

Lista 7

Curso de Ciências Atuariais
Disciplina Probabilidade 1- Professora Cristina
12/08/2022 - Exercícios de função de v.a. e distribuição conjunta

1) O Quadro abaixo dá a distribuição de probabilidade conjunta das v. a. X e Y.

Y	X		
	1	2	3
0	0,1	0,1	0,1
1	0,2	0	0,3
2	0	0,1	0,1

a) Obtenha as distribuições de $X+Y$ e de XY

$$S_{x+y} = \{1, 2, 3, 4, 5\}$$

$$\mathbb{P}(X+Y = 1) = 0,1$$

$$\mathbb{P}(X+Y = 2) = 0,1 + 0,2 = 0,3$$

$$\mathbb{P}(X+Y = 3) = 0,1 + 0 + 0 = 0,1$$

$$\mathbb{P}(X+Y = 4) = 0,3 + 0,1 = 0,4$$

$$\mathbb{P}(X+Y = 5) = 0,1$$

$$S_{xy} = \{0, 1, 2, 3, 4, 6\}$$

$$\mathbb{P}(XY = 0) = 0,1 + 0,1 + 0,1 = 0,3$$

$$\mathbb{P}(XY = 1) = 0,2$$

$$\mathbb{P}(XY = 2) = 0 + 0 = 0$$

$$\mathbb{P}(XY = 3) = 0,3$$

$$\mathbb{P}(XY = 4) = 0,1$$

$$\mathbb{P}(XY = 6) = 0,1$$

b) Calcule $E(X+Y)$, $E(XY)$, $V(X+Y)$ e $V(XY)$

$$E(X+Y) = 1 * 0,1 + 2 * 0,3 + 3 * 0,1 + 4 * 0,4 + 5 * 0,1$$

$$E(X+Y) = 0,1 + 0,6 + 0,3 + 1,6 + 0,5 = 3,1$$

$$E[(X+Y)^2] = 1^2 * 0,1 + 2^2 * 0,3 + 3^2 * 0,1 + 4^2 * 0,4 + 5^2 * 0,1$$

$$E[(X+Y)^2] = 0,1 + 1,2 + 0,9 + 6,4 + 2,5 = 11,1$$

$$V(X+Y) = 11,1 - 3,1^2 = 11,1 - 9,61 = 1,49$$

$$E(XY) = 0 * 0,3 + 1 * 0,2 + 2 * 0 + 3 * 0,3 + 4 * 0,1 + 6 * 0,1$$

$$E(XY) = 0 + 0,2 + 0 + 0,9 + 0,4 + 0,6 = 2,1$$

$$E[(XY)^2] = 0^2 * 0,3 + 1^2 * 0,2 + 2^2 * 0 + 3^2 * 0,3 + 4^2 * 0,1 + 6^2 * 0,1$$

$$E[(XY)^2] = 0 + 0,2 + 0 + 2,7 + 1,6 + 3,6 = 8,1$$

$$V(X+Y) = 8,1 - 2,1^2 = 8,1 - 4,41 = 3,69$$

2) Numa urna tem cinco bolas marcadas com os seguintes números: -1, 0, 0, 0, 1. Retiram-se 3 bolas simultaneamente. X indica a soma dos números obtidos e Y o maior valor da trinca.

a) Determine a distribuição de probabilidade conjunta de (X,Y)

$$S_x = \begin{cases} (0, 0, -1); (0, -1, 0); (-1, 0, 0) = -1 \\ (0, 0, 0); (0, 1, -1); (1, 0, -1); (1, -1, 0); (0, -1, 1); (-1, 0, 1); (-1, 1, 0) = 0 \\ (0, 0, 1); (0, 1, 0); (1, 0, 0) = 1 \end{cases}$$

$$\mathbb{P}(X = -1) = \left(\frac{3}{5} * \frac{2}{4} * \frac{1}{3}\right) * 3 = \left(\frac{6}{60}\right) * 3 = \frac{18}{60}$$

