LISTA 5 - PROBABILIDADE 1

(1)

$3x + x = 1 \rightarrow 4x = 1 \rightarrow x = 1/4$

$P(\text{SAIR CARA}) = 3x \rightarrow 3 \cdot \frac{1}{4} = \frac{3}{4}$

$P(\text{SAIR COROA}) = x \rightarrow \frac{1}{4}$

$P(X=0) = \frac{1}{4} \cdot \frac{1}{4} = \boxed{\frac{1}{16}}$

$P(X=1) = \frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16} + \frac{3}{16} = \boxed{\frac{6}{16}}$

$P(X=2) = \frac{3}{4} \cdot \frac{3}{4} = \boxed{\frac{9}{16}}$

(2)

a) COM REPOSIÇÃO

$P(X=0) = \frac{15}{20} \cdot \frac{15}{20} \cdot \frac{15}{20} = \boxed{\frac{3.375}{8000}}$

$P(X=1) = \frac{15}{20} \cdot \frac{15}{20} \cdot \frac{5}{20} + \frac{15}{20} \cdot \frac{5}{20} \cdot \frac{15}{20} + \frac{5}{20} \cdot \frac{15}{20} \cdot \frac{15}{20}$

$= 3 \cdot \left(\frac{1.125}{8000}\right) = \boxed{\frac{3.375}{8000}}$

$P(X=2) = \frac{15}{20} \cdot \frac{5}{20} \cdot \frac{5}{20} + \frac{5}{20} \cdot \frac{15}{20} \cdot \frac{5}{20} + \frac{5}{20} \cdot \frac{5}{20} \cdot \frac{15}{20}$

$= 3\left(\frac{375}{8000}\right) = \frac{1125}{8000}$

$P(X=3) = \frac{5}{20} \cdot \frac{5}{20} \cdot \frac{5}{20} = \frac{125}{8000}$

b) SEM REPOSIÇÃO

$P(X=0) = \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{13}{18} = \boxed{\frac{2730}{6840}}$

$P(X=1) = \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{5}{18} + \frac{15}{20} \cdot \frac{5}{19} \cdot \frac{14}{18} + \frac{5}{20} \cdot \frac{15}{19} \cdot \frac{14}{18}$

$= 3\left(\frac{1050}{6.840}\right) = \boxed{\frac{3150}{6.840}}$

$P(X=2) = \frac{15}{20} \cdot \frac{5}{19} \cdot \frac{4}{18} + \frac{5}{20} \cdot \frac{15}{19} \cdot \frac{4}{18} + \frac{5}{20} \cdot \frac{4}{19} \cdot \frac{15}{18}$

$= 3 \cdot \left(\frac{300}{6.840}\right) = \boxed{\frac{900}{6.840}}$

$P(X=3) = \frac{5}{20} \cdot \frac{4}{19} \cdot \frac{3}{18} = \boxed{\frac{60}{6840}}$

(3)

$P(X=2) = \frac{2}{5} \cdot \frac{1}{4} + \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} + \frac{2}{20} = \boxed{\frac{4}{20}}$

$P(X=3) = \frac{2}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{2}{4} = \frac{4}{20} + \frac{4}{20} = \boxed{\frac{8}{20}}$

$P(X=4) = \frac{2}{5} \cdot \frac{1}{4} + \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} + \frac{2}{20} = \boxed{\frac{4}{20}}$

$P(X=5) = \frac{2}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{2}{4} = \frac{2}{20} + \frac{2}{20} = \boxed{\frac{4}{20}}$

(4)

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

$X = \max(a, b)$

$Y = a + b$

$P(Y=2) = \frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$

$P(Y=3) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} = \boxed{\frac{2}{36}}$

$P(Y=4) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{3}{36}}$

$P(Y=5) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{4}{36}}$

$P(Y=6) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \boxed{\frac{5}{36}}$

$P(Y=7) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \boxed{\frac{6}{36}}$

$P(Y=8) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \boxed{\frac{5}{36}}$

