Lista 7

Curso de Ciências Atuariais
Disciplina Probabilidade 1- Professora Cristina
12/08/2022 - Exercícios de função de v.a. e distribuição conjunta

1) O Quadro abaixo dá a distribuição de probabilidade conjunta das v. a. X e Y.

Y	X		
	1	2	3
0	0,1	0,1	0,1
1	0,2	0	0,3
2	0	0,1	0,1

a) Obtenha as distribuições de X+Y e de XY

$S_{x+y} = \{1, 2, 3, 4, 5\}$

$\mathbb{P}(X+Y = 1) = 0{,}1$ $\quad\quad$ $\mathbb{P}(X+Y = 2) = 0{,}1 + 0{,}2 = 0{,}3$

$\mathbb{P}(X+Y = 3) = 0{,}1 + 0 + 0 = 0{,}1$ $\quad\quad$ $\mathbb{P}(X+Y = 4) = 0{,}3 + 0{,}1 = 0{,}4$

$\mathbb{P}(X+Y = 5) = 0{,}1$

$S_{xy} = \{0, 1, 2, 3, 4, 6\}$

$\mathbb{P}(XY = 0) = 0{,}1 + 0{,}1 + 0{,}1 = 0{,}3$ $\quad\quad$ $\mathbb{P}(XY = 1) = 0{,}2$ $\quad\quad$ $\mathbb{P}(XY = 2) = 0 + 0 = 0$

$\mathbb{P}(XY = 3) = 0{,}3$ $\quad\quad$ $\mathbb{P}(XY = 4) = 0{,}1$ $\quad\quad$ $\mathbb{P}(XY = 6) = 0{,}1$

b) Calcule E(X+Y), E(XY), V(X+Y) e V(XY)

$$\text{E(X+Y)} = 1 * 0,1 + 2 * 0,3 + 3 * 0,1 + 4 * 0,4 + 5 * 0,1$$
$$\text{E(X+Y)} = 0,1 + 0,6 + 0,3 + 1,6 + 0,5 = 3,1$$

$$E[(X+Y)^2] = 1^2 * 0,1 + 2^2 * 0,3 + 3^2 * 0,1 + 4^2 * 0,4 + 5^2 * 0,1$$
$$E[(X+Y)^2] = 0,1 + 1,2 + 0,9 + 6,4 + 2,5 = 11,1$$

$$\text{V(X+Y)} = 11,1 - 3,1^2 = 11,1 - 9,61 = 1,49$$

$$\text{E(XY)} = 0 * 0,3 + 1 * 0,2 + 2 * 0 + 3 * 0,3 + 4 * 0,1 + 6 * 0,1$$
$$\text{E(XY)} = 0 + 0,2 + 0 + 0,9 + 0,4 + 0,6 = 2,1$$

$$E[(XY)^2] = 0^2 * 0,3 + 1^2 * 0,2 + 2^2 * 0 + 3^2 * 0,3 + 4^2 * 0,1 + 6^2 * 0,1$$
$$E[(XY)^2] = 0 + 0,2 + 0 + 2,7 + 1,6 + 3,6 = 8,1$$

$$\text{V(X+Y)} = 8,1 - 2,1^2 = 8,1 - 4,41 = 3,69$$

2) Numa urna tem cinco bolas marcadas com os seguintes números: -1, 0, 0, 0, 1. Retiram-se 3 bolas simultaneamente. X indica a soma dos números obtidos e Y o maior valor da trinca.
a) Determine a distribuição de probabilidade conjunta de (X,Y)

$$S_x = \begin{cases} (0,0,-1);(0,-1,0);(-1,0,0) = -1 \\ (0,0,0);(0,1,-1);(1,0,-1);(1,-1,0);(0,-1,1);(-1,0,1);(-1,1,0) = 0 \\ (0,0,1);(0,1,0);(1,0,0) = 1 \end{cases}$$

$$\mathbb{P}(\text{X = -1}) = (\frac{3}{5} * \frac{2}{4} * \frac{1}{3}) * 3 = (\frac{6}{60}) * 3 = \frac{18}{60}$$

