

## Lista Extra 5

### Exercício de Variável Bidimensional Discreta Independente 10/10/2022

Existe numa urna 5 bolas numeradas: -1, -1, 0, 1, 1. Serão retiradas 2 bolas simultaneamente (uma a uma sem reposição). Considere:

$X$  = Soma dos números

$Y$  = diferença em módulo (valor absoluto dos números)

$W = X - Y$

a) Encontre  $E(X \mid Y = 0)$

$$X = B_1 + B_2$$

$$Y = |B_1| - |B_2|$$

$B_1$	$B_2$	$X$	$Y$	$\mathbb{P}(B_1 = b_1 \cap B_2 = b_2)$
-1	-1	-2	0	$\frac{2}{5} * \frac{1}{4} = \frac{2}{20}$
-1	0	-1	1	$\frac{2}{5} * \frac{1}{4} = \frac{2}{20}$
-1	1	0	0	$\frac{2}{5} * \frac{2}{4} = \frac{4}{20}$
0	-1	-1	-1	$\frac{1}{5} * \frac{2}{4} = \frac{2}{20}$
0	1	1	-1	$\frac{1}{5} * \frac{2}{4} = \frac{2}{20}$
1	-1	-2	0	$\frac{2}{5} * \frac{2}{4} = \frac{4}{20}$
1	0	1	1	$\frac{2}{5} * \frac{1}{4} = \frac{2}{20}$
1	1	2	0	$\frac{2}{5} * \frac{1}{4} = \frac{2}{20}$

Y	X					$\mathbb{P}(Y = y)$
	-2	-1	0	1	2	
-1	0	$\frac{2}{20}$	0	$\frac{2}{20}$	0	$\frac{4}{20}$
0	$\frac{6}{20}$	0	$\frac{4}{20}$	0	$\frac{2}{20}$	$\frac{12}{20}$
1	0	$\frac{2}{20}$	0	$\frac{2}{20}$	0	$\frac{4}{20}$
$\mathbb{P}(X = x)$	$\frac{6}{20}$	$\frac{4}{20}$	$\frac{4}{20}$	$\frac{4}{20}$	$\frac{2}{20}$	1

X	$\mathbb{P}(X \mid Y = 0)$
-2	$\frac{\frac{6}{20}}{\frac{12}{20}} = \frac{3}{6}$
-1	$\frac{0}{\frac{12}{20}} = 0$
0	$\frac{\frac{4}{20}}{\frac{12}{20}} = \frac{2}{6}$
1	$\frac{0}{\frac{12}{20}} = 0$
2	$\frac{\frac{2}{20}}{\frac{12}{20}} = \frac{1}{6}$

$$E(X \mid Y = 0) = -2 * \frac{3}{6} + (-1) * 0 + 0 * \frac{2}{6} + 1 * 0 + 2 * \frac{1}{6}$$

$$E(X \mid Y = 0) = -\frac{6}{6} + 0 + 0 + 0 + \frac{2}{6} = -\frac{4}{6}$$

b) Encontre  $E(W)$  e  $V(W)$

X	Y	W
-2	0	-2
-1	1	-2
0	0	0
-1	-1	0
1	-1	2
-2	0	-2
1	1	0
2	0	2

$$E(W) = -2 * (\frac{2}{20} + \frac{2}{20} + \frac{4}{20}) + 0 * (\frac{4}{20} + \frac{2}{20} + \frac{2}{20}) + 2 * (\frac{2}{20} + \frac{2}{20})$$

$$E(W) = -\frac{16}{20} + 0 + \frac{8}{20} = -\frac{8}{16}$$

$$E(W^2) = (-2)^2 * (\frac{2}{20} + \frac{2}{20} + \frac{4}{20}) + 0^2 * (\frac{4}{20} + \frac{2}{20} + \frac{2}{20}) + 2^2 * (\frac{2}{20} + \frac{2}{20})$$

$$E(W^2) = \frac{32}{20} + 0 + \frac{16}{20} = \frac{48}{16} = 3$$

$$V(W) = E(W^2) - E^2(W)$$

$$V(W) = 3 - \left(-\frac{8}{16}\right)^2 = 3 - \frac{64}{256} = 3 - 0,25 = 2,75$$