Lista 6 - Probabilidade 1

(1)

$P(X=k) = 3P(X=c)$

$3x + x = 1 \rightarrow 4x = 1 \rightarrow x = \frac{1}{4}$

$P(X=k) = 3x \rightarrow P(X=k) = 3 \cdot \frac{1}{4} \rightarrow P(X=k) = \frac{3}{4}$

$P(X=c) = x \rightarrow P(X=c) = \frac{1}{4}$

$S_x = \{0, 1, 2\}$

a) $P(X=0) = P(X_1=c) \cdot P(X_2=c) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

$P(X=1) = P(X_1=c) \cdot P(X_2=k) + P(X_1=k) \cdot P(X_2=c)$

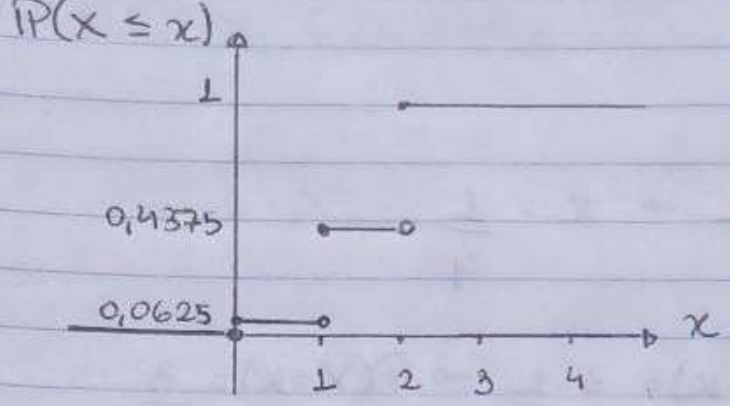
$P(X=1) = \frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16} + \frac{3}{16} = \frac{6}{16}$

$P(X=2) = P(X_1=k) \cdot P(X_2=k) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

b) $P(X \leq 0) = P(X=0) = \frac{1}{16}$

$P(X \leq 1) = P(X \leq 0) + P(X=1) = \frac{1}{16} + \frac{6}{16} = \frac{7}{16}$

$$P(X \leq 2) = P(X \leq 1) + P(X = 2) = \frac{7}{16} + \frac{9}{16} = \frac{16}{16}$$



c) $E(X) = \sum x \cdot P(X = x)$

$$E(X) = 0 \cdot \frac{1}{16} + 1 \cdot \frac{6}{16} + 2 \cdot \frac{9}{16} = 0 + \frac{6}{16} + \frac{18}{16} = \frac{24}{16}$$

$$E(X) = \frac{3}{2}$$

$V(X) = \sum x^2 \cdot P(X = x)$

$$V(X) = 0^2 \cdot \frac{1}{16} + 1^2 \cdot \frac{6}{16} + 2^2 \cdot \frac{9}{16} = 0 \cdot \frac{1}{16} + 1 \cdot \frac{6}{16} + 4 \cdot \frac{9}{16}$$

$$V(X) = 0 + \frac{6}{16} + \frac{36}{16} = \frac{42}{16} = \frac{21}{8}$$

(2)

a) Com reposição

$S_X = \{0, 1, 2, 3\}$

$$P(X = 0) = \frac{15}{20} \cdot \frac{15}{20} \cdot \frac{15}{20} = \frac{3\,375}{8000}$$

$$P(x=1) = \frac{15}{20}\cdot\frac{15}{20}\cdot\frac{5}{20} + \frac{15}{20}\cdot\frac{5}{20}\cdot\frac{15}{20} + \frac{5}{20}\cdot\frac{15}{20}\cdot\frac{15}{20} = 3\left(\frac{1.125}{8000}\right)$$

$$P(x=1) = \frac{3.375}{8000}$$

$$P(x=2) = \frac{15}{20}\cdot\frac{5}{20}\cdot\frac{5}{20} + \frac{5}{20}\cdot\frac{15}{20}\cdot\frac{5}{20} + \frac{5}{20}\cdot\frac{5}{20}\cdot\frac{15}{20} = 3\left(\frac{375}{8000}\right)$$

$$P(x=2) = \frac{1.125}{8000}$$

$$P(x=3) = \frac{5}{20}\cdot\frac{5}{20}\cdot\frac{5}{20} = \frac{125}{8000}$$

b) SEM REPOSIÇÃO

$$S_x = \{0, 1, 2, 3\}$$

$$P(x=0) = \frac{15}{20}\cdot\frac{14}{19}\cdot\frac{13}{18} = \frac{2.730}{6.840}$$

$$P(x=1) = \frac{15}{20}\cdot\frac{14}{19}\cdot\frac{5}{18} + \frac{15}{20}\cdot\frac{5}{19}\cdot\frac{14}{18} + \frac{5}{20}\cdot\frac{15}{19}\cdot\frac{14}{18} = 3\left(\frac{1050}{6840}\right)$$

$$P(x=1) = \frac{3.150}{6840}$$

$$P(x=2) = \frac{15}{20}\cdot\frac{5}{19}\cdot\frac{4}{18} + \frac{5}{20}\cdot\frac{15}{19}\cdot\frac{4}{18} + \frac{5}{20}\cdot\frac{4}{19}\cdot\frac{15}{18} = 3\left(\frac{300}{6840}\right)$$

$$P(x=2) = \frac{900}{6840}$$

$$P(x=3) = \frac{5}{20}\cdot\frac{4}{19}\cdot\frac{3}{18} = \frac{60}{6840}$$

c)

