

Lista Extra 4

Exercício de Variável Bidimensional Discreta Independente 30/09/2022

Considere a distribuição conjunta (X, Y) dada por:

Y	X			$\mathbb{P}(Y = y)$
	-1	0	1	
-1	$\frac{1}{12}$			
0				$\frac{1}{3}$
1	$\frac{1}{4}$		$\frac{1}{4}$	
$\mathbb{P}(X = x)$				1

a) Sabendo que X e Y são independentes, complete a tabela da distribuição de probabilidade.

Y	X			$\mathbb{P}(Y = y)$
	-1	0	1	
-1	$\frac{1}{12}$	x	t	c
0	u	y	v	$\frac{1}{3}$
1	$\frac{1}{4}$	z	$\frac{1}{4}$	d
$\mathbb{P}(X = x)$	a	b	a	1

$$\begin{aligned}\frac{1}{12} + u + \frac{1}{4} &= a \\ \frac{1}{12} + \frac{1}{3} * a + \frac{1}{4} &= a \\ \frac{2a}{3} &= \frac{4}{12} \\ a &= \frac{2}{4} = \frac{1}{2}\end{aligned}$$

$$u = \frac{1}{3} * a$$

$$u = \frac{1}{3} * \frac{1}{2} = \frac{1}{6}$$

$$v = \frac{1}{3} * a$$

$$v = \frac{1}{3} * \frac{1}{2} = \frac{1}{6}$$

$$t = c * a$$

$$t = \frac{1}{6} * \frac{1}{2} = \frac{1}{12}$$

$$a + b + a = 1$$

$$\frac{1}{2} + b + \frac{1}{2} = 1$$

$$b = 1 - \frac{2}{2} = 1 - 1 = 0$$

$$a * d = \frac{1}{4}$$

$$\frac{1}{2} * d = \frac{1}{4}$$

$$d = \frac{1}{4} * \frac{2}{1} = \frac{2}{4} = \frac{1}{2}$$

$$c + \frac{1}{3} + d = 1$$

$$c + \frac{1}{3} + \frac{1}{2} = 1$$

$$c = 1 - \frac{5}{6} = \frac{1}{6}$$

$$c = \frac{1}{12} + x + t$$

$$\frac{1}{6} = \frac{1}{12} + x + \frac{1}{12}$$

$$x = \frac{1}{6} - \frac{2}{12} = \frac{1}{6} - \frac{1}{6} = 0$$

$$\frac{1}{3} = u + y + v$$

$$\frac{1}{3} = \frac{1}{6} + y + \frac{1}{6}$$

$$y = \frac{1}{3} - \frac{2}{6} = \frac{1}{3} - \frac{1}{3} = 0$$

$$d = \frac{1}{4} + z + \frac{1}{4}$$

$$\frac{1}{2} = \frac{2}{4} + z$$

$$z = \frac{1}{2} - \frac{1}{2} = 0$$

Y	X			$\mathbb{P}(Y = y)$
	-1	0	1	
-1	$\frac{1}{12}$	0	$\frac{1}{12}$	$\frac{1}{6}$
0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{3}$
1	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$
$\mathbb{P}(X = x)$	$\frac{1}{2}$	0	$\frac{1}{2}$	1

b) Encontre $E(X \mid Y = 0)$

$$E(X \mid Y = 0) = -1 * \frac{1}{2} + 0 * 0 + 1 * \frac{1}{2} = -\frac{1}{2} + \frac{1}{2} = 0$$