$$\mathbb{P}(X = 0) = \frac{3}{5} * \frac{2}{4} * \frac{1}{3} + \left(\frac{3}{5} * \frac{1}{4} * \frac{1}{3}\right) * 6 = \frac{6}{60} + \left(\frac{3}{60}\right) * 6 = \frac{6}{60} + \frac{18}{60} = \frac{24}{60}$$

$$\mathbb{P}(X = 1) = \left(\frac{3}{5} * \frac{2}{4} * \frac{1}{3}\right) * 3 = \left(\frac{6}{60}\right) * 3 = \frac{18}{60}$$

$$S_y = \begin{cases} (0, 0, 0)(0, 0, -1); (0, -1, 0); (-1, 0, 0) = 0 \\ (0, 1, -1); (1, 0, -1); (1, -1, 0); (0, -1, 1); \\ (-1, 0, 1); (-1, 1, 0), (0, 0, 1); (0, 1, 0); (1, 0, 0) = 1 \end{cases}$$

$$\mathbb{P}(X = 0) = \left(\frac{3}{5} * \frac{2}{4} * \frac{1}{3}\right) * 4 = \left(\frac{6}{60}\right) * 4 = \frac{24}{60}$$

$$\mathbb{P}(X = 1) = \left(\frac{3}{5} * \frac{2}{4} * \frac{1}{3}\right) * 3 + \left(\frac{3}{5} * \frac{1}{4} * \frac{1}{3}\right) * 6 = \left(\frac{6}{60}\right) * 3 + \left(\frac{3}{60}\right) * 6 = \frac{18}{60} + \frac{18}{60} = \frac{36}{60}$$

Y	X			$\mathbb{P}(Y = y)$
	-1	0	1	
0	$\frac{18}{60}$	$\frac{6}{60}$	0	$\frac{24}{60}$
1	0	$\frac{18}{60}$	$\frac{18}{60}$	$\frac{36}{60}$
$\mathbb{P}(X = x)$	$\frac{18}{60}$	$\frac{24}{60}$	$\frac{18}{60}$	1

$$\mathbb{P}(X = -1 \cap Y = 0) = \left(\frac{3}{5} * \frac{2}{4} * \frac{1}{3}\right) * 3 = \left(\frac{6}{60}\right) * 3 = \frac{18}{60}$$

$$\mathbb{P}(X = -1) = \mathbb{P}(X = -1 \cap Y = 0) + \mathbb{P}(X = -1 \cap Y = 1)$$

$$\frac{18}{60} = \frac{18}{60} + \mathbb{P}(X = -1 \cap Y = 1)$$

$$\mathbb{P}(X = -1 \cap Y = 1) = \frac{18}{60} - \frac{18}{60} = 0$$

$$\mathbb{P}(X = 0 \cap Y = 0) = \left(\frac{3}{5} * \frac{2}{4} * \frac{1}{3}\right) * 3 = \frac{6}{60}$$

$$\mathbb{P}(X = 0) = \mathbb{P}(X = 0 \cap Y = 0) + \mathbb{P}(X = 0 \cap Y = 1)$$

$$\frac{24}{60} = \frac{6}{60} + \mathbb{P}(X = 0 \cap Y = 1)$$

$$\mathbb{P}(X = 0 \cap Y = 1) = \frac{24}{60} - \frac{6}{60} = \frac{18}{60}$$

$$\mathbb{P}(Y = 0) = \mathbb{P}(X = -1 \cap Y = 0) + \mathbb{P}(X = 0 \cap Y = 0) + \mathbb{P}(X = 1 \cap Y = 0)$$

$$\frac{24}{60} = \frac{18}{60} + \frac{6}{60} + \mathbb{P}(X = 1 \cap Y = 0)$$

$$\mathbb{P}(X = 1 \cap Y = 0) = \frac{24}{60} - \frac{18}{60} - \frac{6}{60} = 0$$

$$\mathbb{P}(Y = 1) = \mathbb{P}(X = -1 \cap Y = 1) + \mathbb{P}(X = 0 \cap Y = 1) + \mathbb{P}(X = 1 \cap Y = 1)$$

