Lista 6

Curso de Ciências atuariais
Disciplina Probabilidade 1 - Professora Cristina
08/08/2022 - Exercícios de variável aleatória discreta, F(X), E(X) e V(X)

1) Uma moeda apresenta cara 3 vezes mais frequente que coroa. Essa moeda é jogada 2 vezes. Encontre a distribuição de probabilidade de X=número de caras que aparece.
a) Encontre a distribuição de probabilidade

$$\mathbb{P}(X=k) = 3\mathbb{P}(X=c)$$

$$\begin{gathered}\mathbb{P}(X=k) + \mathbb{P}(X=c) = 1 \\ 3\mathbb{P}(X=c) + \mathbb{P}(X=c) = 1 \\ 4\mathbb{P}(X=c) = 1 \\ \mathbb{P}(X=c) = \frac{1}{4}\end{gathered}$$

$$S_x = \begin{cases} (c,c) = 0 \\ (c,k);(k,c) = 1 \\ (k,k) = 2 \end{cases}$$

$$\mathbb{P}(X=0) = \frac{1}{4} * \frac{1}{4} = \frac{1}{16}$$

$$\mathbb{P}(X=1) = \frac{1}{4} * \frac{3}{4} + \frac{3}{4} * \frac{1}{4} = \frac{3}{16} + \frac{3}{16} = \frac{6}{16}$$

$$\mathbb{P}(X=2) = \frac{9}{4} * \frac{9}{4} = \frac{9}{16}$$

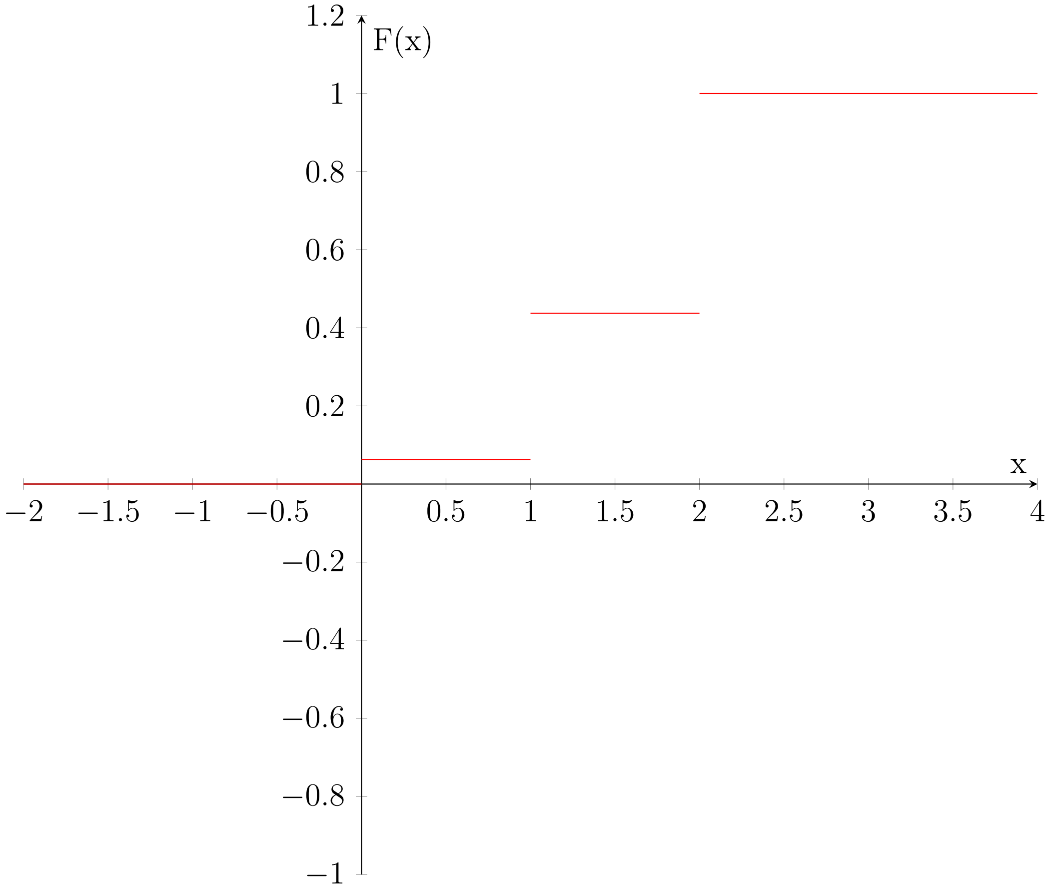
b) Encontre a função distribuição acumulada e faça sua representação gráfica;

$$\mathbb{P}(X \leq 0) = \mathbb{P}(X=0) = \frac{1}{16}$$

$$\mathbb{P}(X \leq 1) = \mathbb{P}(X \leq 0) + \mathbb{P}(X = 1) = \frac{1}{16} + \frac{6}{16} = \tfrac{7}{16}$$

$$\mathbb{P}(X \leq 2) = \mathbb{P}(X \leq 1) + \mathbb{P}(X = 2) = \frac{7}{16} + \frac{9}{16} = \frac{16}{16} = 1$$

$$F(x) = \begin{cases} 0, 0 < x \\ \frac{1}{16}, 0 \leq x < 1 \\ \frac{7}{16}, 1 \leq x < 2 \\ 1, x \geq 2 \end{cases}$$



c) Determine os valores de E(X) e V(X)

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 0 * \frac{1}{16} + 1 * \frac{6}{16} + 2 * \frac{9}{16} = 0 + \frac{6}{16} + \frac{18}{16} = \frac{24}{16}$$

$$E(X^2) = 0^2 * \frac{1}{16} + 1^2 * \frac{6}{16} + 2^2 * \frac{9}{16} = 0 + \frac{6}{16} + \frac{36}{16} = \frac{42}{16}$$

$$V(X) = E(x^2) - E^2(x)$$

$$V(X) = \frac{42}{16} - (\frac{24}{16})^2 = \frac{672}{256} - \frac{576}{256} = \frac{96}{256}$$

2) Um lote que contem 20 peças das quais 5 são defeituosas serão retiradas 3 peças. Encontre a distribuição de probabilidade de X=número de peças defeituosas encontradas, nos seguintes casos:

a) Retirada um a um com reposição

$$S_x = \begin{cases} (P,P,P) = 0 \\ (P,P,D); (P,D,P); (D,P,P) = 1 \\ (P,D,D); (D,P,D); (D,D,P) = 2 \\ (D,D,D) = 3 \end{cases}$$

$$\mathbb{P}(X = 0) = \frac{15}{20} * \frac{15}{20} * \frac{15}{20} = \frac{3.375}{8.000}$$

$$\mathbb{P}(X = 1) = \frac{15}{20} * \frac{15}{20} * \frac{5}{20} * 3 = \frac{1.125}{8000} * 3 = \frac{3.375}{8.000}$$

$$\mathbb{P}(X = 2) = \frac{15}{20} * \frac{5}{20} * \frac{5}{20} * 3 = \frac{375}{8.000} * 3 = \frac{1.125}{8.000}$$

$$\mathbb{P}(X = 3) = \frac{5}{20} * \frac{5}{20} * \frac{5}{20} = \frac{125}{8.000}$$

b) Retirada um a um sem reposição

$$S_x = \begin{cases} (P,P,P) = 0 \\ (P,P,D); (P,D,P); (D,P,P) = 1 \\ (P,D,D); (D,P,D); (D,D,P) = 2 \\ (D,D,D) = 3 \end{cases}$$

$$\mathbb{P}(X = 0) = \frac{15}{20} * \frac{14}{19} * \frac{13}{18} = \frac{2.730}{6.840}$$

$$\mathbb{P}(X = 1) = \frac{15}{20} * \frac{14}{19} * \frac{5}{18} * 3 = \frac{1.050}{6.840} * 3 = \frac{3.150}{6.840}$$

$$\mathbb{P}(X = 2) = \frac{15}{20} * \frac{5}{19} * \frac{4}{18} * 3 = \frac{300}{6.840} * 3 = \frac{900}{6.840}$$

$$\mathbb{P}(X = 3) = \frac{5}{20} * \frac{4}{19} * \frac{3}{18} = \frac{60}{6.840}$$

Em cada caso:
c) Encontre a função distribuição acumulada e faça sua representação gráfica

COM REPOSIÇÃO

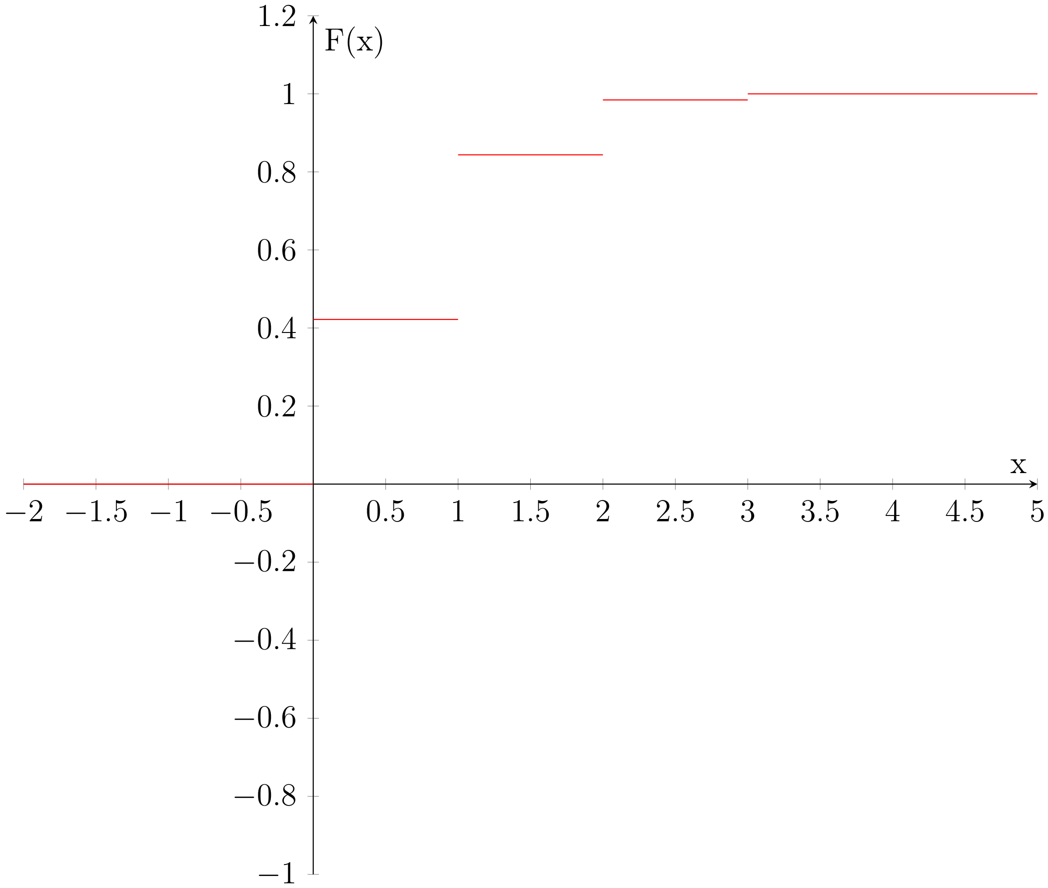
$$\mathbb{P}(X \leq 0) = \mathbb{P}(X = 0) = \frac{3.375}{8.000}$$