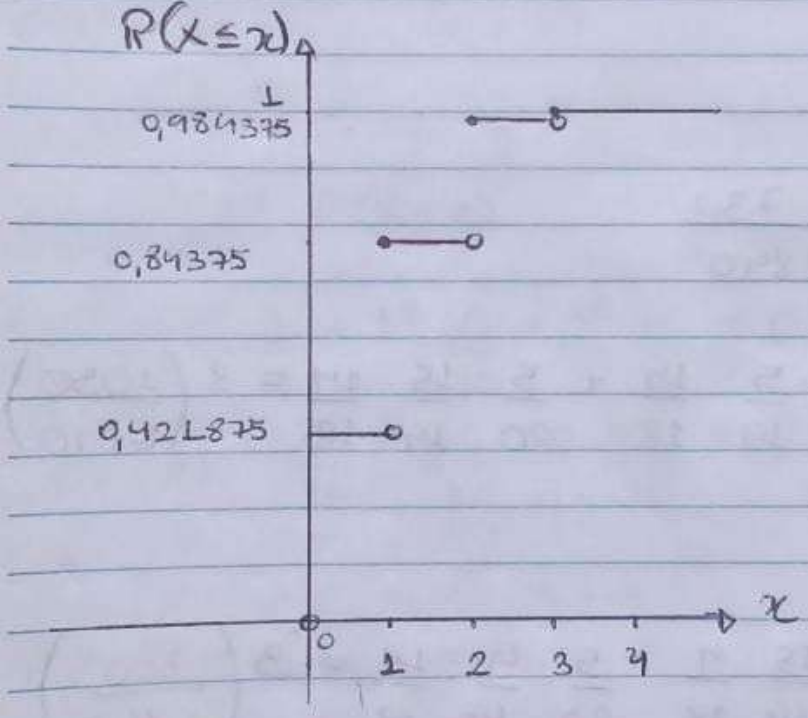
* COM REPOSIÇÃO

$P(X \leq 0) = P(X=0) = \frac{3.375}{8.000}$

$P(X \leq 1) = P(X \leq 0) + P(X=1) = \frac{3.375}{8000} + \frac{3.375}{8000} = \frac{6.750}{8000}$

$P(X \leq 2) = P(X \leq 1) + P(X=2) = \frac{6.750}{8000} + \frac{1.125}{8000} = \frac{7.875}{8000}$

$P(X \leq 3) = P(X \leq 2) + P(X=3) = \frac{7.875}{8.000} + \frac{125}{8000} = \frac{8000}{8000}$



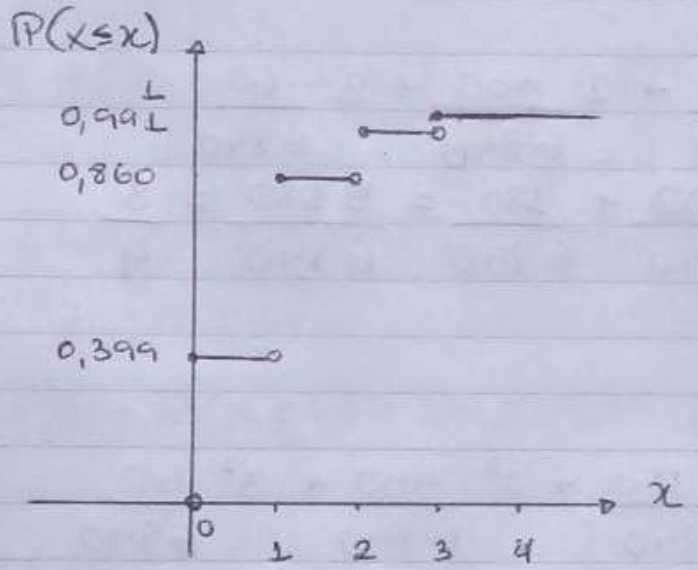
* SEM REPOSIÇÃO

$P(X \leq 0) = P(X=0) = \frac{2.730}{6.840}$

$$P(X \leq 1) = P(X \leq 0) + P(X = 1) = \frac{2730}{6.840} + \frac{3150}{6.840} = \frac{5.880}{6.840}$$

$$P(X \leq 2) = P(X \leq 1) + P(X = 2) = \frac{5.880}{6.840} + \frac{900}{6.840} = \frac{6.780}{6.840}$$

$$P(X \leq 3) = P(X \leq 2) + P(X = 3) = \frac{6.780}{6.840} + \frac{60}{6.840} = \frac{6840}{6.840}$$



d) [illegible]

* Com Reposição

$$E(X) = \sum x \, P(X = x)$$

$$E(X) = 0 \cdot \frac{3\,375}{8.000} + 1 \cdot \frac{3.375}{8.000} + 2 \cdot \frac{1.125}{8.000} + 3 \cdot \frac{125}{8.000}$$

$$E(X) = 0 + \frac{3375}{8.000} + \frac{2\,250}{8.000} + \frac{375}{8.000} = \frac{6.000}{8.000} = \frac{3}{4}$$

$$V(X) = \sum x^2 \cdot P(X = x)$$

$$V(x) = 0^2 \cdot \frac{3\,375}{8.000} + 1^2 \cdot \frac{3.375}{8.000} + 2^2 \cdot \frac{1.125}{8.000} + 3^2 \cdot \frac{125}{8000}$$

$$V(x) = 0 + \frac{3.375}{8.000} + \frac{4500}{8.000} + \frac{1125}{8.000} = \frac{9.000}{8.000} = \frac{9}{8}$$

* SEM REPOSIÇÃO

$$E(x) = \sum x \cdot P(X = x)$$

$$E(x) = 0 \cdot \frac{2.730}{6.840} + 1 \cdot \frac{3.150}{6.840} + 2 \cdot \frac{900}{6.840} + 3 \cdot \frac{60}{6.840}$$

$$E(x) = 0 + \frac{3.150}{6.840} + \frac{1800}{6.840} + \frac{180}{6.840} = \frac{5130}{6.840} = \frac{3}{4}$$

$$V(x) = \sum x^2 \cdot P(x = x)$$

$$V(x) = 0^2 \cdot \frac{2.730}{6.840} + 1^2 \cdot \frac{3.150}{6.840} + 2^2 \cdot \frac{900}{6.840} + 3^2 \cdot \frac{60}{6.840}$$

$$V(x) = 0 + \frac{3.150}{6.840} + \frac{3.600}{6.840} + \frac{540}{6.840} = \frac{7290}{6.840} = \frac{81}{76}$$