$$\mathbb{P}(\text{X = 0}) = \frac{3}{5} * \frac{2}{4} * \frac{1}{3} + (\frac{3}{5} * \frac{1}{4} * \frac{1}{3}) * 6 = \frac{6}{60} + (\frac{3}{60}) * 6 = \frac{6}{60} + \frac{18}{60} = \frac{24}{60}$$

$$\mathbb{P}(\text{X = 1}) = (\frac{3}{5} * \frac{2}{4} * \frac{1}{3}) * 3 = (\frac{6}{60}) * 3 = \frac{18}{60}$$

$$S_y = \begin{cases} (0,0,0)(0,0,-1);(0,-1,0);(-1,0,0) = 0 \\ (0,1,-1);(1,0,-1);(1,-1,0);(0,-1,1); \\ (-1,0,1);(-1,1,0),(0,0,1);(0,1,0);(1,0,0) = 1 \end{cases}$$

$$\mathbb{P}(\text{Y = 0}) = (\frac{3}{5} * \frac{2}{4} * \frac{1}{3}) * 4 = (\frac{6}{60}) * 4 = \frac{24}{60}$$

$$\mathbb{P}(\mathrm{Y}=1)=(\frac{3}{5}*\frac{2}{4}*\frac{1}{3})*3+(\frac{3}{5}*\frac{1}{4}*\frac{1}{3})*6=(\frac{6}{60})*3+(\frac{3}{60})*6=\frac{18}{60}+\frac{18}{60}=\frac{36}{60}$$

Y	X			$\mathbb{P}(\mathrm{Y}=\mathrm{y})$
	-1	0	1	
0	$\frac{18}{60}$	$\frac{6}{60}$	0	$\frac{24}{60}$
1	0	$\frac{18}{60}$	$\frac{18}{60}$	$\frac{36}{60}$
$\mathbb{P}(\mathrm{X}=\mathrm{x})$	$\frac{18}{60}$	$\frac{24}{60}$	$\frac{18}{60}$	1

$$\mathbb{P}(\mathrm{X}=\text{-}1\cap\mathrm{Y}=0)=(\frac{3}{5}*\frac{2}{4}*\frac{1}{3})*3=(\frac{6}{60})*3=\frac{18}{60}$$

$$\mathbb{P}(\mathrm{X}=\text{-}1)=\mathbb{P}(\mathrm{X}=\text{-}1\cap\mathrm{Y}=0)+\mathbb{P}(\mathrm{X}=\text{-}1\cap\mathrm{Y}=1)$$
$$\frac{18}{60}=\frac{18}{60}+\mathbb{P}(\mathrm{X}=\text{-}1\cap\mathrm{Y}=1)$$
$$\mathbb{P}(\mathrm{X}=\text{-}1\cap\mathrm{Y}=1)=\frac{18}{60}-\frac{18}{60}=0$$

$$\mathbb{P}(\mathrm{X}=0\cap\mathrm{Y}=0)=(\frac{3}{5}*\frac{2}{4}*\frac{1}{3})*3=\frac{6}{60}$$

$$\mathbb{P}(\mathrm{X}=0)=\mathbb{P}(\mathrm{X}=0\cap\mathrm{Y}=0)+\mathbb{P}(\mathrm{X}=0\cap\mathrm{Y}=1)$$
$$\frac{24}{60}=\frac{6}{60}+\mathbb{P}(\mathrm{X}=0\cap\mathrm{Y}=1)$$
$$\mathbb{P}(\mathrm{X}=0\cap\mathrm{Y}=1)=\frac{24}{60}-\frac{6}{60}=\frac{18}{60}$$

$$\mathbb{P}(\mathrm{Y}=0)=\mathbb{P}(\mathrm{X}=\text{-}1\cap\mathrm{Y}=0)+\mathbb{P}(\mathrm{X}=0\cap\mathrm{Y}=0)+\mathbb{P}(\mathrm{X}=1\cap\mathrm{Y}=0)$$
$$\frac{24}{60}=\frac{18}{60}+\frac{6}{60}+\mathbb{P}(\mathrm{X}=1\cap\mathrm{Y}=0)$$
$$\mathbb{P}(\mathrm{X}=1\cap\mathrm{Y}=0)=\frac{24}{60}-\frac{18}{60}-\frac{6}{60}=0$$