$$\mathbb{P}(X \leq 1) = \mathbb{P}(X \leq 0) + \mathbb{P}(X = 1) = \frac{3.375}{8.000} + \frac{3.375}{8.000} = \frac{6.750}{8.000}$$

$$\mathbb{P}(X \leq 2) = \mathbb{P}(X \leq 1) + \mathbb{P}(X = 2) = \frac{6.750}{8.000} + \frac{1.125}{8.000} = \frac{7.875}{8.000}$$

$$\mathbb{P}(X \leq 3) = \mathbb{P}(X \leq 2) + \mathbb{P}(X = 3) = \frac{7.875}{8.000} + \frac{125}{8.000} = \frac{8.000}{8.000} = 1$$

$$F(x) = \begin{cases} 0, 0 < x \\ \frac{3.375}{8.000}, 0 \leq x < 1 \\ \frac{6.750}{8.000}, 1 \leq x < 2 \\ \frac{7.875}{8.000}, 2 \leq x < 3 \\ 1, x \geq 3 \end{cases}$$



SEM REPOSIÇÃO

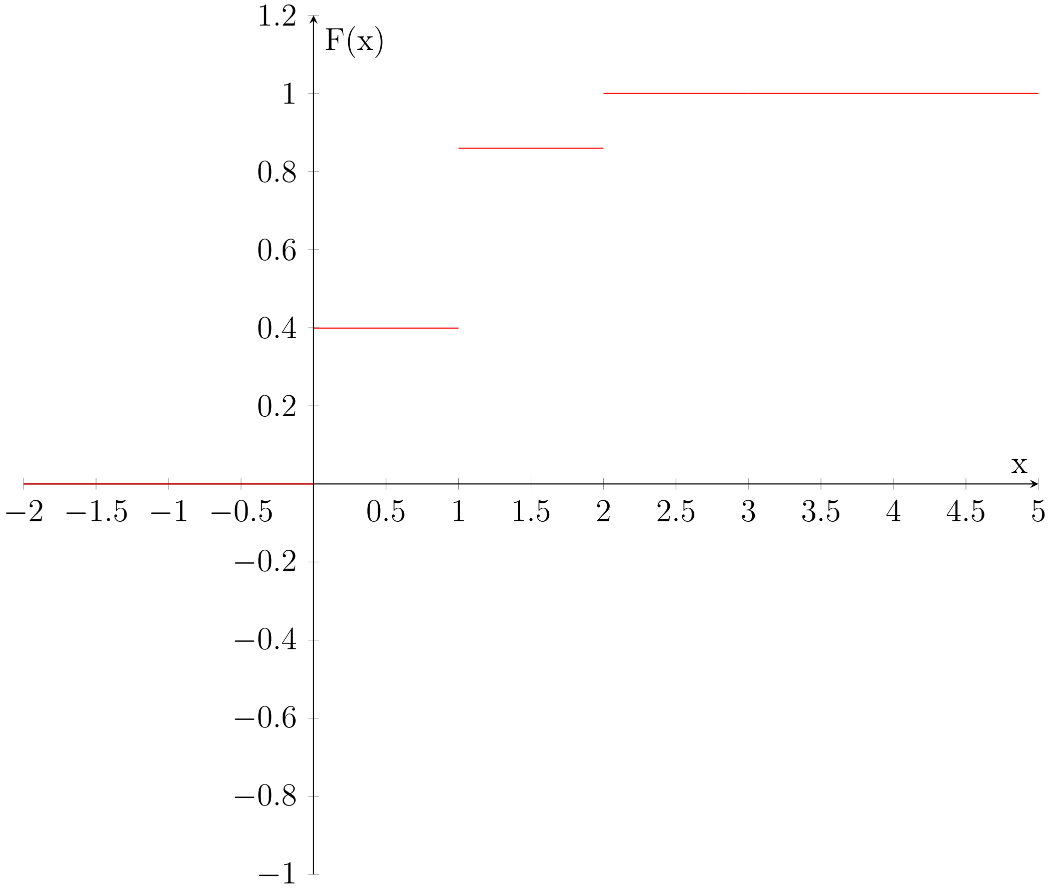
$$\mathbb{P}(X \leq 0) = \mathbb{P}(X = 0) = \frac{2.730}{6.840}$$

$$\mathbb{P}(X \leq 1) = \mathbb{P}(X \leq 0) + \mathbb{P}(X = 1) = \frac{2.730}{6.840} + \frac{3.150}{6.840} = \frac{5.880}{6.840}$$

$$\mathbb{P}(X \leq 2) = \mathbb{P}(X \leq 1) + \mathbb{P}(X = 2) = \frac{5.880}{6.840} + \frac{900}{6.840} = \frac{6.780}{6.840}$$

$$\mathbb{P}(X \leq 3) = \mathbb{P}(X \leq 2) + \mathbb{P}(X = 3) = \frac{6.780}{6.840} + \frac{60}{6.840} = \frac{6.840}{6.840} = 1$$

$$F(x) = \begin{cases} 0, 0 < x \\ \frac{2.730}{6.840}, 0 \leq x < 1 \\ \frac{5.880}{6.840}, 1 \leq x < 2 \\ \frac{6.780}{6.840}, 2 \leq x < 3 \\ 1, x \geq 3 \end{cases}$$



d) Determine os valores de E(X) e V(X)

COM REPOSIÇÃO

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 0*\frac{3.375}{8.000}+1*\frac{3.375}{8.000}+2*\frac{1.125}{8.000}+3*\frac{125}{8.000} = 0+\frac{3.375}{8.000}+\frac{2.250}{8.000}+\frac{375}{8.000} = \frac{6.000}{8.000} = \frac{3}{4}$$

$$E(X^2) = 0^2 * \frac{3.375}{8.000} + 1^2 * \frac{3.375}{8.000} + 2^2 * \frac{1.125}{8.000} + 3^2 * \frac{125}{8.000} = 0 + \frac{3.375}{8.000} + \frac{4.500}{8.000} + \frac{1.125}{8.000}$$

$$E(X^2) = \frac{9.000}{8.000} = \frac{9}{8}$$

$$V(X) = E(X^2) - E^2(X)$$

$$V(X) = \frac{9}{8} - \left(\frac{3}{4}\right)^2 = \frac{18}{16} - \tfrac{9}{16} = \frac{9}{16}$$

SEM REPOSIÇÃO

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 0*\frac{2.730}{6.840}+1*\frac{3.150}{6.840}+2*\frac{900}{6.840}+3*\frac{60}{6.840} = 0+\frac{3.150}{6.840}+\frac{1.800}{6.840}+\frac{180}{6.840} = \frac{5.130}{6.840} = \frac{57}{76}$$

$$E(X^2) = 0^2 * \frac{2.730}{6.840} + 1^2 * \frac{3.150}{6.840} + 2^2 * \frac{900}{6.840} + 3^2 * \frac{60}{6.840} = 0 + \frac{3.150}{6.840} + \frac{3.600}{6.840} + \frac{540}{6.840}$$

$$E(X^2) = \frac{7.290}{6.840} = \frac{81}{76}$$

$$V(X) = E(X^2) - E^2(X)$$

$$V(X) = \frac{81}{76} - (\frac{57}{76})^2 = \frac{6.156}{5.776} - \frac{3.249}{5.776} = \frac{2.907}{50.776}$$

3) Duas cartas são selecionadas aleatoriamente, sem reposição, de uma caixa que contem 5 cartas numeradas: 1, 1, 2, 2, 3. Seja X=soma das duas cartas selecionadas. Encontre a distribuição de probabilidade de X.