(3)

a)

$$Sx = \begin{cases} (1,1) = 2 \\ (1,2);(2,1) = 3 \\ (1,3);(3,1);(2,2) = 4 \\ (2,3);(3,2) = 5 \end{cases}$$

$$P(x = 2) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$$

$$P(x = 3) = \frac{2}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{2}{4} = \frac{4}{20} + \frac{4}{20} = \frac{8}{20}$$

$$P(X=4) = \frac{2}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{2}{4} + \frac{2}{5}\cdot\frac{1}{4} = \frac{2}{20} + \frac{2}{20} + \frac{2}{20} = \frac{6}{20}$$

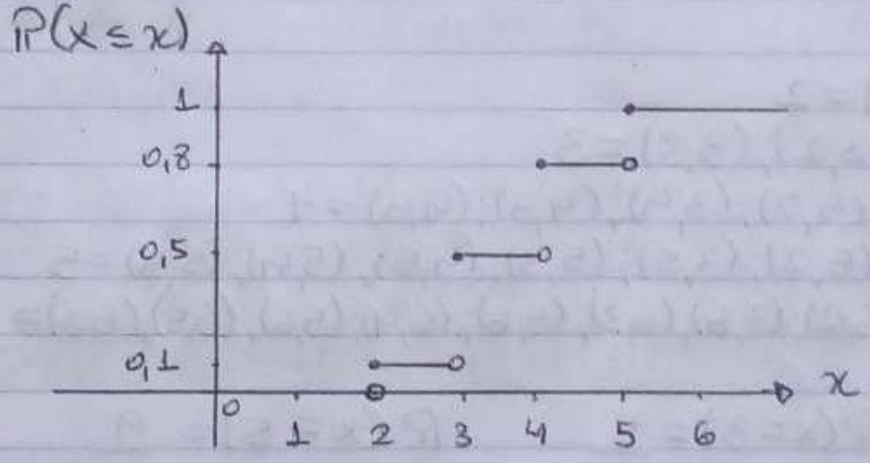
$$P(X=5) = \frac{2}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{2}{4} = \frac{2}{20} + \frac{2}{20} = \frac{4}{20}$$

b) $P(X \leq 2) = P(X=2) = \frac{2}{20}$

$$P(X \leq 3) = P(X=2) + P(X=3) = \frac{2}{20} + \frac{8}{20} = \frac{10}{20}$$

$$P(X \leq 4) = P(X \leq 3) + P(X=4) = \frac{10}{20} + \frac{6}{20} = \frac{16}{20}$$

$$P(X \leq 5) = P(X \leq 4) + P(X=5) = \frac{16}{20} + \frac{4}{20} = \frac{20}{20}$$



c) $E(X) = \sum x \cdot P(X=x)$

$$E(X) = 2\cdot\frac{2}{20} + 3\cdot\frac{8}{20} + 4\cdot\frac{6}{20} + 5\cdot\frac{4}{20} = \frac{4}{20} + \frac{24}{20} + \frac{24}{20} + \frac{20}{20}$$

$$E(x) = \frac{72}{20} = \frac{18}{5}$$

$$V(x) = \sum x^2 \cdot P(x = x)$$

$$V(x) = 2^2 \cdot \frac{2}{20} + 3^2 \cdot \frac{8}{20} + 4^2 \cdot \frac{6}{20} + 5^2 \cdot \frac{4}{20} = \frac{8}{20} + \frac{72}{20} + \frac{96}{20} + \frac{100}{20}$$

$$V(x) = \frac{348}{20} = \frac{87}{5}$$

d) $P(x \geq 2) = 1 - P(x < 2) = 1 - 0 = 1$

$P(x > 1) = P(x \geq 2) = 1$

④

$X = \max(a, b)$

$Y = a + b$

a) $S_x = \begin{cases} (1,1) = 1 \\ (1,2)(2,1);(2,2) = 2 \\ (1,3)(3,1)(2,3);(3,2);(3,3) = 3 \\ (1,4)(4,1)(2,4);(4,2);(3,4);(4,3);(4,4) = 4 \\ (1,5);(5,1);(2,5);(5,2);(3,5);(5,3);(4,5);(5,4);(5,5) = 5 \\ (1,6);(6,1);(2,6);(6,2);(3,6);(6,3);(4,6);(6,4);(5,6);(6,5);(6,6) = 6 \end{cases}$

$P(x = 1) = \frac{1}{36}$ $P(x = 3) = \frac{5}{36}$ $P(x = 5) = \frac{9}{36}$

$P(x = 2) = \frac{3}{36}$ $P(x = 4) = \frac{7}{36}$ $P(x = 6) = \frac{11}{36}$

$S_Y = \begin{cases} (1,1) = 2 \\ (1,2);(2,1) = 3 \\ (1,3);(3,1);(2,2) = 4 \\ (1,4);(4,1);(2,3);(3,2) = 5 \\ (1,5);(5,1);(2,4);(4,2);(3,3) = 6 \\ (1,6);(6,1);(2,5);(5,2);(3,4);(4,3) = 7 \\ (2,6);(6,2);(3,5);(5,3);(4,4) = 8 \\ (3,6);(6,3);(4,5);(5,4) = 9 \\ (4,6);(6,4);(5,5) = 10 \\ (5,6);(6,5) = 11 \\ (6,6) = 12 \end{cases}$