$$\frac{36}{60} = 0 + \frac{18}{60} + \mathbb{P}(X = 1 \cap Y = 1)$$

$$\mathbb{P}(X = 1 \cap Y = 1) = \frac{36}{60} - \frac{18}{60} = \frac{18}{60}$$

b) Encontre $E(X)$ e $V(X)$

$$E(X) = (-1) * \frac{18}{60} + 0 * \frac{24}{60} + 1 * \frac{18}{60} = \frac{-18}{60} + 0 + \frac{18}{60} = 0$$

$$E(X^2) = (-1)^2 * \frac{18}{60} + 0^2 * \frac{24}{60} + 1^2 * \frac{18}{60} = \frac{18}{60} + 0 + \frac{18}{60} = \frac{36}{60}$$

$$V(X) = \frac{36}{60} - 0^2 = \frac{36}{60}$$

c) Encontre a distribuição de probabilidade de $X+Y$

$$S_{x+y} = \begin{cases} (-1, 0) = -1 \\ (-1, 1); (0, 0) = 0 \\ (1, 0); (0, 1) = 1 \\ (1, 1) = 2 \end{cases}$$

$$\mathbb{P}(X + Y = -1) = \frac{18}{60}$$

$$\mathbb{P}(X + Y = 0) = 0 + \frac{6}{60} = \frac{6}{60}$$

$$\mathbb{P}(X + Y = 1) = 0 + \frac{18}{60} = \frac{18}{60}$$

$$\mathbb{P}(X + Y = 2) = \frac{18}{60} = \frac{18}{60}$$

d) Encontre $E(X+Y)$ e $V(X+Y)$

$$E(X+Y) = (-1) * \frac{18}{60} + 0 * \frac{6}{60} + 1 * \frac{18}{60} + 2 * \frac{18}{60} = \frac{-18}{60} + 0 + \frac{18}{60} + \frac{36}{60} = \frac{36}{60} = \frac{3}{5}$$

$$E[(X+Y)^2] = (-1)^2 * \frac{18}{60} + 0^2 * \frac{6}{60} + 1^2 * \frac{18}{60} + 2^2 * \frac{18}{60} = \frac{18}{60} + 0 + \frac{18}{60} + \frac{72}{60} = \frac{108}{60} = \frac{9}{5}$$

$$V(X+Y) = \frac{9}{5} - \left(\frac{3}{5}\right)^2 = \frac{45}{25} - \frac{9}{25} = \frac{36}{25}$$

3) Numa urna tem cinco tiras de papel, numeradas 1, 3, 5, 5, 7. Uma tira é sorteada e recolocada na urna, Depois uma segunda tira é retirada. Sejam as v.a. X_1 e X_2 que representam o primeiro e o segundo número sorteado.

a) Determine a distribuição conjunta de (X_1, X_2)

$$\mathbb{P}(X_1 = 1) = \frac{1}{5} \quad \mathbb{P}(X_2 = 1) = \frac{1}{5}$$

$$\mathbb{P}(X_1 = 3) = \frac{1}{5} \quad \mathbb{P}(X_2 = 3) = \frac{1}{5}$$

$$\mathbb{P}(X_1 = 5) = \frac{2}{5} \quad \mathbb{P}(X_2 = 5) = \frac{2}{5}$$

$$\mathbb{P}(X_1 = 7) = \frac{1}{5} \quad \mathbb{P}(X_2 = 7) = \frac{1}{5}$$

X ₂	X ₁				P(X ₂ = x)
	1	3	5	7	
1	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{1}{5}$
3	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{1}{5}$
5	$\frac{2}{25}$	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{2}{25}$	$\frac{2}{5}$
7	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{1}{5}$
P(X ₁ = x)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	1

$$\mathbb{P}(X_1 = 1 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25} \quad \mathbb{P}(X_1 = 3 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 1) = \frac{2}{5} * \frac{1}{5} = \frac{2}{25} \quad \mathbb{P}(X_1 = 7 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