a) Encontre a distribuição de probabilidade

$$S_x = \begin{cases} (1,1) = 2 \\ (1,2); (2,1) = 3 \\ (1,3); (3,1); (2,2) = 4 \\ (2,3); (3,2) = 5 \end{cases}$$

$$\mathbb{P}(X = 2) = \frac{2}{5} * \frac{1}{4} = \frac{2}{20}$$

$$\mathbb{P}(X = 3) = \frac{2}{5} * \frac{2}{4} + \frac{2}{5} * \frac{2}{4} = \frac{4}{20} + \frac{4}{20} = \frac{8}{20}$$

$$\mathbb{P}(X = 4) = \frac{2}{5} * \frac{1}{4} + \frac{1}{5} * \frac{2}{4} + \frac{2}{5} * \frac{1}{4} = \frac{2}{20} + \frac{2}{20} + \frac{2}{20} = \frac{6}{20}$$

$$\mathbb{P}(X = 5) = \frac{2}{5} * \frac{1}{4} + \frac{1}{5} * \frac{2}{4} = \frac{2}{20} + \frac{2}{20} = \frac{4}{20}$$

b) Encontre a função distribuição acumulada e faça sua representação gráfica

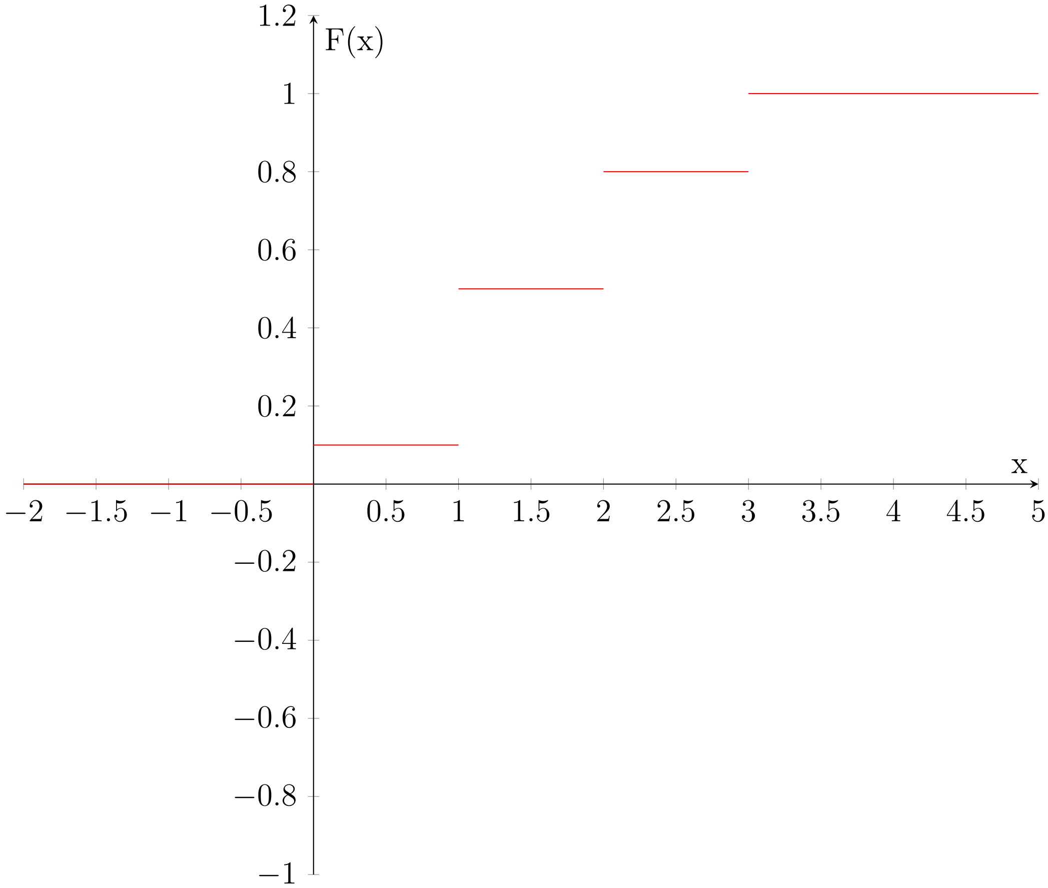
$$\mathbb{P}(X \leq 0) = \mathbb{P}(X = 0) = \frac{2}{20}$$

$$\mathbb{P}(X \leq 1) = \mathbb{P}(X \leq 0) + \mathbb{P}(X = 1) = \frac{2}{20} + \frac{8}{20} = \frac{10}{20}$$

$$\mathbb{P}(X \leq 2) = \mathbb{P}(X \leq 1) + \mathbb{P}(X = 2) = \frac{10}{20} + \frac{6}{20} = \frac{16}{20}$$

$$\mathbb{P}(X \leq 3) = \mathbb{P}(X \leq 2) + \mathbb{P}(X = 3) = \frac{16}{20} + \frac{4}{20} = \frac{20}{20} = 1$$

$$F(x) = \begin{cases} 0, 0 < x \\ \frac{2}{20}, 0 \leq x < 1 \\ \frac{10}{20}, 1 \leq x < 2 \\ \frac{16}{20}, 2 \leq x < 3 \\ 1, x \geq 3 \end{cases}$$



c) Determine os valores de E(X) e V(X)

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 2 * \frac{2}{20} + 3 * \frac{8}{20} + 4 * \frac{6}{20} + 5 * \frac{4}{20} = \frac{4}{20} + \frac{24}{20} + \frac{24}{20} + \frac{20}{20} = \frac{72}{20} = \frac{18}{5}$$

$$E(X^2) = 2^2 * \frac{2}{20} + 3^2 * \frac{8}{20} + 4^2 * \frac{6}{20} + 5^2 * \frac{4}{20} = \frac{8}{20} + \frac{72}{20} + \frac{96}{20} + \frac{100}{20} = \frac{276}{20} = \frac{69}{5}$$

$$V(X) = E(X^2) - E^2(X)$$

$$V(X) = \frac{69}{5} - (\frac{18}{5})^2 = \frac{345}{25} - \frac{324}{25} = \frac{21}{25}$$

d) Determine $\mathbb{P}(X \geq 2)$ e $\mathbb{P}(X > 1)$

$$\mathbb{P}(X \geq 2) = \mathbb{P}(X = 2) + \mathbb{P}(X = 3) + \mathbb{P}(X = 4) + \mathbb{P}(X = 5)$$
$$\mathbb{P}(X \geq 2) = \frac{2}{20} + \frac{8}{20} + \frac{6}{20} + \frac{4}{20} = 1$$

$$\mathbb{P}(X > 1) = \mathbb{P}(X \geq 2) = 1$$

4) Um dado é lançado duas vezes. Seja X= max(a,b) e Y=a+b; onde a é o resultado do primeiro lançamento e b é o resultado do segundo lançamento. Encontre as distribuições de probabilidade de X e de Y.

Em cada caso:

a) Encontre a distribuição de probabilidade

$$S_x = \begin{cases} (1,1) = 1 \\ (1,2);(2,1);(2,2) = 2 \\ (1,3);(3,1);(2,3);(3,2);(3,3) = 3 \\ (1,4);(4,1);(2,4);(4,2);(3,4);(4,3);(4,4) = 4 \\ (1,5);(5,1);(2,5);(5,2);(3,5);(5,3);(4,5);(5,4);(5,5) = 5 \\ (1,6);(6,1);(2,6);(6;2);(3,6);(6,3);(4;6);(6,4);(5;6);(6;5);(6;6) = 6 \end{cases}$$

$$\mathbb{P}(X = 1) = \frac{1}{36}$$

$$\mathbb{P}(X = 2) = 3 * \frac{1}{36} = \frac{3}{36}$$

$$\mathbb{P}(X = 3) = 5 * \frac{1}{36} = \frac{5}{36}$$

$$\mathbb{P}(X = 4) = 7 * \frac{1}{36} = \frac{7}{36}$$

$$\mathbb{P}(X = 5) = 9 * \frac{1}{36} = \frac{9}{36}$$

$$\mathbb{P}(X = 6) = 11 * \frac{1}{36} = \frac{11}{36}$$

$$S_y = \begin{cases} (1,1) = 2 \\ (1,2);(2,1) = 3 \\ (1,3);(3,1);(2,2) = 4 \\ (1,4);(4,1);(2,3);(3,2) = 5 \\ (1,5);(5,1);(2,4);(4,2);(3,3) = 6 \\ (1,6);(6,1);(2,5);(5;2);(3,4);(4,3) = 7 \\ (2,6);(6,2);(3,5);(5,3);(4,4) = 8 \\ (3,6);(6,3);(4,5);(5,4) = 9 \\ (4,6);(6,4);(5,5) = 10 \\ (5,6);(6,5) = 11 \\ (6,6) = 12 \end{cases}$$

$$\mathbb{P}(Y = 2) = \frac{1}{36}$$

$$\mathbb{P}(Y = 3) = 2 * \frac{1}{36} = \frac{2}{36}$$

$$\mathbb{P}(Y = 4) = 3 * \frac{1}{36} = \frac{3}{36}$$

$$\mathbb{P}(Y = 5) = 4 * \frac{1}{36} = \frac{4}{36}$$

$$\mathbb{P}(Y = 6) = 5 * \frac{1}{36} = \frac{5}{36}$$

$$\mathbb{P}(Y = 7) = 6 * \frac{1}{36} = \frac{6}{36}$$

$$\mathbb{P}(Y = 8) = 5 * \frac{1}{36} = \frac{5}{36}$$

$$\mathbb{P}(Y = 9) = 4 * \frac{1}{36} = \frac{4}{36}$$

$$\mathbb{P}(Y = 10) = 3 * \frac{1}{36} = \frac{3}{36}$$

$$\mathbb{P}(Y = 11) = 2 * \frac{1}{36} = \frac{2}{36}$$

$$\mathbb{P}(Y = 12) = \frac{1}{36}$$

b) Encontre a função distribuição acumulada e faça sua representação gráfica

$$\mathbb{P}(X \leq 1) = \mathbb{P}(X = 1) = \frac{1}{36}$$

$$\mathbb{P}(X \leq 2) = \mathbb{P}(X \leq 1) + \mathbb{P}(X = 2) = \frac{1}{36} + \frac{3}{36} = \frac{4}{36}$$