$$\mathbb{P}(\mathrm{Y}=1)=\mathbb{P}(\mathrm{X}=\text{-}1\cap\mathrm{Y}=1)+\mathbb{P}(\mathrm{X}=0\cap\mathrm{Y}=1)+\mathbb{P}(\mathrm{X}=1\cap\mathrm{Y}=1)$$
$$\frac{36}{60}=0+\frac{18}{60}+\mathbb{P}(\mathrm{X}=1\cap\mathrm{Y}=1)$$
$$\mathbb{P}(\mathrm{X}=1\cap\mathrm{Y}=1)=\frac{36}{60}-\frac{18}{60}=\frac{18}{60}$$

b) Encontre E(X) e V(X)

$$\text{E(X)} = (-1) * \frac{18}{60} + 0 * \frac{24}{60} + 1 * \frac{18}{60} = \frac{-18}{60} + 0 + \frac{18}{60} = 0$$

$$\text{E(X}^2\text{)} = (-1)^2 * \frac{18}{60} + 0^2 * \frac{24}{60} + 1^2 * \frac{18}{60} = \frac{18}{60} + 0 + \frac{18}{60} = \frac{36}{60}$$

$$\text{V(X)} = \frac{36}{60} - 0^2 = \frac{36}{60}$$

c) Encontre a distribuição de probabilidade de X+Y

$$S_{x+y} = \begin{cases} (-1, 0) = -1 \\ (-1, 1); (0, 0) = 0 \\ (1, 0); (0, 1) = 1 \\ (1, 1) = 2 \end{cases}$$

$$\mathbb{P}\text{(X + Y = -1)} = \frac{18}{60}$$

$$\mathbb{P}\text{(X + Y = 0)} = 0 + \frac{6}{60} = \frac{6}{60}$$

$$\mathbb{P}\text{(X + Y = 1)} = 0 + \frac{18}{60} = \frac{18}{60}$$

$$\mathbb{P}\text{(X + Y = 2)} = \frac{18}{60} = \frac{18}{60}$$

d) Encontre E(X+Y) e V(X+Y)

$$\text{E(X+Y)} = (-1) * \frac{18}{60} + 0 * \frac{6}{60} + 1 * \frac{18}{60} + 2 * \frac{18}{60} = \frac{-18}{60} + 0 + \frac{18}{60} + \frac{36}{60} = \frac{36}{60} = \frac{3}{5}$$

$$\text{E[(X+Y)}^2\text{]} = (-1)^2 * \frac{18}{60} + 0^2 * \frac{6}{60} + 1^2 * \frac{18}{60} + 2^2 * \frac{18}{60} = \frac{18}{60} + 0 + \frac{18}{60} + \frac{72}{60} = \frac{108}{60} = \frac{9}{5}$$

$$\text{V(x+Y)} = \frac{9}{5} - (\frac{3}{5})^2 = \frac{45}{25} - \frac{9}{25} = \frac{36}{25}$$

3) Numa urna tem cinco tiras de papel, numeradas 1,3 5, 5. 7. Uma tira é sorteada e recolocada na urna, Depois uma segunda tira é retirada. Sejam as v.a. X_1 e X_2 que representam o primeiro e o segundo número sorteado.
a) Determine a distribuição conjunta de (X_1, X_2)

$$\mathbb{P}(X_1 = 1) = \frac{1}{5} \qquad \mathbb{P}(X_1 = 1) = \frac{1}{5}$$

$$\mathbb{P}(X_1 = 3) = \frac{1}{5} \qquad \mathbb{P}(X_2 = 3) = \frac{1}{5}$$

$$\mathbb{P}(X_1 = 5) = \frac{2}{5} \qquad \mathbb{P}(X_2 = 5) = \frac{2}{5}$$

$$\mathbb{P}(X_1 = 7) = \frac{1}{5} \qquad \mathbb{P}(X_2 = 7) = \frac{1}{5}$$

X_2	X_1				$\mathbb{P}(X_2 = x)$
	1	3	5	7	
1	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{1}{5}$
3	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{1}{5}$
5	$\frac{2}{25}$	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{2}{25}$	$\frac{2}{5}$
7	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{1}{5}$
$\mathbb{P}(X_1 = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	1

$$\mathbb{P}(X_1 = 1 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25} \qquad \mathbb{P}(X_1 = 3 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 1) = \frac{2}{5} * \frac{1}{5} = \frac{2}{25} \qquad \mathbb{P}(X_1 = 7 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