$$\mathbb{P}(X_1 = 1 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25} \quad \mathbb{P}(X_1 = 3 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 3) = \frac{2}{5} * \frac{1}{5} = \frac{2}{25} \quad \mathbb{P}(X_1 = 7 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

$$\mathbb{P}(X_1 = 1 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{5} = \frac{2}{25} \quad \mathbb{P}(X_1 = 3 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{5} = \frac{2}{25}$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 5) = \frac{2}{5} * \frac{2}{5} = \frac{4}{25} \quad \mathbb{P}(X_1 = 7 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{5} = \frac{2}{25}$$

$$\begin{aligned} \mathbb{P}(X_1 = 1) &= \mathbb{P}(X_1 = 1 \cap X_2 = 1) + \mathbb{P}(X_1 = 1 \cap X_2 = 3) \\ &\quad + \mathbb{P}(X_1 = 1 \cap X_2 = 5) + \mathbb{P}(X_1 = 1 \cap X_2 = 7) \end{aligned}$$

$$\frac{1}{5} = \frac{1}{25} + \frac{1}{25} + \frac{2}{25} + \mathbb{P}(X_1 = 1 \cap X_2 = 7)$$

$$\mathbb{P}(X_1 = 1 \cap X_2 = 7) = \frac{5}{25} - \left(\frac{1}{25} + \frac{1}{25} + \frac{2}{25} \right) = \frac{1}{25}$$

$$\begin{aligned}
\mathbb{P}(X_1 = 3) &= \mathbb{P}(X_1 = 3 \cap X_2 = 1) + \mathbb{P}(X_1 = 3 \cap X_2 = 3) \\
&\quad + \mathbb{P}(X_1 = 3 \cap X_2 = 5) + \mathbb{P}(X_1 = 3 \cap X_2 = 7) \\
\frac{1}{5} &= \frac{1}{25} + \frac{1}{25} + \frac{2}{25} + \mathbb{P}(X_1 = 3 \cap X_2 = 7) \\
\mathbb{P}(X_1 = 3 \cap X_2 = 7) &= \frac{5}{25} - \left(\frac{1}{25} + \frac{1}{25} + \frac{2}{25}\right) = \frac{1}{25}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(X_1 = 5) &= \mathbb{P}(X_1 = 5 \cap X_2 = 1) + \mathbb{P}(X_1 = 5 \cap X_2 = 3) \\
&\quad + \mathbb{P}(X_1 = 5 \cap X_2 = 5) + \mathbb{P}(X_1 = 5 \cap X_2 = 7) \\
\frac{2}{5} &= \frac{2}{25} + \frac{2}{25} + \frac{4}{25} + \mathbb{P}(X_1 = 5 \cap X_2 = 7) \\
\mathbb{P}(X_1 = 5 \cap X_2 = 7) &= \frac{10}{25} - \left(\frac{2}{25} + \frac{2}{25} + \frac{4}{25}\right) = \frac{2}{25}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(X_1 = 7) &= \mathbb{P}(X_1 = 7 \cap X_2 = 1) + \mathbb{P}(X_1 = 7 \cap X_2 = 3) \\
&\quad + \mathbb{P}(X_1 = 7 \cap X_2 = 5) + \mathbb{P}(X_1 = 7 \cap X_2 = 7) \\
\frac{1}{5} &= \frac{1}{25} + \frac{1}{25} + \frac{2}{25} + \mathbb{P}(X_1 = 7 \cap X_2 = 7) \\
\mathbb{P}(X_1 = 7 \cap X_2 = 7) &= \frac{5}{25} - \left(\frac{1}{25} + \frac{1}{25} + \frac{2}{25}\right) = \frac{1}{25}
\end{aligned}$$

b) Encontre $E(X_1 + X_2)$ e $V(X_1 + X_2)$

$$S_{x+y} = \begin{cases} (1, 1) = 2 \\ (1, 3); (3, 1) = 4 \\ (1, 5); (5, 1); (3, 3) = 6 \\ (1, 7); (7, 1); (3, 5); (5, 3) = 8 \\ (3, 7); (7, 3); (5, 5) = 10 \\ (5, 7); (7, 5) = 12 \\ (7, 7) = 14 \end{cases}$$