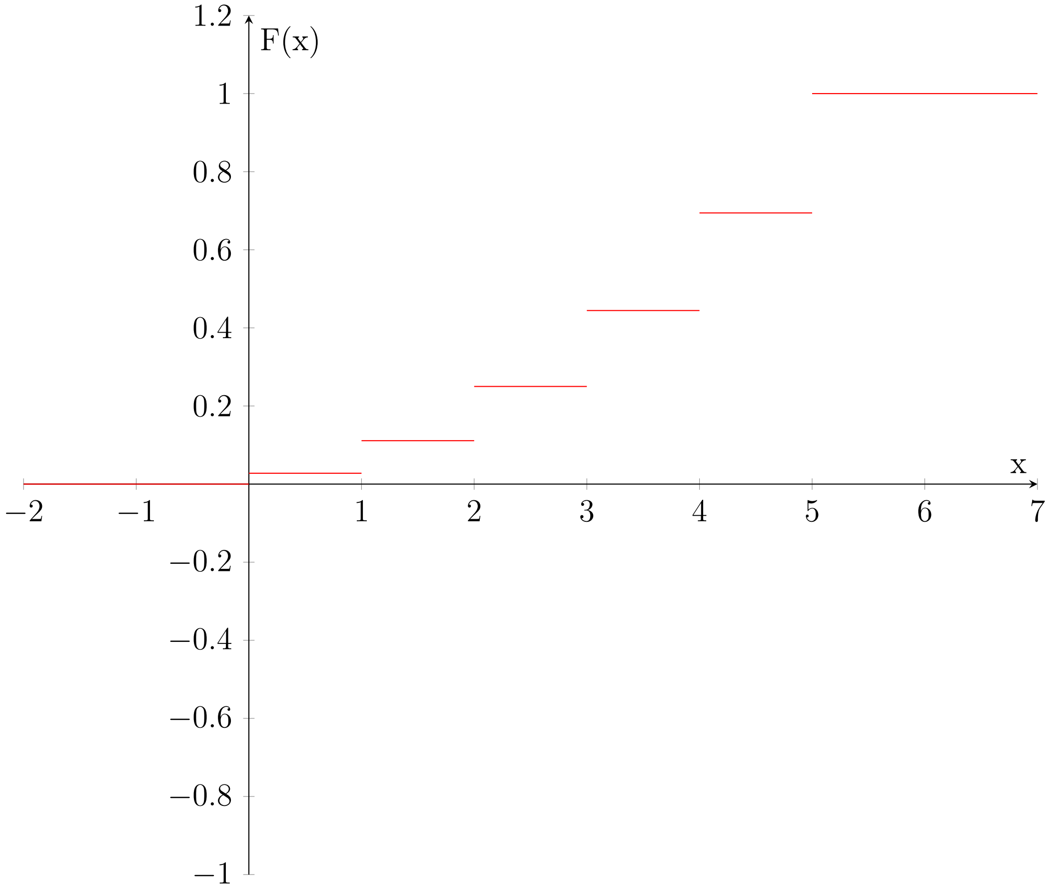
$$\mathbb{P}(X \leq 3) = \mathbb{P}(X \leq 2) + \mathbb{P}(X = 3) = \frac{4}{36} + \frac{5}{36} = \frac{9}{36}$$

$$\mathbb{P}(X \leq 4) = \mathbb{P}(X \leq 3) + \mathbb{P}(X = 4) = \frac{9}{36} + \frac{7}{36} = \frac{16}{36}$$

$$\mathbb{P}(X \leq 5) = \mathbb{P}(X \leq 4) + \mathbb{P}(X = 5) = \frac{16}{36} + \frac{9}{36} = \frac{25}{36}$$

$$\mathbb{P}(X \leq 6) = \mathbb{P}(X \leq 5) + \mathbb{P}(X = 6) = \frac{25}{36} + \frac{11}{36} = \frac{36}{36} = 1$$

$$F(x) = \begin{cases} 0, 0 < x \\ \frac{1}{36}, 0 \leq x < 1 \\ \frac{4}{36}, 1 \leq x < 2 \\ \frac{9}{36}, 2 \leq x < 3 \\ \frac{16}{36}, 3 \leq x < 4 \\ \frac{25}{36}, 4 \leq x < 5 \\ 1, x \geq 6 \end{cases}$$



$$\mathbb{P}(Y \leq 2) = \mathbb{P}(Y = 1) = \frac{1}{36}$$

$$\mathbb{P}(Y \leq 3) = \mathbb{P}(Y \leq 2) + \mathbb{P}(X = 3) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$$

$$\mathbb{P}(Y \leq 4) = \mathbb{P}(Y \leq 3) + \mathbb{P}(Y = 4) = \frac{3}{36} + \frac{3}{36} = \frac{6}{36}$$

$$\mathbb{P}(Y \leq 5) = \mathbb{P}(Y \leq 4) + \mathbb{P}(Y = 5) = \frac{6}{36} + \frac{4}{36} = \frac{10}{36}$$

$$\mathbb{P}(Y \leq 6) = \mathbb{P}(Y \leq 5) + \mathbb{P}(Y = 6) = \frac{10}{36} + \frac{5}{36} = \frac{15}{36}$$

$$\mathbb{P}(Y \leq 7) = \mathbb{P}(Y \leq 6) + \mathbb{P}(Y = 7) = \frac{15}{36} + \frac{6}{36} = \frac{21}{36}$$

$$\mathbb{P}(Y \leq 8) = \mathbb{P}(Y \leq 7) + \mathbb{P}(Y = 8) = \frac{21}{36} + \frac{5}{36} = \frac{26}{36}$$

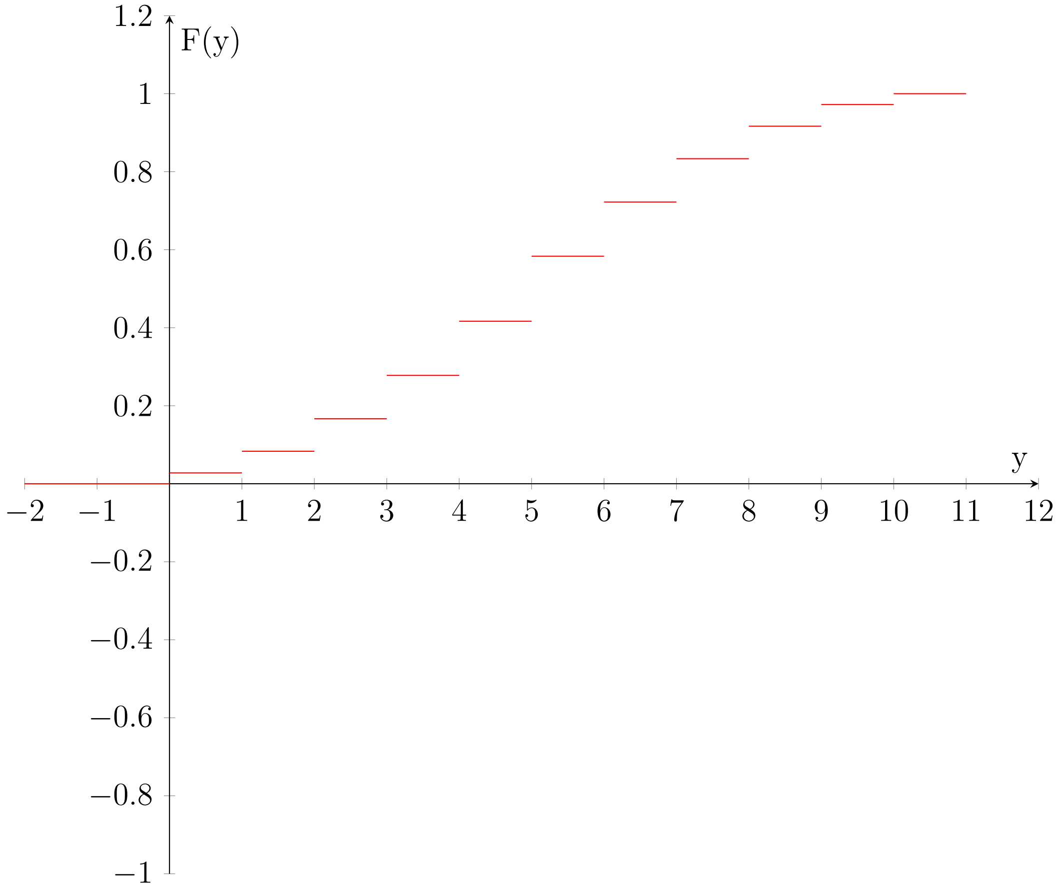
$$\mathbb{P}(Y \leq 9) = \mathbb{P}(Y \leq 8) + \mathbb{P}(Y = 9) = \frac{26}{36} + \frac{4}{36} = \frac{30}{36}$$

$$\mathbb{P}(Y \leq 10) = \mathbb{P}(Y \leq 9) + \mathbb{P}(Y = 10) = \frac{30}{36} + \frac{3}{36} = \frac{33}{36}$$

$$\mathbb{P}(Y \leq 11) = \mathbb{P}(Y \leq 10) + \mathbb{P}(Y = 11) = \frac{33}{36} + \frac{2}{36} = \frac{35}{36}$$

$$\mathbb{P}(Y \leq 12) = \mathbb{P}(Y \leq 11) + \mathbb{P}(Y = 12) = \frac{35}{36} + \frac{1}{36} = \frac{36}{36} = 1$$

$$F(y) = \begin{cases} 0, 1 < y \\ \frac{1}{36}, 1 \leq y < 2 \\ \frac{3}{36}, 2 \leq y < 3 \\ \frac{6}{36}, 3 \leq y < 4 \\ \frac{10}{36}, 4 \leq y < 5 \\ \frac{15}{36}, 5 \leq y < 6 \\ \frac{21}{36}, 6 \leq y < 7 \\ \frac{26}{36}, 7 \leq y < 8 \\ \frac{30}{36}, 8 \leq y < 9 \\ \frac{33}{36}, 9 \leq y < 10 \\ \frac{35}{36}, 10 \leq y < 11 \\ 1, x \geq 11 \end{cases}$$



c) Determine os valores de E(X) e V(X)