$$\mathbb{P}(X_1 = 1 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25} \qquad \mathbb{P}(X_1 = 3 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 3) = \frac{2}{5} * \frac{1}{5} = \frac{2}{25} \qquad \mathbb{P}(X_1 = 7 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

$$\mathbb{P}(X_1 = 1 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{5} = \frac{2}{25} \qquad \mathbb{P}(X_1 = 3 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{5} = \frac{2}{25}$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 5) = \frac{2}{5} * \frac{2}{5} = \frac{4}{25} \qquad \mathbb{P}(X_1 = 7 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{5} = \frac{2}{25}$$

$$\mathbb{P}(X_1 = 1) = \mathbb{P}(X_1 = 1 \cap X_2 = 1) + \mathbb{P}(X_1 = 1 \cap X_2 = 3)$$
$$+ \mathbb{P}(X_1 = 1 \cap X_2 = 5) + \mathbb{P}(X_1 = 1 \cap X_2 = 7)$$
$$\frac{1}{5} = \frac{1}{25} + \frac{1}{25} + \frac{2}{25} + \mathbb{P}(X_1 = 1 \cap X_2 = 7)$$
$$\mathbb{P}(X_1 = 1 \cap X_2 = 7) = \frac{5}{25} - \left(\frac{1}{25} + \frac{1}{25} + \frac{2}{25}\right) = \frac{1}{25}$$

$$\mathbb{P}(\mathrm{X}_1 = 3) = \mathbb{P}(\mathrm{X}_1 = 3 \cap \mathrm{X}_2 = 1) + \mathbb{P}(\mathrm{X}_1 = 3 \cap \mathrm{X}_2 = 3) + \mathbb{P}(\mathrm{X}_1 = 3 \cap \mathrm{X}_2 = 5) + \mathbb{P}(\mathrm{X}_1 = 3 \cap \mathrm{X}_2 = 7)$$

$$\frac{1}{5} = \frac{1}{25} + \frac{1}{25} + \frac{2}{25} + \mathbb{P}(\mathrm{X}_1 = 3 \cap \mathrm{X}_2 = 7)$$

$$\mathbb{P}(\mathrm{X}_1 = 3 \cap \mathrm{X}_2 = 7) = \frac{5}{25} - (\frac{1}{25} + \frac{1}{25} + \frac{2}{25}) = \frac{1}{25}$$

$$\mathbb{P}(\mathrm{X}_1 = 5) = \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 1) + \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 3) + \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 5) + \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 7)$$

$$\frac{2}{5} = \frac{2}{25} + \frac{2}{25} + \frac{4}{25} + \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 7)$$

$$\mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 7) = \frac{10}{25} - (\frac{2}{25} + \frac{2}{25} + \frac{4}{25}) = \frac{2}{25}$$

$$\mathbb{P}(\mathrm{X}_1 = 7) = \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 1) + \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 3) + \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 5) + \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 7)$$

$$\frac{1}{5} = \frac{1}{25} + \frac{1}{25} + \frac{2}{25} + \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 7)$$

$$\mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 7) = \frac{5}{25} - (\frac{1}{25} + \frac{1}{25} + \frac{2}{25}) = \frac{1}{25}$$

b) Encontre $\mathrm{E}(\mathrm{X}_1 + \mathrm{X}_2)$ e $\mathrm{V}(\mathrm{X}_1 + \mathrm{X}_2)$

$$S_{x+y} = \begin{cases} (1,1) = 2 \\ (1,3); (3,1) = 4 \\ (1,5); (5,1); (3,3) = 6 \\ (1,7); (7,1); (3,5); (5,3) = 8 \\ (3,7); (7,3); (5,5) = 10 \\ (5,7); (7,5) = 12 \\ (7,7) = 14 \end{cases}$$

$$\mathrm{E}(\mathrm{X}_1 + \mathrm{X}_2) = 2 * \frac{1}{25} + 4 * \frac{2}{25} + 6 * \frac{5}{25} + 8 * \frac{6}{25} + 10 * \frac{6}{25} + 12 * \frac{4}{25} + 14 * \frac{1}{25}$$