$P(Y=2) = \frac{1}{36}$ $P(Y=6) = \frac{5}{36}$ $P(Y=10) = \frac{3}{36}$

$P(Y=3) = \frac{2}{36}$ $P(Y=7) = \frac{6}{36}$ $P(Y=11) = \frac{2}{36}$

$P(Y=4) = \frac{3}{36}$ $P(Y=8) = \frac{5}{36}$ $P(Y=12) = \frac{1}{36}$

$P(Y=5) = \frac{4}{36}$ $P(Y=9) = \frac{4}{36}$

b) $P(X \leq 1) = P(X=1) = \frac{1}{36}$

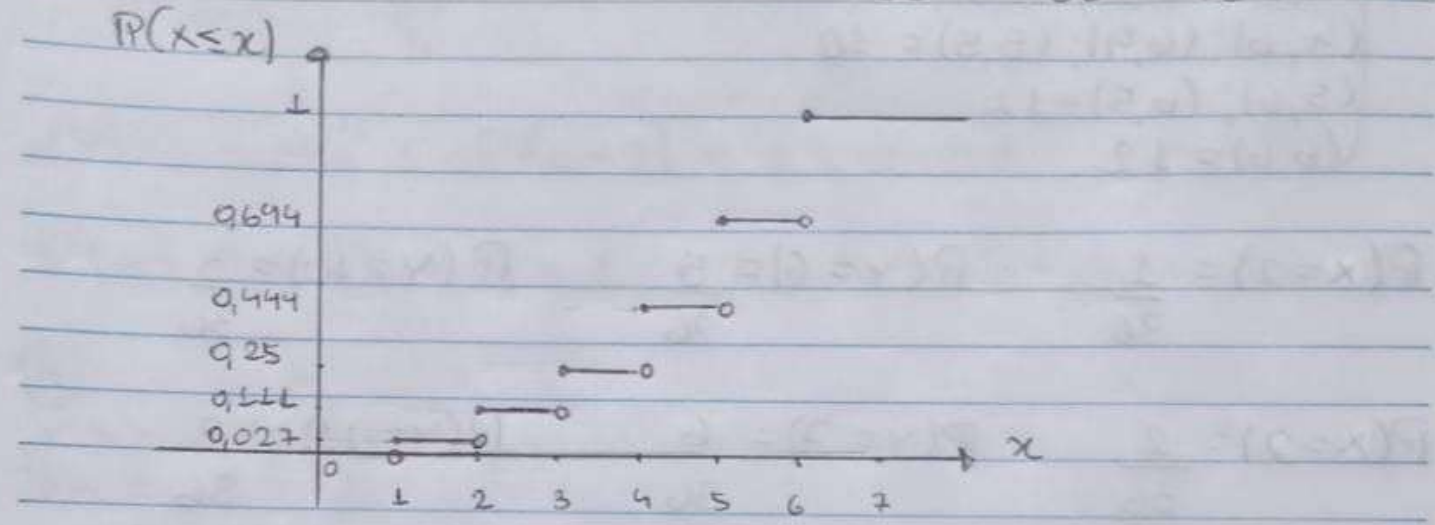
$P(X \leq 2) = P(X \leq 1) + P(X=2) = \frac{1}{36} + \frac{3}{36} = \frac{4}{36}$

$P(X \leq 3) = P(X \leq 2) + P(X=3) = \frac{4}{36} + \frac{5}{36} = \frac{9}{36}$

$$P(X \leq 4) = P(X \leq 3) + P(X = 4) = \frac{9}{36} + \frac{7}{36} = \frac{16}{36}$$

$$P(X \leq 5) = P(X \leq 4) + P(X = 5) = \frac{16}{36} + \frac{9}{36} = \frac{25}{36}$$

$$P(X \leq 6) = P(X \leq 5) + P(X = 6) = \frac{25}{36} + \frac{11}{36} = \frac{36}{36}$$



$$P(Y \leq 2) = P(Y = 2) = \frac{1}{36}$$

$$P(Y \leq 3) = P(Y \leq 2) + P(Y = 3) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$$

$$P(Y \leq 4) = P(Y \leq 3) + P(Y = 4) = \frac{3}{36} + \frac{3}{36} = \frac{6}{36}$$

$$P(Y \leq 5) = P(Y \leq 4) + P(Y = 5) = \frac{6}{36} + \frac{4}{36} = \frac{10}{36}$$

$$P(Y \leq 6) = P(Y \leq 5) + P(Y = 6) = \frac{10}{36} + \frac{5}{36} = \frac{15}{36}$$

$$P(Y \leq 7) = P(Y \leq 6) + P(Y = 7) = \frac{15}{36} + \frac{6}{36} = \frac{21}{36}$$

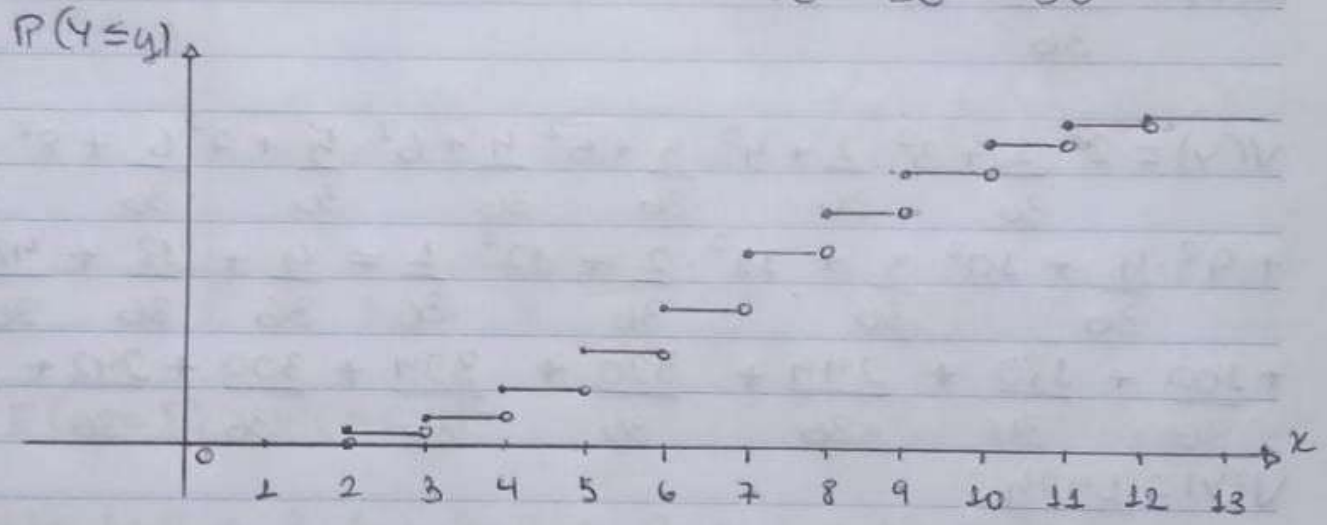
$$P(Y \leq 8) = P(Y \leq 7) + P(Y = 8) = \frac{21}{36} + \frac{5}{36} = \frac{26}{36}$$