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 1 * \frac{1}{36} + 2 * \frac{3}{20} + 3 * \frac{5}{20} + 4 * \frac{7}{20} + 5 * \frac{9}{36} + 6 * \frac{11}{36}$$
$$E(X) = \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36} = \frac{161}{36}$$

$$E(X^2) = 1^2 * \frac{1}{36} + 2^2 * \frac{3}{36} + 3^2 * \frac{5}{36} + 4^2 * \frac{7}{36} + 5^2 * \frac{9}{36} + 6^2 * \frac{11}{36}$$
$$E(X^2) = \frac{1}{36} + \frac{12}{36} + \frac{45}{36} + \frac{112}{36} + \frac{225}{36} + \frac{396}{36} = \frac{791}{36}$$

$$V(X) = E(X^2) - E^2(X)$$

$$V(X) = \frac{791}{36} - \left(\frac{161}{36}\right)^2 = \frac{28.476}{1.296} - \frac{25.921}{1.296} = \frac{2.555}{1.296}$$

$$E(Y) = \sum_i^n y_i * \mathbb{P}(Y = y_i)$$

$$E(Y) = 2*\frac{1}{36}+3*\frac{2}{20}+4*\frac{3}{20}+5*\frac{4}{20}+6*\frac{5}{36}+7*\frac{6}{36}+8*\frac{5}{36}+9*\frac{4}{36}+10*\frac{3}{36}+11*\frac{2}{36}+12*\frac{1}{36}$$
$$E(Y) = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36} = \frac{251}{36}$$

$$E(Y^2) =$$
$$2^2*\frac{1}{36}+3^2*\frac{2}{36}+4^2*\frac{3}{36}+5^2*\frac{4}{36}+6^2*\frac{5}{36}+7^2*\frac{6}{36}+8^2*\frac{5}{36}+9^2*\frac{4}{36}+10^2*\frac{3}{36}+11^2*\frac{2}{36}+12^2*\frac{1}{36}$$
$$E(X^2) = \frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{295}{36} + \frac{320}{36} + \frac{324}{320} + \frac{242}{36} + \frac{144}{36} = \frac{1974}{36}$$

$$V(Y) = E(Y^2) - E^2(Y)$$

$$V(X) = \frac{1974}{36} - \left(\frac{251}{36}\right)^2 = \frac{71.064}{1.296} - \frac{63.001}{1.296} = \frac{8.023}{1.296}$$

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5) A distribuição de probabilidade de uma variável aleatória discreta é dada por:

x	1	2	3
P(X=x)	2a	a	4a

a) Determine o valor de a

$$\mathbb{P}(X=1)+\mathbb{P}(X=2)+\mathbb{P}(X=3)=1$$
$$2a+a+4a=1$$
$$7a=1$$
$$a=\frac{1}{7}$$

b) Calcule as seguintes probabilidades: $\mathbb{P}(0 \leq x \leq 3)$ e $\mathbb{P}(0 < x < 2)$

$$\mathbb{P}(0 \leq x \leq 3)=\mathbb{P}(X=1)+\mathbb{P}(X=2)+\mathbb{P}(X=3)$$
$$\mathbb{P}(0 \leq x \leq 3)=1$$

$$\mathbb{P}(0 < x < 2)=\mathbb{P}(X=1)=2*\frac{1}{7}=\frac{2}{7}$$

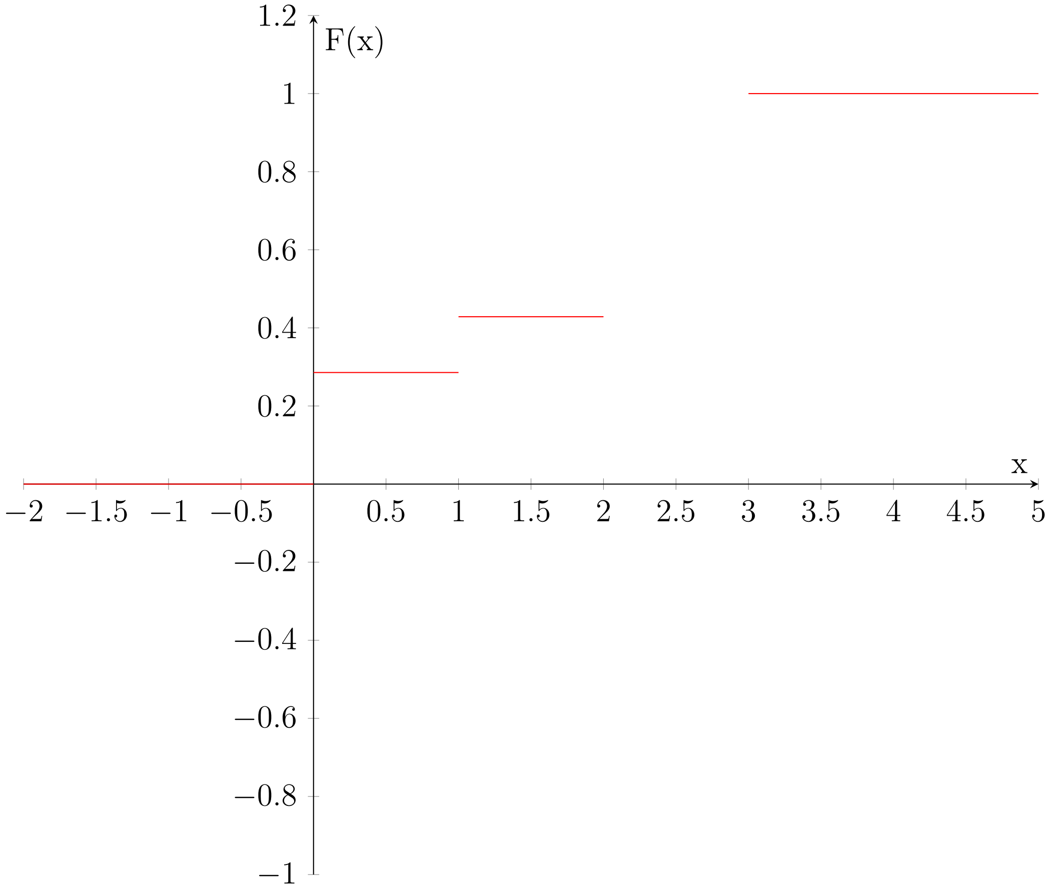
c) Encontre a função distribuição acumulada e faça sua representação gráfica

$$\mathbb{P}(X \leq 1)=\mathbb{P}(X=1)=\frac{2}{7}$$

$$\mathbb{P}(X \leq 2)=\mathbb{P}(X \leq 1)+\mathbb{P}(X=2)=\frac{2}{7}+\frac{1}{7}=\frac{3}{7}$$

$$\mathbb{P}(X \leq 3)=\mathbb{P}(X \leq 2)+\mathbb{P}(X=3)=\frac{3}{7}+\frac{4}{7}=1$$

$$F(x)=\begin{cases} 0, 1 < x \\ \frac{2}{7}, 1 \leq x < 2 \\ \frac{3}{7}, 2 \leq x < 3 \\ 1, x \geq 3 \end{cases}$$



d) Determine os valores de E(X) e V(X)

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 1 * \frac{2}{7} + 2 * \frac{1}{7} + 3 * \frac{4}{7} = \frac{2}{7} + \frac{2}{7} + \frac{12}{7} = \frac{16}{7}$$

$$E(X^2) = 1^2 * \frac{2}{7} + 2^2 * \frac{1}{7} + 3^2 * \frac{4}{7} = \frac{2}{7} + \frac{4}{7} + \frac{36}{7} = \frac{42}{7}$$

$$V(X) = E(x^2) - E^2(x)$$

$$V(X) = \frac{42}{7} - \left(\frac{16}{7}\right)^2 = \frac{294}{49} - \frac{256}{49} = \frac{38}{49}$$

e) Definindo-se Y=3X, determine E(Y) e V(Y)

$$E(Y) = E(3X) = 3 * \frac{16}{7} = \frac{48}{7}$$

$$V(Y) = V(3X) = 3^2 * V(X) = 9 * \frac{38}{49} = \frac{2390}{49}$$

6) De um lote com 5 bolas brancas, 3 verdes e 6 azuis serão retiradas 2 bolas. Encontre a distribuição de probabilidade da variável aleatória X=número de bolas brancas retiradas e de Y=número de bolas verdes retiradas, nos seguintes casos:

a) Retirada um a um com reposição

$$S_x = \begin{cases} (B^c, B^c) = 0 \\ (B^c, B); (B, B^c) = 1 \\ (B, B) = 2 \end{cases}$$

$$\mathbb{P}(X = 0) = \frac{9}{14} * \frac{9}{14} = \frac{81}{196}$$

$$\mathbb{P}(X = 1) = \frac{9}{14} * \frac{5}{14} + \frac{5}{14} * \frac{9}{14} = \frac{45}{196} + \frac{45}{196} = \frac{90}{196}$$

$$\mathbb{P}(X = 2) = \frac{5}{14} * \frac{5}{14} = \frac{25}{196}$$

$$S_y = \begin{cases} (V^c, V^c) = 0 \\ (V^c, V); (V, V^c) = 1 \\ (V, V) = 2 \end{cases}$$

$$\mathbb{P}(X = 0) = \frac{11}{14} * \frac{11}{14} = \frac{121}{196}$$

$$\mathbb{P}(X = 1) = \frac{11}{14} * \frac{3}{14} + \frac{3}{14} * \frac{11}{14} = \frac{33}{196} + \frac{3}{196} = \frac{66}{196}$$

$$\mathbb{P}(X = 2) = \frac{3}{14} * \frac{3}{14} = \frac{9}{196}$$

b) Retirada um a um sem reposição

$$S_x = \begin{cases} (B^c, B^c) = 0 \\ (B^c, B); (B, B^c) = 1 \\ (B, B) = 2 \end{cases}$$

$$\mathbb{P}(X=0) = \frac{9}{14} * \frac{8}{13} = \frac{72}{182}$$

$$\mathbb{P}(X=1) = \frac{9}{14} * \frac{5}{13} + \frac{5}{14} * \frac{9}{13} = \frac{45}{182} + \frac{45}{182} = \frac{90}{182}$$

$$\mathbb{P}(X=2) = \frac{5}{14} * \frac{4}{13} = \frac{20}{182}$$

$$S_y = \begin{cases} (V^c, V^c) = 0 \\ (V^c, V); (V, V^c) = 1 \\ (V, V) = 2 \end{cases}$$

$$\mathbb{P}(X=0) = \frac{11}{14} * \frac{10}{13} = \frac{110}{182}$$

$$\mathbb{P}(X=1) = \frac{11}{14} * \frac{3}{13} + \frac{3}{14} * \frac{11}{13} = \frac{33}{182} + \frac{3}{182} = \frac{66}{182}$$

$$\mathbb{P}(X=2) = \frac{3}{14} * \frac{2}{13} = \frac{6}{182}$$

Em cada caso:
c) Encontre a função distribuição acumulada e faça sua representação gráfica