$$\mathrm{E}(\mathrm{X}_1 + \mathrm{X}_2) = \frac{2}{25} + \frac{8}{25} + \frac{30}{25} + \frac{48}{25} + \frac{60}{25} + \frac{48}{25} + \frac{14}{25} = \frac{210}{25} = \frac{42}{5}$$

$$\mathrm{E}[(\mathrm{X}_1 + \mathrm{X}_2)^2] = 2^2 * \frac{1}{25} + 4^2 * \frac{2}{25} + 6^2 * \frac{5}{25} + 8^2 * \frac{6}{25} + 10^2 * \frac{6}{25} + 12^2 * \frac{4}{25} + 14^2 * \frac{1}{25}$$

$$\mathrm{E}[(\mathrm{X}_1 + \mathrm{X}_2)^2] = \frac{4}{25} + \frac{32}{25} + \frac{180}{25} + \frac{384}{25} + \frac{600}{25} + \frac{576}{25} + \frac{196}{25} = \frac{1.972}{25}$$

$$V(X_1 + X_2) = \frac{1.972}{25} - (\frac{42}{5})^2 = \frac{1.972}{25} - \frac{1.764}{25} = \frac{208}{25}$$

c) Refaça a questão se a retirada for sem reposição

X_2	X_1				$\mathbb{P}(X_2 = x)$
	1	3	5	7	
1	0	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{1}{20}$	$\frac{1}{5}$
3	$\frac{1}{20}$	0	$\frac{2}{20}$	$\frac{1}{20}$	$\frac{1}{5}$
5	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{2}{20}$	$\frac{2}{5}$
7	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	0	$\frac{1}{5}$
$\mathbb{P}(X_1 = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	1

$$\mathbb{P}(X_1 = 1 \cap X_2 = 1) = \frac{1}{5} * 0 = 0 \qquad \mathbb{P}(X_1 = 3 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{4} = \frac{1}{20}$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 1) = \frac{2}{5} * \frac{1}{4} = \frac{2}{20} \qquad \mathbb{P}(X_1 = 7 \cap X_2 = 1) = \frac{1}{5} * \frac{1}{4} = \frac{1}{20}$$

$$\mathbb{P}(X_1 = 1 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{4} = \frac{1}{20} \qquad \mathbb{P}(X_1 = 3 \cap X_2 = 3) = \frac{1}{5} * 0 = 0$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 3) = \frac{2}{5} * \frac{1}{4} = \frac{2}{20} \qquad \mathbb{P}(X_1 = 7 \cap X_2 = 3) = \frac{1}{5} * \frac{1}{4} = \frac{1}{20}$$

$$\mathbb{P}(X_1 = 1 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{4} = \frac{2}{20} \qquad \mathbb{P}(X_1 = 3 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{4} = \frac{2}{20}$$

$$\mathbb{P}(X_1 = 5 \cap X_2 = 5) = \frac{2}{5} * \frac{1}{4} = \frac{2}{20} \qquad \mathbb{P}(X_1 = 7 \cap X_2 = 5) = \frac{1}{5} * \frac{2}{4} = \frac{2}{20}$$

$$\mathbb{P}(X_1 = 1) = \mathbb{P}(X_1 = 1 \cap X_2 = 1) + \mathbb{P}(X_1 = 1 \cap X_2 = 3)$$
$$+ \mathbb{P}(X_1 = 1 \cap X_2 = 5) + \mathbb{P}(X_1 = 1 \cap X_2 = 7)$$
$$\frac{1}{5} = 0 + \frac{1}{20} + \frac{2}{20} + \mathbb{P}(X_1 = 1 \cap X_2 = 7)$$
$$\mathbb{P}(X_1 = 1 \cap X_2 = 7) = \frac{4}{20} \text{ - } (\frac{1}{20} + \frac{2}{20}) = \frac{1}{20}$$

$$\mathbb{P}(X_1 = 3) = \mathbb{P}(X_1 = 3 \cap X_2 = 1) + \mathbb{P}(X_1 = 3 \cap X_2 = 3)$$
$$+ \mathbb{P}(X_1 = 3 \cap X_2 = 5) + \mathbb{P}(X_1 = 3 \cap X_2 = 7)$$