$$\begin{aligned}
E(X_1 + X_2) &= 2 * \frac{1}{25} + 4 * \frac{2}{25} + 6 * \frac{5}{25} + 8 * \frac{6}{25} + 10 * \frac{6}{25} + 12 * \frac{4}{25} + 14 * \frac{1}{25} \\
E(X_1 + X_2) &= \frac{2}{25} + \frac{8}{25} + \frac{30}{25} + \frac{48}{25} + \frac{60}{25} + \frac{48}{25} + \frac{14}{25} = \frac{210}{25} = \frac{42}{5}
\end{aligned}$$

$$\begin{aligned}
E[(X_1 + X_2)^2] &= 2^2 * \frac{1}{25} + 4^2 * \frac{2}{25} + 6^2 * \frac{5}{25} + 8^2 * \frac{6}{25} + 10^2 * \frac{6}{25} + 12^2 * \frac{4}{25} + 14^2 * \frac{1}{25} \\
E[(X_1 + X_2)^2] &= \frac{4}{25} + \frac{32}{25} + \frac{180}{25} + \frac{384}{25} + \frac{600}{25} + \frac{576}{25} + \frac{196}{25} = \frac{1.972}{25}
\end{aligned}$$

$$V(X_1 + X_2) = \frac{1.972}{25} - \left(\frac{42}{5}\right)^2 = \frac{1.972}{25} - \frac{1.764}{25} = \frac{208}{25}$$

c) Refaça a questão se a retirada for sem reposição

X ₂	X ₁				P(X ₂ = x)
	1	3	5	7	
1	0	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{1}{20}$	$\frac{1}{5}$
3	$\frac{1}{20}$	0	$\frac{2}{20}$	$\frac{1}{20}$	$\frac{1}{5}$
5	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{1}{5}$
7	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	0	$\frac{1}{5}$
P(X ₁ = x)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	1

$$P(X_1 = 1 \cap X_2 = 1) = \frac{1}{5} * 0 = 0 \quad P(X_1 = 3 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{4} = \frac{1}{20}$$

$$P(X_1 = 5 \cap X_2 = 1) = \frac{2}{5} * \frac{1}{4} = \frac{2}{20} \quad P(X_1 = 7 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{4} = \frac{1}{20}$$

$$P(X_1 = 1 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{4} = \frac{1}{20} \quad P(X_1 = 3 \cap X_2 = 3) = \frac{1}{5} * 0 = 0$$

$$P(X_1 = 5 \cap X_2 = 3) = \frac{2}{5} * \frac{1}{4} = \frac{2}{20} \quad P(X_1 = 7 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{4} = \frac{1}{20}$$

$$P(X_1 = 1 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{4} = \frac{2}{20} \quad P(X_1 = 3 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{4} = \frac{2}{20}$$

$$P(X_1 = 5 \cap X_2 = 5) = \frac{2}{5} * \frac{1}{4} = \frac{2}{20} \quad P(X_1 = 7 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{4} = \frac{2}{20}$$

$$P(X_1 = 1) = P(X_1 = 1 \cap X_2 = 1) + P(X_1 = 1 \cap X_2 = 3) \\ + P(X_1 = 1 \cap X_2 = 5) + P(X_1 = 1 \cap X_2 = 7)$$

$$\frac{1}{5} = 0 + \frac{1}{20} + \frac{2}{20} + P(X_1 = 1 \cap X_2 = 7)$$

$$P(X_1 = 1 \cap X_2 = 7) = \frac{4}{20} - \left(\frac{1}{20} + \frac{2}{20}\right) = \frac{1}{20}$$

$$P(X_1 = 3) = P(X_1 = 3 \cap X_2 = 1) + P(X_1 = 3 \cap X_2 = 3) \\ + P(X_1 = 3 \cap X_2 = 5) + P(X_1 = 3 \cap X_2 = 7)$$