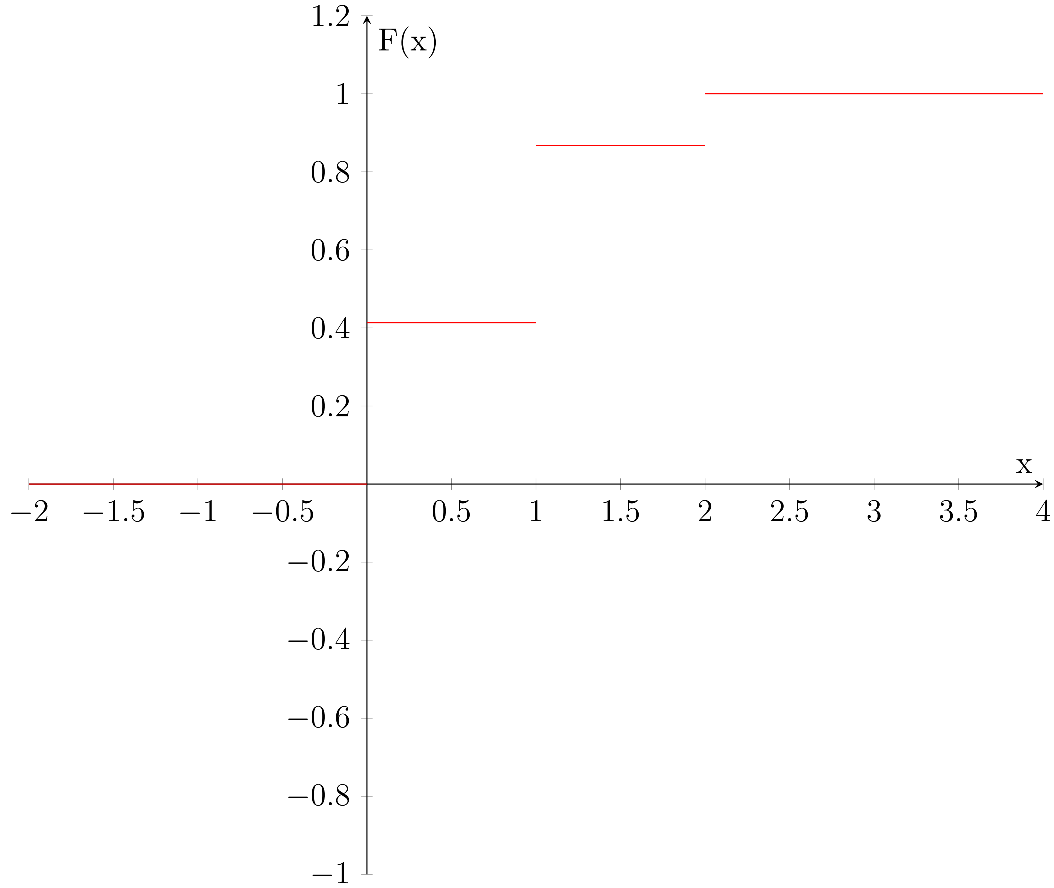
COM REPOSIÇÃO

$$\mathbb{P}(X \leq 0) = \mathbb{P}(X = 0) = \frac{81}{196}$$

$$\mathbb{P}(X \leq 1) = \mathbb{P}(X \leq 0) + \mathbb{P}(X = 1) = \frac{81}{196} + \frac{90}{196} = \frac{171}{196}$$

$$\mathbb{P}(X \leq 2) = \mathbb{P}(X \leq 1) + \mathbb{P}(X = 2) = \frac{171}{196} + \frac{25}{196} = 1$$

$$F(y) = \begin{cases} 0, 0 < x \\ \frac{81}{196}, 0 \leq x < 1 \\ \frac{171}{196}, 1 \leq x < 2 \\ 1, x \geq 2 \end{cases}$$

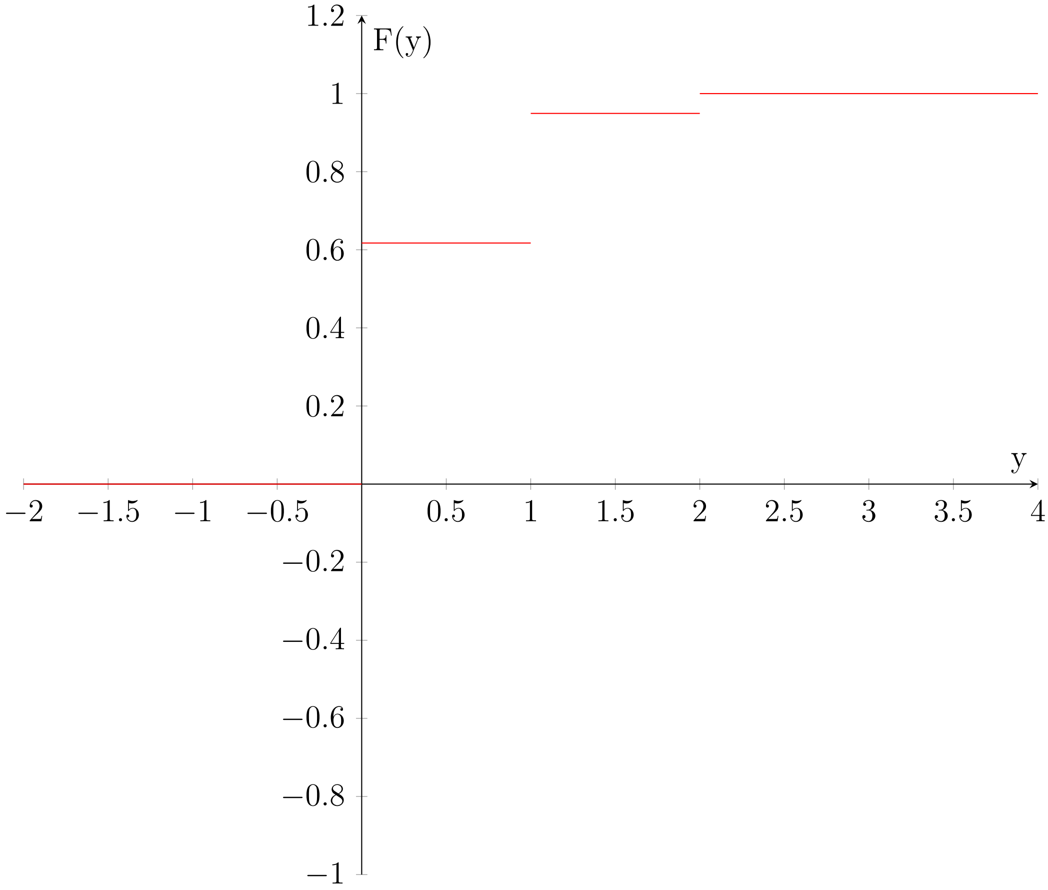


$$\mathbb{P}(Y \leq 0) = \mathbb{P}(Y = 0) = \frac{121}{196}$$

$$\mathbb{P}(Y \leq 1) = \mathbb{P}(Y \leq 0) + \mathbb{P}(Y = 1) = \frac{121}{196} + \frac{66}{196} = \frac{187}{196}$$

$$\mathbb{P}(Y \leq 2) = \mathbb{P}(Y \leq 1) + \mathbb{P}(Y = 2) = \frac{187}{196} + \frac{9}{196} = 1$$

$$F(y) = \begin{cases} 0, 0 < y \\ \frac{121}{196}, 0 \leq y < 1 \\ \frac{187}{196}, 1 \leq y < 2 \\ 1, y \geq 2 \end{cases}$$