$$\frac{1}{5} = \frac{1}{20} + 0 + \frac{2}{20} + \mathbb{P}(\mathrm{X}_1 = 3 \cap \mathrm{X}_2 = 7)$$
$$\mathbb{P}(\mathrm{X}_1 = 3 \cap \mathrm{X}_2 = 7) = \frac{4}{20} \text{ - } (\frac{1}{20} + \frac{2}{20}) = \frac{1}{20}$$

$$\mathbb{P}(\mathrm{X}_1 = 5) = \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 1) + \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 3)$$
$$+ \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 5) + \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 7)$$
$$\frac{2}{5} = \frac{2}{20} + \frac{2}{20} + \frac{2}{20} + \mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 7)$$
$$\mathbb{P}(\mathrm{X}_1 = 5 \cap \mathrm{X}_2 = 7) = \frac{8}{20} \text{ - } (\frac{2}{20} + \frac{2}{20} + \frac{2}{20}) = \frac{2}{20}$$

$$\mathbb{P}(\mathrm{X}_1 = 7) = \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 1) + \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 3)$$
$$+ \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 5) + \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 7)$$
$$\frac{1}{5} = \frac{1}{20} + \frac{1}{20} + \frac{2}{20} + \mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 7)$$
$$\mathbb{P}(\mathrm{X}_1 = 7 \cap \mathrm{X}_2 = 7) = \frac{4}{20} \text{ - } (\frac{1}{25} + \frac{1}{25} + \frac{2}{25}) = 0$$

b) Encontre $\mathrm{E}(\mathrm{X}_1 + \mathrm{X}_2)$ e $\mathrm{V}(\mathrm{X}_1 + \mathrm{X}_2)$

$$S_{x+y} = \begin{cases} (1,1) = 2 \\ (1,3); (3,1) = 4 \\ (1,5); (5,1); (3,3) = 6 \\ (1,7); (7,1); (3,5); (5,3) = 8 \\ (3,7); (7,3); (5,5) = 10 \\ (5,7); (7,5) = 12 \\ (7,7) = 14 \end{cases}$$

$$\mathrm{E}(\mathrm{X}_1 + \mathrm{X}_2) = 2 * 0 + 4 * \frac{2}{20} + 6 * \frac{4}{20} + 8 * \frac{6}{20} + 10 * \frac{4}{20} + 12 * \frac{2}{20} + 14 * 0$$
$$\mathrm{E}(\mathrm{X}_1 + \mathrm{X}_2) = 0 + \frac{8}{20} + \frac{24}{20} + \frac{48}{20} + \frac{40}{20} + \frac{24}{20} + 0 = \frac{140}{20} = \frac{35}{5}$$

$$\mathrm{E}[(\mathrm{X}_1 + \mathrm{X}_2)^2] = 2^2 * 0 + 4^2 * \frac{2}{20} + 6^2 * \frac{4}{20} + 8^2 * \frac{6}{20} + 10^2 * \frac{4}{20} + 12^2 * \frac{2}{20} + 14^2 * 0$$
$$\mathrm{E}[(\mathrm{X}_1 + \mathrm{X}_2)^2] = 0 + \frac{32}{20} + \frac{144}{20} + \frac{384}{20} + \frac{400}{20} + \frac{288}{20} + 0 = \frac{1.248}{20} = \frac{312}{5}$$

$$\mathrm{V}(\mathrm{X}_1 + \mathrm{X}_2) = \frac{312}{5} - (\frac{35}{5})^2 = \frac{1.560}{25} - \frac{1.225}{25} = \frac{335}{25} = \frac{67}{5}$$

d) Verifique, em ambos os casos se X_1 e X_2 são independentes

$$E(X_1) = 1 * \frac{1}{5} + 3 * \frac{1}{5} + 5 * \frac{2}{5} + 7 * \frac{1}{5} = \frac{1}{5} + \frac{3}{5} + \frac{10}{5} + \frac{7}{5} = \frac{21}{5}$$

$$E(X_1)^2 = 1^2 * \frac{1}{5} + 3^2 * \frac{1}{5} + 5^2 * \frac{2}{5} + 7^2 * \frac{1}{5} = \frac{1}{5} + \frac{9}{5} + \frac{50}{5} + \frac{49}{5} = \frac{109}{5}$$