$$\frac{1}{5} = \frac{1}{20} + 0 + \frac{2}{20} + \mathbb{P}(X_1 = 3 \cap X_2 = 7)$$

$$\mathbb{P}(X_1 = 3 \cap X_2 = 7) = \frac{4}{20} - (\frac{1}{20} + \frac{2}{20}) = \frac{1}{20}$$

$$\mathbb{P}(X_1 = 5) = \mathbb{P}(X_1 = 5 \cap X_2 = 1) + \mathbb{P}(X_1 = 5 \cap X_2 = 3)$$

$$+ \mathbb{P}(X_1 = 5 \cap X_2 = 5) + \mathbb{P}(X_1 = 5 \cap X_2 = 7)$$

$$\frac{2}{5} = \frac{2}{20} + \frac{2}{20} + \frac{2}{20} + \mathbb{P}(X_1 = 5 \cap X_2 = 7)$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 7) = \frac{8}{20} - (\frac{2}{20} + \frac{2}{20} + \frac{2}{20}) = \frac{2}{20}$$

$$\mathbb{P}(X_1 = 7) = \mathbb{P}(X_1 = 7 \cap X_2 = 1) + \mathbb{P}(X_1 = 7 \cap X_2 = 3)$$

$$+ \mathbb{P}(X_1 = 7 \cap X_2 = 5) + \mathbb{P}(X_1 = 7 \cap X_2 = 7)$$

$$\frac{1}{5} = \frac{1}{20} + \frac{1}{20} + \frac{2}{20} + \mathbb{P}(X_1 = 7 \cap X_2 = 7)$$

$$\mathbb{P}(X_1 = 7 \cap X_2 = 7) = \frac{4}{20} - (\frac{1}{25} + \frac{1}{25} + \frac{2}{25}) = 0$$

b) Encontre $E(X_1 + X_2)$ e $V(X_1 + X_2)$

$$S_{x+y} = \begin{cases} (1, 1) = 2 \\ (1, 3); (3, 1) = 4 \\ (1, 5); (5, 1); (3, 3) = 6 \\ (1, 7); (7, 1); (3, 5); (5, 3) = 8 \\ (3, 7); (7, 3); (5, 5) = 10 \\ (5, 7); (7, 5) = 12 \\ (7, 7) = 14 \end{cases}$$

$$E(X_1 + X_2) = 2 * 0 + 4 * \frac{2}{20} + 6 * \frac{4}{20} + 8 * \frac{6}{20} + 10 * \frac{4}{20} + 12 * \frac{2}{20} + 14 * 0$$

$$E(X_1 + X_2) = 0 + \frac{8}{20} + \frac{24}{20} + \frac{48}{20} + \frac{40}{20} + \frac{24}{20} + 0 = \frac{140}{20} = \frac{35}{5}$$

$$E[(X_1 + X_2)^2] = 2^2 * 0 + 4^2 * \frac{2}{20} + 6^2 * \frac{4}{20} + 8^2 * \frac{6}{20} + 10^2 * \frac{4}{20} + 12^2 * \frac{2}{20} + 14^2 * 0$$

$$E[(X_1 + X_2)^2] = 0 + \frac{32}{20} + \frac{144}{20} + \frac{384}{20} + \frac{400}{20} + \frac{288}{20} + 0 = \frac{1.248}{20} = \frac{312}{5}$$

$$V(X_1 + X_2) = \frac{312}{5} - (\frac{35}{5})^2 = \frac{1.560}{25} - \frac{1.225}{25} = \frac{335}{25} = \frac{67}{5}$$

d) Verifique, em ambos os casos se X_1 e X_2 são independentes

$$E(X_1) = 1 * \frac{1}{5} + 3 * \frac{1}{5} + 5 * \frac{2}{5} + 7 * \frac{1}{5} = \frac{1}{5} + \frac{3}{5} + \frac{10}{5} + \frac{7}{5} = \frac{21}{5}$$

$$E(X_1)^2 = 1^2 * \frac{1}{5} + 3^2 * \frac{1}{5} + 5^2 * \frac{2}{5} + 7^2 * \frac{1}{5} = \frac{1}{5} + \frac{9}{5} + \frac{50}{5} + \frac{49}{5} = \frac{109}{5}$$