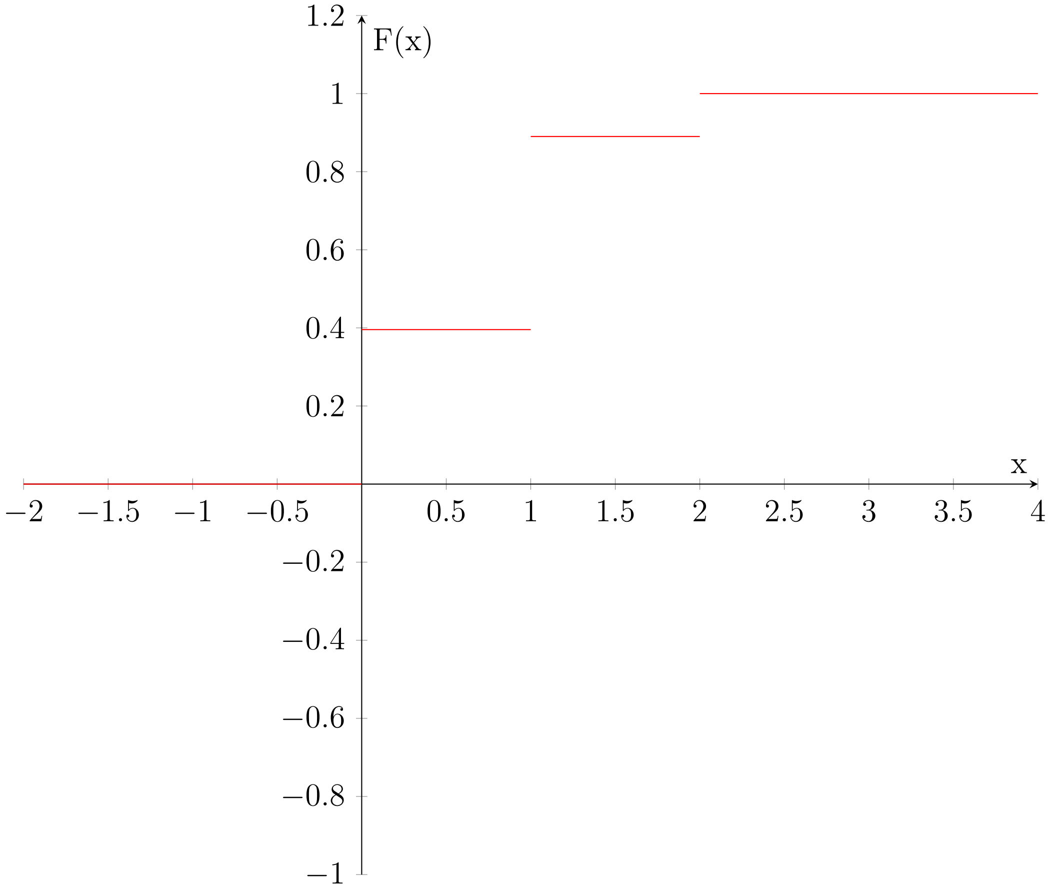
SEM REPOSIÇÃO

$$\mathbb{P}(X \leq 0) = \mathbb{P}(X = 0) = \frac{72}{182}$$

$$\mathbb{P}(X \leq 1) = \mathbb{P}(X \leq 0) + \mathbb{P}(X = 1) = \frac{72}{182} + \frac{90}{182} = \frac{162}{182}$$

$$\mathbb{P}(X \leq 2) = \mathbb{P}(X \leq 1) + \mathbb{P}(X = 2) = \frac{162}{182} + \frac{20}{182} = 1$$

$$F(x) = \begin{cases} 0, 0 < x \\ \frac{72}{182}, 0 \leq x < 1 \\ \frac{162}{182}, 1 \leq x < 2 \\ 1, x \geq 2 \end{cases}$$

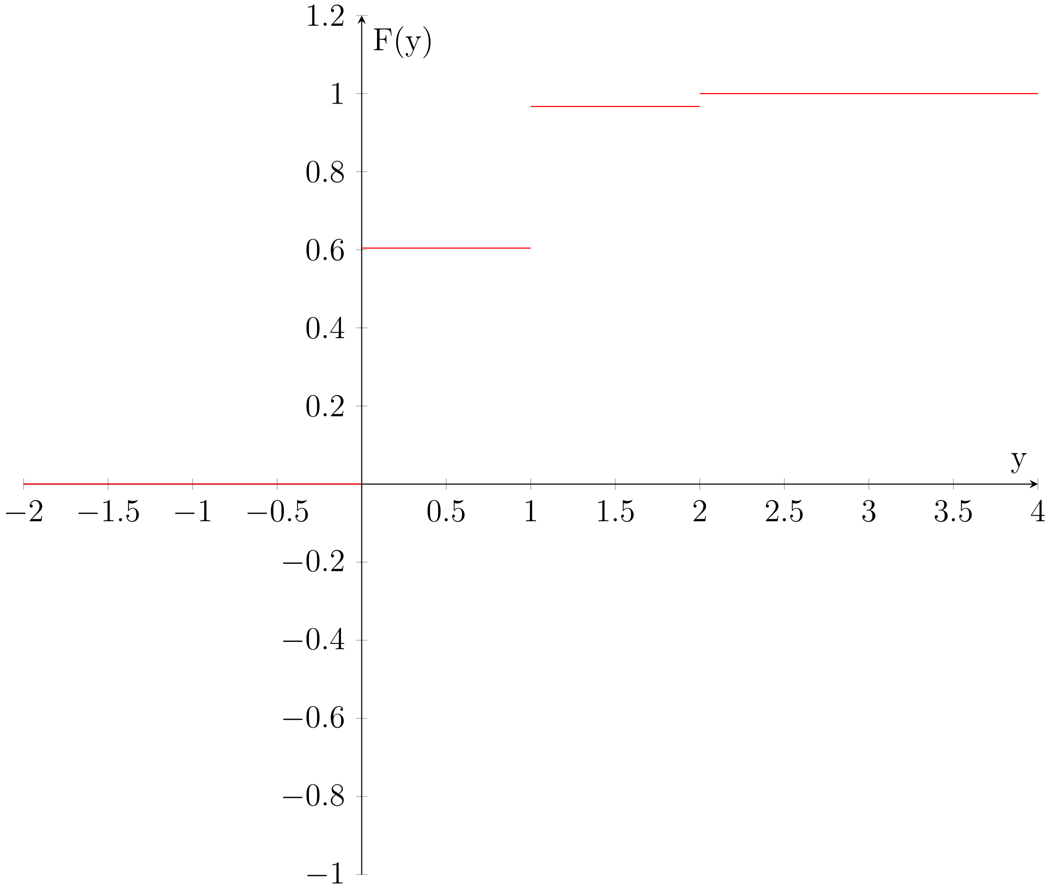


$$\mathbb{P}(Y \leq 0) = \mathbb{P}(Y = 0) = \frac{110}{182}$$

$$\mathbb{P}(Y \leq 1) = \mathbb{P}(Y \leq 0) + \mathbb{P}(Y = 1) = \frac{110}{182} + \frac{66}{182} = \frac{176}{182}$$

$$\mathbb{P}(Y \leq 2) = \mathbb{P}(Y \leq 1) + \mathbb{P}(Y = 2) = \frac{176}{182} + \frac{6}{182} = 1$$

$$F(y) = \begin{cases} 0, 0 < y \\ \frac{110}{182}, 0 \leq y < 1 \\ \frac{176}{182}, 1 \leq y < 2 \\ 1, y \geq 2 \end{cases}$$



d) Determine os valores de E(X) e V(X)

COM REPOSIÇÃO

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 0 * \frac{81}{196} + 1 * \frac{90}{196} + 2 * \frac{25}{196} = 0 + \frac{90}{196} + \frac{50}{196} = \frac{140}{196} = \frac{70}{98}$$

$$E(X^2) = 0^2 * \frac{81}{196} + 1^2 * \frac{90}{196} + 2^2 * \frac{25}{196} = 0 + \frac{90}{196} + \frac{100}{196} = \frac{190}{196} = \frac{95}{98}$$

$$V(X) = E(X^2) - E^2(X)$$

$$V(X) = \frac{95}{98} - (\frac{70}{98})^2 = \frac{9.310}{9.604} - \frac{6.860}{9.604} = \frac{2.450}{9.604}$$

$$E(Y) = \sum_i^n y_i * \mathbb{P}(Y = y_i)$$

$$E(Y) = 0 * \frac{121}{196} + 1 * \frac{66}{196} + 2 * \frac{9}{196} = 0 + \frac{66}{196} + \frac{18}{196} = \frac{84}{196} = \frac{42}{98}$$

$$E(Y^2) = 0^2 * \frac{121}{196} + 1^2 * \frac{66}{196} + 2^2 * \frac{9}{196} = 0 + \frac{66}{196} + \frac{36}{196} = \frac{102}{196} = \frac{51}{98}$$

$$V(Y) = E(Y^2) - E^2(Y)$$

$$V(Y) = \frac{51}{98} - (\frac{42}{98})^2 = \frac{4.998}{9.604} - \frac{1.764}{9.604} = \frac{3.234}{9.604}$$

SEM REPOSIÇÃO

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 0 * \frac{72}{182} + 1 * \frac{90}{182} + 2 * \frac{20}{182} = 0 + \frac{90}{182} + \frac{40}{182} = \frac{130}{182} = \frac{65}{91}$$

$$E(X^2) = 0^2 * \frac{72}{182} + 1^2 * \frac{90}{182} + 2^2 * \frac{20}{182} = 0 + \frac{90}{182} + \frac{80}{182} = \frac{170}{182} = \frac{85}{91}$$

$$V(X) = E(X^2) - E^2(X)$$

$$V(X) = \frac{85}{91} - (\frac{65}{91})^2 = \frac{7.735}{8.281} - \frac{4.225}{8.281} = \frac{3.510}{8.281}$$

$$E(Y) = \sum_i^n y_i * \mathbb{P}(Y = y_i)$$

$$E(Y) = 0 * \frac{110}{182} + 1 * \frac{66}{182} + 2 * \frac{6}{196} = 0 + \frac{66}{192} + \frac{12}{182} = \frac{74}{182} = \frac{37}{91}$$

$$E(Y^2) = 0^2 * \frac{110}{182} + 1^2 * \frac{66}{182} + 2^2 * \frac{6}{182} = 0 + \frac{66}{182} + \frac{24}{182} = \frac{90}{182} = \frac{45}{91}$$

$$V(Y) = E(Y^2) - E^2(Y)$$

$$V(Y) = \frac{45}{91} - (\frac{37}{91})^2 = \frac{4.085}{8.281} - \frac{1.369}{8.281} = \frac{2.716}{8.281}$$

7) Uma moeda não viciada é lançada 3 vezes. Encontre a distribuição de probabilidade da variável aleatória X= número de coroas que apareceram.

