0) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{8}}$

$P(x=1) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \boxed{\frac{3}{8}}$

$P(x=1) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \boxed{\frac{3}{8}}$

$P(x=3) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{8}}$

(9) $p + 2p = 1 \rightarrow 3p = 1 \rightarrow p = \frac{1}{3}$

$P(x>5) = P(x=10) = 2p \rightarrow 2 \cdot \frac{1}{3} = \frac{2}{3}$