$$V(X_1) = \frac{109}{5} - \left(\frac{21}{5}\right)^2 = \frac{545}{25} - \frac{441}{25} = \frac{104}{25}$$

$$E(X_2) = 1 * \frac{1}{5} + 3 * \frac{1}{5} + 5 * \frac{2}{5} + 7 * \frac{1}{5} = \frac{1}{5} + \frac{3}{5} + \frac{10}{5} + \frac{7}{5} = \frac{21}{5}$$

$$E(X_2)^2 = 1^2 * \frac{1}{5} + 3^2 * \frac{1}{5} + 5^2 * \frac{2}{5} + 7^2 * \frac{1}{5} = \frac{1}{5} + \frac{9}{5} + \frac{50}{5} + \frac{49}{5} = \frac{109}{5}$$

$$V(X_2) = \frac{109}{5} - \left(\frac{21}{5}\right)^2 = \frac{545}{25} - \frac{441}{25} = \frac{104}{25}$$

COM REPOSIÇÃO

$$V(X_1 + X_2) = \frac{208}{25} = V(X_1) + V(X_2) = \frac{104}{25} + \frac{104}{25} = \frac{208}{25}$$

Os eventos X_1 e X_2 são independentes

SEM REPOSIÇÃO

$$V(X_1 + X_2) = \frac{67}{5} \neq V(X_1) + V(X_2) = \frac{104}{25} + \frac{104}{25} = \frac{208}{25}$$

Os eventos X_1 e X_2 não são independentes

4) Considerando a distribuição de probabilidade conjunta de (X, Y) da questão 1.

a) Encontre as distribuições de probabilidade das variáveis: $X+3$ e $2Y$

$$U = X+3$$

$$S_u = \{4, 5, 6\}$$

$$\mathbb{P}(U = 4) = 0,1 + 0,2 + 0 = 0,3$$

$$\mathbb{P}(U = 5) = 0,1 + 0 + 0,1 = 0,2$$

$$\mathbb{P}(X = 1) = 0,1 + 0,3 + 0,1 = 0,5$$

$$V = 2Y$$

$$S_v = \{0, 2, 4\}$$

$$\mathbb{P}(V = 0) = 0,1 + 0,1 + 0,1 = 0,3$$

$$\mathbb{P}(V = 2) = 0,2 + 0 + 0,3 = 0,5$$

$$\mathbb{P}(V = 4) = 0 + 0,1 + 0,1 = 0,2$$

b) Calcule a esperança e variância de $X+3$ e de $2Y$

$$U = X+3$$

$$E(U) = 4*0,3 + 5*0,2 + 6*0,5 = 1,2 + 1 + 3 = 5,2$$

$$E(U^2) = 4^2 * 0,3 + 5^2 * 0,2 + 6^2 * 0,5 = 4,8 + 5 + 18 = 27,8$$

$$V(U) = 27,8 - 5,2^2 = 27,8 - 27,04 = 0,76$$

$$V = 2Y$$

$$E(V) = 0*0,3 + 2*0,5 + 4*0,2 = 0 + 1 + 0,8 = 1,8$$

$$E(U^2) = 0^2 * 0,3 + 2^2 * 0,5 + 4^2 * 0,2 = 0 + 2 + 3,2 = 5,2$$

$$V(U) = 5,2 - 1,8^2 = 5,2 - 3,24 = 1,96$$

5) Considere a distribuição conjunta de X e Y , parcialmente conhecida, dada por:

a) Complete a Tabela

Y	X			$\mathbb{P}(Y = y)$
	-1	0	1	
-1	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{7}{12}$
0	$\frac{0}{12}$	$\frac{1}{12}$	$\frac{0}{12}$	$\frac{1}{12}$
1	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{4}{12}$
$\mathbb{P}(X = x)$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	1