$$P(Y \leq 9) = P(Y \leq 8) + P(Y = 9) = \frac{26}{36} + \frac{4}{36} = \frac{30}{36}$$

$$P(Y \leq 10) = P(Y \leq 9) + P(Y = 10) = \frac{30}{36} + \frac{3}{36} = \frac{33}{36}$$

$$P(Y \leq 11) = P(Y \leq 10) + P(Y = 11) = \frac{33}{36} + \frac{2}{36} = \frac{35}{36}$$

$$P(Y \leq 12) = P(Y \leq 11) + P(Y = 12) = \frac{35}{36} + \frac{1}{36} = \frac{36}{36}$$



d) $E(x) = \sum x \cdot P(X = x)$

$$E(x) = 1 \cdot \frac{1}{36} + 2 \cdot \frac{3}{36} + 3 \cdot \frac{5}{36} + 4 \cdot \frac{7}{36} + 5 \cdot \frac{9}{36} + 6 \cdot \frac{11}{36}$$

$$E(X) = \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36} = \frac{161}{36}$$

$$V(X) = 1^2 \cdot \frac{1}{36} + 2^2 \cdot \frac{3}{36} + 3^2 \cdot \frac{5}{36} + 4^2 \cdot \frac{7}{36} + 5^2 \cdot \frac{9}{36} + 6^2 \cdot \frac{11}{36}$$

$$V(X) = \frac{1}{36} + \frac{12}{36} + \frac{45}{36} + \frac{112}{36} + \frac{225}{36} + \frac{396}{36} = \frac{791}{36}$$

$$E(Y) = \sum y \cdot P(Y = y)$$

$$E(Y) = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36}$$
$$+ 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} = \frac{1}{36} + \frac{6}{36} + \frac{12}{36}$$
$$+ \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$
$$E(Y) = \frac{251}{36}$$

$$V(Y) = 2^2 \cdot \frac{1}{36} + 3^2 \cdot \frac{2}{36} + 4^2 \cdot \frac{3}{36} + 5^2 \cdot \frac{4}{36} + 6^2 \cdot \frac{5}{36} + 7^2 \cdot \frac{6}{36} + 8^2 \cdot \frac{5}{36}$$
$$+ 9^2 \cdot \frac{4}{36} + 10^2 \cdot \frac{3}{36} + 11^2 \cdot \frac{2}{36} + 12^2 \cdot \frac{1}{36} = \frac{4}{36} + \frac{18}{36} + \frac{48}{36}$$
$$+ \frac{100}{36} + \frac{180}{36} + \frac{294}{36} + \frac{320}{36} + \frac{324}{36} + \frac{300}{36} + \frac{242}{36} + \frac{144}{36}$$
$$V(Y) = \frac{1974}{36}$$

(5)

a) $2a + a + 4a = 1 \rightarrow 7a = 1 \rightarrow a = \frac{1}{7}$

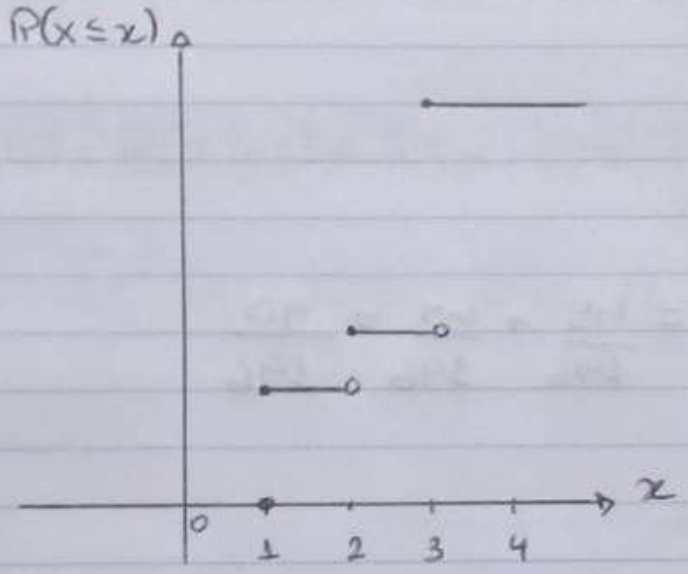
b) $P(0 \leq x \leq 3) = P(x=1) + P(x=2) + P(x=3) = 1$

$P(0 < x < 2) = P(x=1) = \frac{2}{7}$

c) $P(x \leq 1) = P(x=1) = \frac{2}{7}$

$P(x \leq 2) = P(x \leq 1) + P(x=2) = \frac{2}{7} + \frac{1}{7} = \frac{3}{7}$

$P(x \leq 3) = P(x \leq 2) + P(x=3) = \frac{3}{7} + \frac{4}{7} = \frac{7}{7}$



d) $E(x) = \sum x\, P(x=x)$

$E(x) = 1 \cdot \frac{2}{7} + 2 \cdot \frac{1}{7} + 3 \cdot \frac{4}{7} = \frac{2}{7} + \frac{2}{7} + \frac{12}{7} = \frac{16}{7}$