$$V(X_1) = \frac{109}{5} - (\frac{21}{5})^2 = \frac{545}{25} - \frac{441}{25} = \frac{104}{25}$$

$$E(X_2) = 1 * \frac{1}{5} + 3 * \frac{1}{5} + 5 * \frac{2}{5} + 7 * \frac{1}{5} = \frac{1}{5} + \frac{3}{5} + \frac{10}{5} + \frac{7}{5} = \frac{21}{5}$$

$$E(X_2)^2 = 1^2 * \frac{1}{5} + 3^2 * \frac{1}{5} + 5^2 * \frac{2}{5} + 7^2 * \frac{1}{5} = \frac{1}{5} + \frac{9}{5} + \frac{50}{5} + \frac{49}{5} = \frac{109}{5}$$

$$V(X_2) = \frac{109}{5} - (\frac{21}{5})^2 = \frac{545}{25} - \frac{441}{25} = \frac{104}{25}$$

COM REPOSIÇÃO

$$V(X_1 + X_2) = \frac{208}{25} = V(X_1) + V(_2) = \frac{104}{25} + \frac{104}{25} = \frac{108}{25}$$

Os eventos X_1 e X_2 são independentes

SEM REPOSIÇÃO

$$V(X_1 + X_2) = \frac{67}{5} \neq V(X_1) + V(_2) = \frac{104}{25} + \frac{104}{25} = \frac{108}{25}$$

Os eventos X_1 e X_2 não são independentes

4) Considerando a distribuição de probabilidade conjunta de (X, Y) da questão 1.
a) Encontre as distribuições de probabilidade das variáveis: X+3 e 2Y

U = X+3

$S_u = \{4, 5, 6\}$

$$\mathbb{P}(\text{U} = 4) = 0{,}1 + 0{,}2 + 0 = 0{,}3$$

$$\mathbb{P}(\text{U} = 5) = 0{,}1 + 0 + 0{,}1 = 0{,}2$$

$$\mathbb{P}(\text{X} = 1) = 0{,}1 + 0{,}3 + 0{,}1 = 0{,}5$$

V = 2Y

$\text{S}_v = \{0, 2, 4\}$

$$\mathbb{P}(\text{V} = 0) = 0{,}1 + 0{,}1 + 0{,}1 = 0{,}3$$

$$\mathbb{P}(\text{V} = 2) = 0{,}2 + 0 + 0{,}3 = 0{,}5$$

$$\mathbb{P}(\text{V} = 4) = 0 + 0{,}1 + 0{,}1 = 0{,}2$$

b) Calcule a esperança e variância de X+3 e de 2Y

U = X+3

$$\text{E(U)} = 4*0{,}3 + 5*0{,}2 + 6*0{,}5 = 1{,}2 + 1 + 3 = 5{,}2$$

$$\text{E(U}^2\text{)} = 4^2 * 0,3 + 5^2 * 0,2 + 6^2 * 0,5 = 4{,}8 + 5 + 18 = 27{,}8$$

$$\text{V(U)} = 27{,}8 \text{ - } 5,2^2 = 27{,}8 \text{ - } 27{,}04 = 0{,}76$$

V = 2Y

$$\text{E(V)} = 0*0{,}3 + 2*0{,}5 + 4*0{,}2 = 0 + 1 + 0{,}8 = 1{,}8$$

$$\text{E(U}^2\text{)} = 0^2 * 0,3 + 2^2 * 0,5 + 4^2 * 0,2 = 0 + 2 + 3{,}2 = 5{,}2$$

$$\text{V(U)} = 5{,}2 \text{ - } 1,8^2 = 5{,}2 \text{ - } 3{,}24 = 1{,}96$$

5)Considere a distribuição conjunta de X e Y, parcialmente conhecida, dada por:
a) Complete a Tabela