$$\mathbb{P}(X = -1) = \mathbb{P}(X = -1 \cap Y = -1) + \mathbb{P}(X = -1 \cap Y = 0) + \mathbb{P}(X = -1 \cap Y = 1)$$

$$\frac{3}{12} = \frac{1}{12} + \mathbb{P}(X = -1 \cap Y = 0) + \frac{2}{12}$$

$$\mathbb{P}(X = -1 \cap Y = 0) = \frac{3}{12} - \left(\frac{1}{12} + \frac{2}{12}\right) = 0$$

$$\mathbb{P}(Y = -1) = \mathbb{P}(X = -1 \cap Y = -1) + \mathbb{P}(X = 0 \cap Y = -1) + \mathbb{P}(X = 1 \cap Y = -1)$$

$$\frac{7}{12} = \frac{1}{12} + \mathbb{P}(X = 0 \cap Y = -1) + \frac{4}{12}$$

$$\mathbb{P}(X = 0 \cap Y = -1) = \frac{7}{12} - \left(\frac{1}{12} + \frac{4}{12}\right) = \frac{2}{12}$$

$$\mathbb{P}(X = 1) = \mathbb{P}(X = 1 \cap Y = -1) + \mathbb{P}(X = 1 \cap Y = 0) + \mathbb{P}(X = 1 \cap Y = 1)$$

$$\frac{5}{12} = \frac{4}{12} + \mathbb{P}(X = 1 \cap Y = 0) + \frac{1}{12}$$

$$\mathbb{P}(X = 1 \cap Y = 0) = \frac{5}{12} - \left(\frac{4}{12} + \frac{1}{12}\right) = 0$$

$$\mathbb{P}(Y = 0) = \mathbb{P}(X = -1 \cap Y = 0) + \mathbb{P}(X = 0 \cap Y = 0) + \mathbb{P}(X = 1 \cap Y = 0)$$

$$\mathbb{P}(Y = 0) = 0 + \frac{1}{12} + 0 = \frac{1}{12}$$

$$\mathbb{P}(X = 0) = \mathbb{P}(X = 0 \cap Y = -1) + \mathbb{P}(X = 0 \cap Y = 0) + \mathbb{P}(X = 0 \cap Y = 1)$$

$$\frac{4}{12} = \frac{2}{12} + \frac{1}{12} + \mathbb{P}(X = 0 \cap Y = 1)$$

$$\mathbb{P}(X = 0 \cap Y = 1) = \frac{4}{12} - \left(\frac{2}{12} + \frac{1}{12}\right) = \frac{1}{12}$$

$$\mathbb{P}(Y = 1) = \mathbb{P}(X = -1 \cap Y = 1) + \mathbb{P}(X = 0 \cap Y = 1) + \mathbb{P}(X = 1 \cap Y = 1)$$

$$\mathbb{P}(Y = 1) = \frac{2}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12}$$

b) Encontre a distribuição de probabilidade de $X + Y$

$$S_{x+y} = \begin{cases} (-1, -1) = -2 \\ (-1, 0); (0, -1) = -1 \\ (-1, 1); (0, 0); (1, -1) = 0 \\ (0, 1); (1, 0) = 1 \\ (1, 1) = 2 \end{cases}$$

$$\mathbb{P}(X + Y = -2) = \frac{1}{12}$$

$$\mathbb{P}(X + Y = -1) = 0 + \frac{2}{12} = \frac{2}{12}$$

$$\mathbb{P}(X + Y = 0) = \frac{2}{12} + \frac{1}{12} + \frac{4}{12} = \frac{7}{12}$$

$$\mathbb{P}(X + Y = 1) = \frac{1}{12} + 0 = \frac{1}{12}$$

$$\mathbb{P}(X + Y = 2) = \frac{1}{12}$$

c) Verifique se X e Y são independentes

$$\mathbb{P}(X = -1 \cap Y = -1) = \frac{1}{12} \neq \mathbb{P}(X = -1) * \mathbb{P}(Y = -1) = \frac{3}{12} * \frac{7}{12} = \frac{21}{144} = \frac{7}{48}$$

Os eventos X e Y não são independentes