$V(x) = 1^2 \cdot \frac{2}{7} + 2^2 \cdot \frac{1}{7} + 3^2 \cdot \frac{4}{7} = \frac{2}{7} + \frac{4}{7} + \frac{36}{7} = \frac{42}{7}$

c) $Y = 3X$

$E(Y) = E(3X) \rightarrow E(Y) = 3\ E(X) \rightarrow E(Y) = 3\ \frac{16}{7} = \frac{48}{7}$

$V(Y) = V(3X) \rightarrow V(Y) = 3^2 \cdot V(X) \rightarrow V(Y) = 9\ \frac{42}{7} = \frac{378}{7}$

(6)

X = Nº DE BOLAS BRANCAS
Y = Nº DE BOLAS VERDES

a) COM REPOSIÇÃO

$S_X = \{0, 1, 2\}$

$P(X=0) = \frac{9}{14}\ \frac{9}{14} = \frac{81}{196}$

$P(X=1) = \frac{9}{14}\ \frac{5}{14} + \frac{5}{14}\ \frac{9}{14} = \frac{45}{196} + \frac{45}{196} = \frac{90}{196}$

$P(X=2) = \frac{5}{14}\ \frac{5}{14} = \frac{25}{196}$

$S_Y = \{0, 1, 2\}$

$P(Y=0) = \frac{11}{14}\ \frac{11}{14} = \frac{121}{196}$

$P(Y=1) = \frac{11}{14}\ \frac{3}{14} + \frac{3}{14}\ \frac{11}{14} = \frac{33}{196} + \frac{33}{196} = \frac{66}{196}$

$P(Y=2) = \frac{3}{14} \cdot \frac{3}{14} = \frac{9}{196}$

b) SEM REPOSIÇÃO

$S_X = \{0, 1, 2\}$

$P(x=0) = \frac{9}{14} \cdot \frac{8}{13} = \frac{72}{182}$

$P(x=1) = \frac{9}{14} \cdot \frac{5}{13} + \frac{5}{14} \cdot \frac{9}{13} = \frac{45}{182} + \frac{45}{182} = \frac{90}{182}$

$P(x=2) = \frac{5}{14} \cdot \frac{4}{13} = \frac{20}{182}$

$S_Y = \{0, 1, 2\}$

$P(Y=0) = \frac{11}{14} \cdot \frac{10}{13} = \frac{110}{182}$

$P(Y=1) = \frac{11}{14} \cdot \frac{3}{13} + \frac{3}{14} \cdot \frac{11}{13} = \frac{33}{182} + \frac{33}{182} = \frac{66}{182}$

$P(Y=2) = \frac{3}{14} \cdot \frac{2}{13} = \frac{6}{182}$

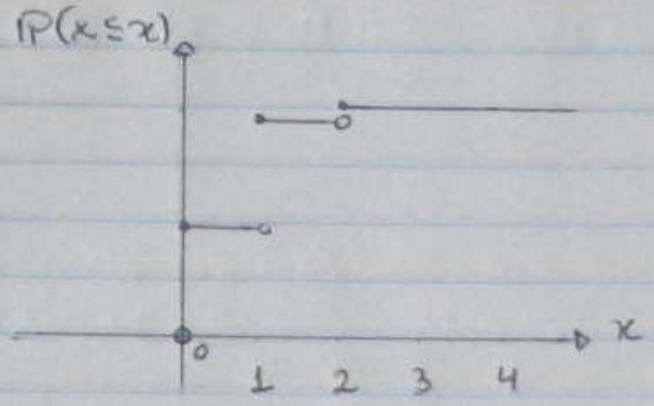
c)

• COM REPOSIÇÃO

$P(x \leq 0) = P(x=0) = \frac{81}{196}$

$P(X \leq 1) = P(X \leq 0) + P(X = 1) = \frac{81}{196} + \frac{90}{196} = \frac{171}{196}$

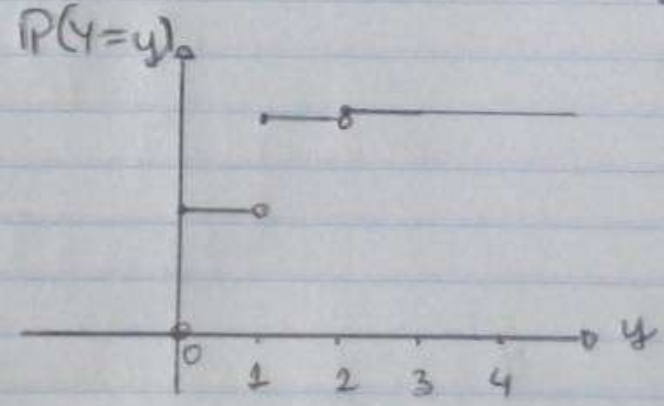
$P(X \leq 2) = P(X \leq 1) + P(X = 2) = \frac{171}{196} + \frac{25}{196} = \frac{196}{196}$



$P(Y \leq 0) = P(Y = 0) = \frac{121}{196}$

$P(Y \leq 1) = P(Y \leq 0) + P(Y = 1) = \frac{121}{196} + \frac{66}{196} = \frac{187}{196}$

$P(Y \leq 2) = P(Y \leq 1) + P(Y = 2) = \frac{187}{196} + \frac{9}{196} = \frac{196}{196}$

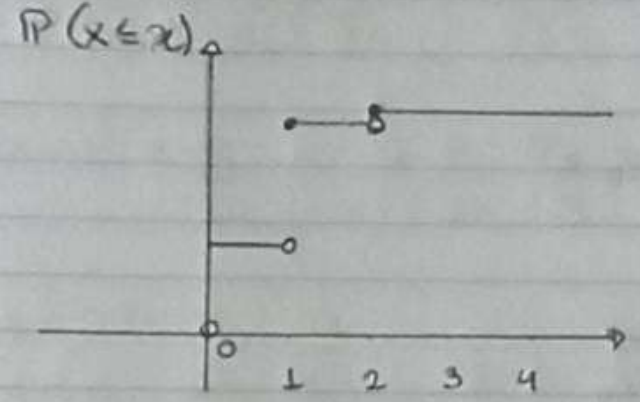


* SEM REPOSIÇÃO

$$P(X \leq 0) = P(X = 0) = \frac{72}{182}$$

$$P(X \leq 1) = P(X \leq 0) + P(X = 1) = \frac{72}{182} + \frac{90}{182} = \frac{162}{182}$$