Y	X			$\mathbb{P}(Y = y)$
	-1	0	1	
-1	$\frac{1}{12}$	$\mathbf{\frac{2}{12}}$	$\frac{4}{12}$	$\frac{7}{12}$
0	$\mathbf{0}$	$\frac{1}{12}$	$\mathbf{0}$	$\mathbf{\frac{1}{12}}$
1	$\frac{2}{12}$	$\mathbf{\frac{1}{12}}$	$\frac{1}{12}$	$\mathbf{\frac{4}{12}}$
$\mathbb{P}(X = x)$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	1

$$\mathbb{P}(X = -1) = \mathbb{P}(X = -1 \cap Y = -1) + \mathbb{P}(X = -1 \cap Y = 0) + \mathbb{P}(X = -1 \cap Y = 1)$$
$$\frac{3}{12} = \frac{1}{12} + \mathbb{P}(X = -1 \cap Y = 0) + \frac{2}{12}$$
$$\mathbb{P}(X = -1 \cap Y = 0) = \frac{3}{12} - (\frac{1}{12} + \frac{2}{12}) = 0$$

$$\mathbb{P}(Y = -1) = \mathbb{P}(X = -1 \cap Y = -1) + \mathbb{P}(X = 0 \cap Y = -1) + \mathbb{P}(X = 1 \cap Y = -1)$$
$$\frac{7}{12} = \frac{1}{12} + \mathbb{P}(X = 0 \cap Y = -1) + \frac{4}{12}$$
$$\mathbb{P}(X = 0 \cap Y = -1) = \frac{7}{12} - (\frac{1}{12} + \frac{4}{12}) = \frac{2}{12}$$

$$\mathbb{P}(X = 1) = \mathbb{P}(X = 1 \cap Y = -1) + \mathbb{P}(X = 1 \cap Y = 0) + \mathbb{P}(X = 1 \cap Y = 1)$$
$$\frac{5}{12} = \frac{4}{12} + \mathbb{P}(X = 1 \cap Y = 0) + \frac{1}{12}$$
$$\mathbb{P}(X = 1 \cap Y = 0) = \frac{5}{12} - (\frac{4}{12} + \frac{1}{12}) = 0$$

$$\mathbb{P}(Y = 0) = \mathbb{P}(X = -1 \cap Y = 0) + \mathbb{P}(X = 0 \cap Y = 0) + \mathbb{P}(X = 1 \cap Y = 0)$$
$$\mathbb{P}(Y = 0) = 0 + \frac{1}{12} + 0 = \frac{1}{12}$$

$$\mathbb{P}(X = 0) = \mathbb{P}(X = 0 \cap Y = -1) + \mathbb{P}(X = 0 \cap Y = 0) + \mathbb{P}(X = 0 \cap Y = 1)$$
$$\frac{4}{12} = \frac{2}{12} + \frac{1}{12} + \mathbb{P}(X = 0 \cap Y = 1)$$
$$\mathbb{P}(X = 0 \cap Y = 1) = \frac{4}{12} - (\frac{2}{12} + \frac{1}{12}) = \frac{1}{12}$$

$$\mathbb{P}(Y = 1) = \mathbb{P}(X = -1 \cap Y = 1) + \mathbb{P}(X = 0 \cap Y = 1) + \mathbb{P}(X = 1 \cap Y = 1)$$
$$\mathbb{P}(Y = 1) = \frac{2}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12}$$

b) Encontre a distribuição de probabilidade de X + Y

$$S_{x+y} = \begin{cases} (-1,-1) = -2 \\ (-1,0);(0,-1) = -1 \\ (-1,1);(0,0);(1,-1) = 0 \\ (0,1);(1,0) = 1 \\ (1,1) = 2 \end{cases}$$

$\mathbb{P}(\text{X + Y = -2}) = \dfrac{1}{12}$

$\mathbb{P}(\text{X + Y = -1}) = 0 + \dfrac{2}{12} = \dfrac{2}{12}$

$\mathbb{P}(\text{X + Y = 0}) = \dfrac{2}{12} + \dfrac{1}{12} + \dfrac{4}{12} = \dfrac{7}{12}$

$\mathbb{P}(\text{X + Y = 1}) = \dfrac{1}{12} + 0 = \dfrac{1}{12}$

$\mathbb{P}(\text{X + Y = 2}) = \dfrac{1}{12}$

c) Verifique se X e Y são independentes

$$\mathbb{P}(\text{X = -1} \cap \text{Y = -1}) = \frac{1}{12} \neq \mathbb{P}(\text{X = -1})*\mathbb{P}(\text{Y = -1}) = \frac{3}{12} * \frac{7}{12} = \frac{21}{144} = \frac{7}{48}$$

Os eventos X e Y não são independentes