a) Encontre a distribuição de probabilidade

$$S_x = \begin{cases} (k,k,k) = 0 \\ (k,k,c);(k,c,k);(c,k,k) = 1 \\ (k,c,c);(c,k,c);(c,c,k) = 2 \\ (k,k,k) = 3 \end{cases}$$

$$\mathbb{P}(X = 0) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8}$$

$$\mathbb{P}(X = 1) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * 3 = \frac{1}{8} * 3 = \frac{3}{8}$$

$$\mathbb{P}(X = 2) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * 3 = \frac{1}{8} * 3 = \frac{3}{8}$$

$$\mathbb{P}(X = 3) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8}$$

b) Encontre a função distribuição acumulada e faça sua representação gráfica

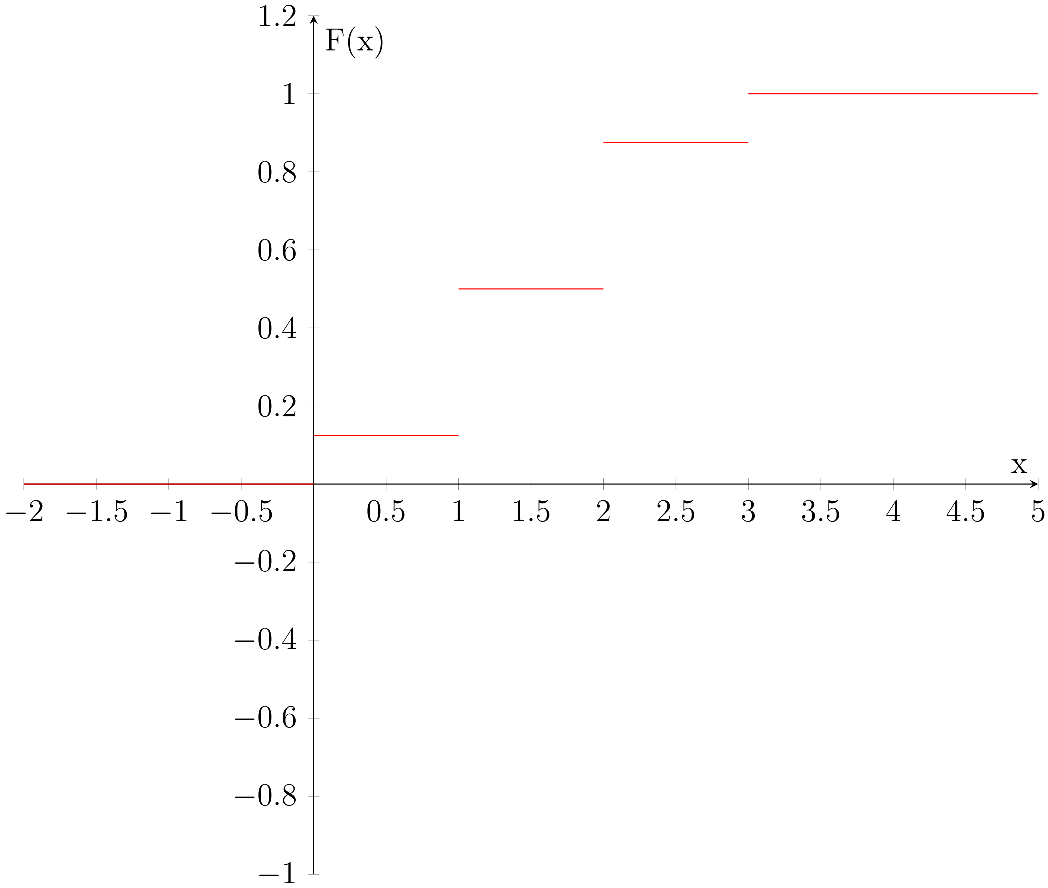
$$\mathbb{P}(X \leq 0) = \mathbb{P}(X = 0) = \frac{1}{8}$$

$$\mathbb{P}(X \leq 1) = \mathbb{P}(X \leq 0) + \mathbb{P}(X = 1) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$$

$$\mathbb{P}(X \leq 2) = \mathbb{P}(X \leq 1) + \mathbb{P}(X = 2) = \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$$

$$\mathbb{P}(X \leq 3) = \mathbb{P}(X \leq 2) + \mathbb{P}(X = 3) = \frac{7}{8} + \frac{1}{8} = \frac{8}{8} = 1$$

$$F(x) = \begin{cases} 0, 0 < x \\ \frac{1}{8}, 0 \leq x < 1 \\ \frac{4}{8}, 1 \leq x < 2 \\ \frac{7}{8}, 2 \leq x < 3 \\ 1, x \geq 3 \end{cases}$$



c) Determine os valores de E(X) e V(X)

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 0 * \frac{1}{8} + 1 * \frac{3}{8} + 2 * \frac{3}{8} + 3 * \frac{1}{8} = 0 + \tfrac{3}{8} + \tfrac{6}{8} + \tfrac{3}{8} = \frac{12}{8} = \frac{3}{2}$$

$$E(X^2) = 0^2 * \frac{1}{8} + 1^2 * \frac{3}{8} + 2^2 * \frac{3}{8} + 3^2 * \frac{1}{8} = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{24}{8} = \frac{6}{2}$$

$$V(X) = E(X^2) - E^2(X)$$

$$V(X) = \frac{6}{2} - (\frac{3}{2})^2 = \frac{12}{4} - \frac{9}{4} = \frac{3}{4}$$

8) Uma variável aleatória discreta X pode assumir os valores 5; 10 e 15. Sabendo que F(5)=0,35, F(10)= 0,70 e F(15)=1, faça a representação gráfica de F(x) e encontre os valores de E(X) e V(X).

$$F(5) = \mathbb{P}(X \geq 5) = \mathbb{P}(X = 5) = 0.35$$

$$F(10) = \mathbb{P}(X \geq 10) = \mathbb{P}(X \geq 5) + \mathbb{P}(X = 10) \Leftrightarrow \mathbb{P}(X = 10) = \mathbb{P}(X \geq 10) - \mathbb{P}(X \geq 5)$$
$$\mathbb{P}(X = 10) = 0.70 - 0.35 = 0.35$$

$$F(15) = \mathbb{P}(X \geq 15) = \mathbb{P}(X \geq 10) + \mathbb{P}(X = 15) \Leftrightarrow \mathbb{P}(X = 15) = \mathbb{P}(X \geq 15) - \mathbb{P}(X \geq 10)$$
$$\mathbb{P}(X = 10) = 1 - 0.7 = 0.3$$

$$E(X) = \sum_i^n x_i * \mathbb{P}(X = x_i)$$

$$E(X) = 5 * 0.35 + 10 * 0.35 + 15 * 0.3 = 1.75 + 3.5 + 4.5 = 9.75$$

$$E(X^2) = 5^2 * 0.35 + 10^2 * 0.35 + 15^2 * 0.3 = 8.75 + 35 + 67.5 = 111.25$$

$$V(X) = E(X^2) - E^2(X)$$

$$V(X) = 111.25 - (9.75)^2 = 111,25 - 95.0625 = 16.1875$$

9) Aos valores da v.a. X da questão 8 foi somado o número 4 definindo-se Y=4+X. Determine E(Y) e V(Y).

$$E(Y) = E(4 + X) = 4 + E(X) = 4 + 9,75 = 13,75$$

$$V(Y) = V(4 + X) = V(X) = 111,75$$

10) Para produzir um determinado componente eletrônico se gasta R$50,00. Este componente é vendido por R$100,00. Um lote de 25 componentes é posto a venda. Sabe-se que no lote tem apenas 2 componentes com defeito. O comprador vai inspecionar 2 componentes e comprará o lote se encontrar no máximo um componente defeituoso. Qual o lucro esperado do produtor?

X = Selecionar componentes perfeitos

$$S_x = \begin{cases} (D, D) = 0 \\ (D, P); (P, D) = 1 \\ (P, P) = 2 \end{cases}$$

Y = Vender o lote

$$S_x = \begin{cases} (P,P);(P,D);(D,P) = s \\ (D,D) = n \end{cases}$$

COM REPOSIÇÃO

$$\mathbb{P}(\text{X} = 0) = \frac{2}{25} * \frac{2}{25} = \frac{4}{625}$$

$$\mathbb{P}(\text{X} = 1) = \frac{2}{25} * \frac{23}{25} + \frac{23}{25} * \frac{2}{25} = \frac{46}{625} + \frac{46}{625} = \frac{92}{625}$$

$$\mathbb{P}(\text{X} = 2) = \frac{23}{25} * \frac{23}{25} = \frac{529}{625}$$

$$\mathbb{P}(\text{Y} = \text{s}) = \frac{92}{625} + \frac{529}{625} = \frac{621}{625}$$

$$\mathbb{P}(\text{Y} = \text{n}) = \frac{4}{625}$$

$$\text{Lucro} = 25\left[(100-50) * \frac{621}{625} + (-50) * \frac{4}{625}\right] = 1.242 - 8 = 1.234$$

SEM REPOSIÇÃO

$$\mathbb{P}(X = 0) = \frac{2}{25} * \frac{1}{24} = \frac{2}{600}$$

$$\mathbb{P}(X = 1) = \frac{2}{25} * \frac{23}{24} + \frac{23}{25} * \frac{2}{23} = \frac{46}{600} + \frac{46}{600} = \frac{92}{600}$$

$$\mathbb{P}(X = 2) = \frac{23}{25} * \frac{22}{24} = \frac{506}{600}$$

$$\mathbb{P}(\text{Y} = \text{s}) = \frac{92}{600} + \frac{506}{600} = \frac{596}{600}$$

$$\mathbb{P}(\text{Y} = \text{n}) = \frac{4}{600}$$

$$\text{Lucro} = 25\left[(100-50)*\frac{596}{600}+(-50)*\frac{4}{600}\right] = 1.241,66 - 8,33 = 1.233,33$$

11) A função distribuição acumulada, F(x), de uma v.a. discreta X é dada por:
F(x) = 0, x < -2
F(x)= 0,25, se $-2 \leq x < 1$
F(x)= 0,40, se $1 \leq x < 3$
F(x)= 0,70, se $3 \leq x < 5$
F(x)= 1 se $x \geq 5$
Encontre: $\mathbb{P}(x=3)$, $\mathbb{P}(x=4)$, F(0) e F(4)

$$\mathbb{P}(X=3) = F(3) - F(1) = 0.7 - 0.4 = 0.3$$

$$\mathbb{P}(X=4) = 0$$

$$F(0) = \mathbb{P}(-2 \leq x < 1) = 0.25$$

$$F(4) = \mathbb{P}(3 \leq x < 5) = 0.7$$