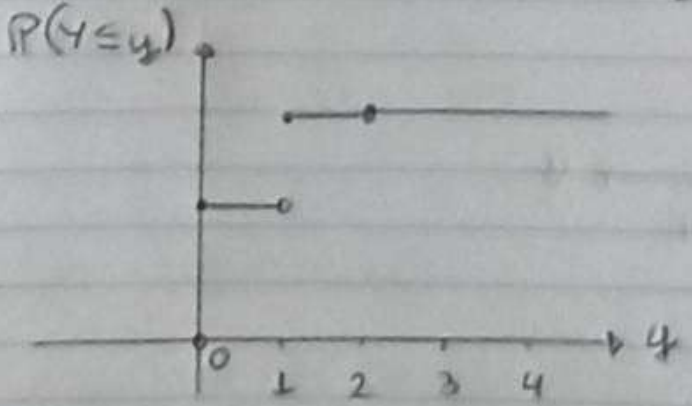
$$P(X \leq 2) = P(X \leq 1) + P(X = 2) = \frac{162}{182} + \frac{20}{182} = \frac{182}{182}$$



$$P(Y \leq 0) = P(Y = 0) = \frac{110}{182}$$

$$P(Y \leq 1) = P(Y \leq 0) + P(Y = 1) = \frac{110}{182} + \frac{66}{182} = \frac{176}{182}$$

$$P(Y \leq 2) = P(Y \leq 1) + P(Y = 2) = \frac{176}{182} + \frac{6}{182} = \frac{182}{182}$$



d)

* Com reposição

$E(X) = \sum x \cdot \mathbb{P}(X = x)$

$E(x) = 0 \cdot \frac{81}{196} + 1 \cdot \frac{90}{196} + 2 \cdot \frac{25}{196} = 0 + \frac{90}{196} + \frac{50}{196} = \frac{140}{196}$

$V(x) = \sum x^2 \cdot \mathbb{P}(x = x)$

$V(x) = 0^2 \cdot \frac{81}{196} + 1^2 \cdot \frac{90}{196} + 2^2 \cdot \frac{25}{196} = 0 + \frac{90}{196} + \frac{100}{196} = \frac{190}{196}$

$E(Y) = \sum y \cdot \mathbb{P}(Y = y)$

$E(Y) = 0 \cdot \frac{121}{196} + 1 \cdot \frac{66}{196} + 2 \cdot \frac{9}{196} = 0 + \frac{66}{196} + \frac{18}{196} = \frac{84}{196}$

$V(Y) = \sum y^2 \cdot \mathbb{P}(Y = y)$

$V(Y) = 0^2 \cdot \frac{121}{196} + 1^2 \cdot \frac{66}{196} + 2^2 \cdot \frac{9}{196} = 0 + \frac{66}{196} + \frac{36}{196} = \frac{102}{192}$

* Sem reposição

$E(X) = \sum x \cdot \mathbb{P}(X = x)$

$E(x) = 0 \cdot \frac{72}{182} + 1 \cdot \frac{90}{182} + 2 \cdot \frac{20}{182} = 0 + \frac{90}{182} + \frac{40}{182} = \frac{130}{182}$

$V(x) = \sum x^2 \cdot \mathbb{P}(X = x)$

$$V(x) = 0^2 \cdot \frac{72}{182} + 1^2 \cdot \frac{90}{182} + 2^2 \cdot \frac{20}{182} = 0 + \frac{90}{182} + \frac{80}{182} = \frac{170}{182}$$

$$E(Y) = \sum y \cdot P(Y = y)$$

$$E(Y) = 0 \cdot \frac{110}{182} + 1 \cdot \frac{66}{182} + 2 \cdot \frac{6}{182} = 0 + \frac{66}{182} + \frac{12}{182} = \frac{78}{182}$$

$$V(Y) = \sum y^2 \cdot P(Y = y)$$

$$V(Y) = 0^2 \cdot \frac{110}{182} + 1^2 \cdot \frac{66}{182} + 2 \cdot \frac{6}{182} = 0 + \frac{66}{182} + \frac{24}{182} = \frac{90}{182}$$

(7)

a)
$$S_x = \begin{cases} (K,K,K) = 0 \\ (K,K,C);(K,C,K);(C,K,K) = 1 \\ (K,C,C);(C,K,C);(C,C,K) = 2 \\ (C,C,C) = 3 \end{cases}$$

$$P(x=0) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

$$P(x=1) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = 3\left(\frac{1}{8}\right) = \frac{3}{8}$$

$$P(x=2) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = 3\left(\frac{1}{8}\right) = \frac{3}{8}$$

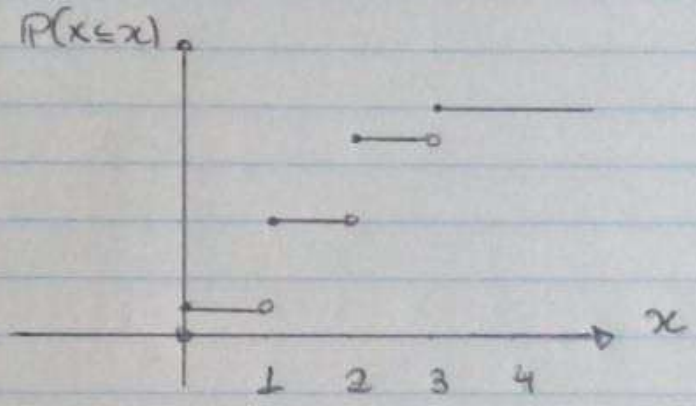
$$P(x=3) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

b) $P(x \leq 0) = P(x=0) = \frac{1}{8}$

$P(x \leq 1) = P(x \leq 0) + P(x=1) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