$P(Y=9) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{4}{36}}$

$P(Y=10) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{3}{36}}$

$P(Y=11) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} + \frac{1}{36} = \boxed{\frac{2}{36}}$

$P(Y=12) = \frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$

$P(X=1) = \frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$

$P(x=2) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{3}{36}}$

$P(x=3) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \boxed{\frac{5}{36}}$

$P(x=4) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{7}{36}}$

$P(x=5) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{9}{36}}$

$P(x=6) = \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \boxed{\frac{11}{36}}$

(5)

a) $2a + a + 4a = 1 \rightarrow 7a = 1 \rightarrow a = \frac{1}{7}$

b) $P(0 \leq x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= 0a + 2a + a + 4a = 7a \rightarrow 7 \cdot \frac{1}{7} = \boxed{\frac{7}{7}}$

$P(0 < x < 2) = P(x=1) = 2a \rightarrow 2 \cdot \frac{1}{7} = \boxed{\frac{2}{7}}$

(6)

X = BOLAS BRANCAS

Y = BOLAS VERDES

a) COM REPOSIÇÃO

$P(X=0) = \frac{9}{14}\cdot\frac{9}{14} = \boxed{\frac{81}{196}}$

$P(X=1) = \frac{5}{14}\cdot\frac{9}{14} + \frac{9}{14}\cdot\frac{5}{14} = \frac{45}{196} + \frac{45}{196} = \boxed{\frac{90}{196}}$

$P(X=2) = \frac{5}{14}\cdot\frac{5}{14} = \boxed{\frac{25}{196}}$

$P(Y=0) = \frac{11}{14}\cdot\frac{11}{14} = \boxed{\frac{121}{196}}$

$P(Y=1) = \frac{11}{14}\cdot\frac{3}{14} + \frac{3}{14}\cdot\frac{11}{14} = \frac{33}{196} + \frac{33}{196} = \boxed{\frac{66}{196}}$

$P(Y=2) = \frac{3}{14}\cdot\frac{3}{14} = \boxed{\frac{9}{196}}$

b) SEM REPOSIÇÃO

$P(X=0) = \frac{9}{14}\cdot\frac{8}{13} = \boxed{\frac{72}{182}}$

$P(X=1) = \frac{5}{14}\cdot\frac{9}{13} + \frac{9}{14}\cdot\frac{5}{13} = \frac{45}{182} + \frac{45}{182} = \boxed{\frac{90}{182}}$

$P(X=2) = \frac{5}{14}\cdot\frac{4}{13} = \boxed{\frac{20}{182}}$

$P(Y=0) = \frac{11}{14}\cdot\frac{10}{13} = \boxed{\frac{110}{182}}$

$P(Y=1) = \frac{3}{14}\cdot\frac{11}{13} = \frac{33}{182} + \frac{33}{182} = \boxed{\frac{66}{182}}$

$P(Y=2) = \frac{3}{11} \cdot \frac{2}{13} = \boxed{\frac{6}{132}}$

(7)

a) $P(x<5) = P(x=2) + P(x=3) + P(x=4)$

$\left(\frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}\right)$

$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \boxed{\frac{6}{36}}$

b) $P(4<x<8) = P(x=5) + P(x=6) + P(x=7)$

$\left(\frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6}\right) + \left(\frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6}\right) +$

$\left(\frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6}\frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}\right) = \frac{4}{36} + \frac{5}{36} + \frac{6}{36}$

$= \boxed{\frac{15}{36}}$

(8)

$x = N^{\circ}$ COROAS

$P(x=0) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{8}}$

$P(x=1) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \boxed{\frac{3}{8}}$

$P(x=2) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \boxed{\frac{3}{8}}$

$P(x=3) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{8}}$

(9) $p + 2p = 1 \rightarrow 3p = 1 \rightarrow p = 1/3$

$P(X>5) = P(X=10) = 2p \rightarrow 2 \cdot \frac{1}{3} = \frac{2}{3}$