$P(x \leq 2) = P(x \leq 1) + P(x=2) = \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$

$P(x \leq 3) = P(x \leq 2) + P(x=3) = \frac{7}{8} + \frac{1}{8} = \frac{8}{8}$



c) $E(x) = \sum x \cdot P(x=x)$

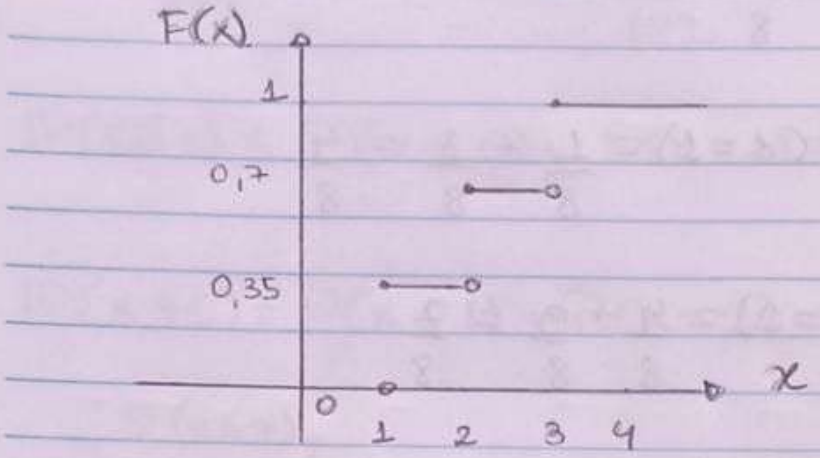
$E(x) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8}$

$V(x) = \sum x^2 \cdot P(x=x)$

$V(x) = 0^2 \cdot \frac{1}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{3}{8} + 3^2 \cdot \frac{1}{8} = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8}$

$V(x) = \frac{24}{8}$

(8)



$F(5) = P(X \leq 5) = P(X=5) = 0,35$

$F(10) = P(X \leq 10) = P(X \leq 5) + P(X=10) \rightarrow P(X=10) = P(X \leq 10) - P(X \leq 5)$
$P(X=10) = 0,7 - 0,35 = 0,35$

$F(15) = P(X \leq 15) = P(X \leq 10) + P(X=15) \rightarrow P(X=15) = P(X \leq 15) - P(X \leq 10)$
$P(X=15) = 1 - 0,7 = 0,3$

$E(X) = \sum_i x \cdot P(X=x)$

$E(X) = 5 \cdot 0,35 + 10 \cdot 0,35 + 15 \cdot 0,3 = 1,75 + 3,5 + 4,5$
$E(X) = 9,75$

$V(X) = \sum x^2 \cdot P(X=x)$

$V(X) = 5^2 \cdot 0,35 + 10^2 \cdot 0,35 + 15^2 \cdot 0,3 = 8,75 + 35 + 67,5$
$V(X) = 111,25$

(9)

$Y = 4 + X$

$E(Y) = E(4+X) \rightarrow E(Y) = 4 + E(X) \rightarrow E(Y) = 4 + 9,75$

$E(Y) = 43,75$

$V(Y) = V(4+X) \rightarrow V(Y) = V(X) \rightarrow V(Y) = 111,75$

(10)

X = ACHAR COMPONENTES PERFEITOS

$$S_x = \begin{cases} (D,D) = 0 \\ (D,P);(P,D) = 1 \\ (P,P) = 2 \end{cases}$$

* COM REPOSIÇÃO

$$P(x=0) = \frac{2}{25} \cdot \frac{2}{25} = \frac{4}{625}$$

$$P(x=1) = \frac{2}{25} \cdot \frac{23}{25} + \frac{23}{25} \cdot \frac{2}{25} = \frac{46}{625} + \frac{46}{625} = \frac{92}{625}$$

$$P(x=2) = \frac{23}{25} \cdot \frac{23}{25} = \frac{529}{625}$$

$$E(x) = \sum x \cdot P(x=x)$$

$$E(x) = 0 \cdot \frac{4}{625} + 1 \cdot \frac{92}{625} + 2 \cdot \frac{529}{625} = 0 + \frac{92}{625} + \frac{1058}{625}$$

$$E(x) = \frac{1150}{625}$$

$$\text{LUCRO ESPERADO} = \frac{1150}{625}(100-50) = 92$$

* SEM REPOSIÇÃO

$$P(X=0) = \frac{2}{25} \cdot \frac{1}{24} = \frac{2}{600}$$

$$P(X=1) = \frac{2}{25} \cdot \frac{23}{24} + \frac{23}{25} \cdot \frac{2}{24} = \frac{46}{600} + \frac{46}{600} = \frac{92}{600}$$

$$P(X=2) = \frac{23}{25} \cdot \frac{22}{24} = \frac{506}{600}$$

$$E(x) = \sum x \cdot P(X=x)$$

$$E(x) = 0 \cdot \frac{2}{600} + 1 \cdot \frac{92}{600} + 2 \cdot \frac{506}{600} = 0 + \frac{92}{600} + \frac{1012}{600}$$

$$E(x) = \frac{1104}{600}$$

LUCRO ESPERADO $= \frac{1104}{600}(100-50) = 92$

(11)

x	$x \leq -2$	$-2 \leq x < 1$	$1 \leq x < 3$	$3 \leq x < 5$	$x \geq 5$
$F(x)$	0	0,25	0,40	0,70	1

$P(X=3) = F(3 \leq x < 5) - F(1 \leq x < 3) = 0,7 - 0,4 = 0,3$

$P(X=4) = F(3 \leq x < 5) - F(1 \leq x < 3) = 0,7 - 0,4 = 0,3$

$F(0) = F(-2 \leq x < 1) = 0,25$

$F(4) = F(3 \leq x < 5) = 0,7$