YeshwantraoChavan College of Engineering
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Department of Applied Mathematics & Humanities

Electronic Question Bank with Solution

Course Name: Differential Equation and Complex Analysis

Course Code: 23GE1103

UNIT-I DIFFERENTIAL- EQUATION I

S. n	Unit No.	Text of Question	Time required in Minutes	Marks allotted	lengthiness	Difficulty level	CO Mapping	Type Descriptive / Numerical
1	Unit 1	$Solve\ (1+x^2)\frac{dy}{dx}+y=e^{tan^{-1}x}$ Solution : - The above equation can be written as $\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}x}}{1+x^2}$ Which is of the form $\frac{dy}{dx}+Py=Q$ $I.F.=e^{\int Pdx}=e^{\int\frac{dx}{1+x^2}}=e^{\tan^{-1}x}$ It's Solution is $y\,(\text{I.F.})=\int \text{Q}\,(\text{I.F.})\text{dx}+C$ $y.e^{\tan^{-1}x}=\int\frac{e^{\tan^{-1}x}}{1+\text{x}^2}(e^{\tan^{-1}x})\,\text{dx}+C$	10	5	low	Easy	CO-1	Descriptive

		$y.e^{\tan^{-1}x} = \int \frac{e^{2\tan^{-1}x}}{1+x^2}\,dx + C \quad put\ \tan^{-1}x = t \rightarrow \frac{1}{1+x^2}dx = dt$ $y.e^{\tan^{-1}x} = \int e^{2t}\,dt + C$ $y.e^{\tan^{-1}x} = \frac{e^{2t}}{2} + C \quad \Rightarrow y.e^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + C$						
2	Unit 1	Solve $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$ **Solution.** $\frac{dy}{dx} - \frac{y}{x+1} = e^x(x+1)$ *Integrating factor* $= e^{-\int \frac{dx}{x+1}} = e^{-\log(x+1)} = e^{\log(x+1)^{-1}} = \frac{1}{x+1}$. *The solution is* $y.\frac{1}{x+1} = \int e^x.(x+1).\frac{1}{x+1}\,dx = \int e^x\,dx$ $\frac{y}{x+1} = e^x + C$	10	5	low	Easy	CO-1	Descriptive
3	Unit 1	Solve $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$	10	5	low	Easy	CO-1	Descriptive

		Soln. We have $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$ $\Rightarrow \frac{dy}{dx} - \frac{y}{x(x-1)} = x(x-1)$ which is a linear equation in y i.e. of the form $\frac{dy}{dx} + Py = Q$ where $P = \frac{-1}{x(x-1)}$ and $Q = x(x-1)$ $\therefore$ I.F. $= e^{\int P\,dx} = e^{-\int \frac{1}{x(x-1)}dx} = e^{-\int \left(\frac{1}{x-1} - \frac{1}{x}\right)dx}$ ......*(partial fraction method)* $= e^{\int \left(\frac{1}{x} - \frac{1}{x-1}\right)dx} = e^{\log x - \log(x-1)}$ $= e^{\log\left(\frac{x}{x-1}\right)} = \frac{x}{x-1}$ $\therefore$ Solution is $y(\text{I.F.}) = \int Q.(\text{I. F.})\,dx + C$ $\Rightarrow y\left(\frac{x}{x-1}\right) = \int x(x-1)\frac{x}{x-1}dx + C = \int x^2 dx + C$ $\therefore$ The required solution is $\boxed{\frac{yx}{x-1} = \frac{x^3}{3} + C}$ **Ans.**							
4	Unit 1	Solve $(x+1)\frac{dy}{dx} - 2y = (x+1)^4$ Ans The given D.E. is $(x+1)\frac{dy}{dx} - 2y = (x+1)^4$ $\Rightarrow \frac{dy}{dx} - \frac{2}{x+1}y = (x+1)^3$ Comparing with L.D.E $\frac{dy}{dx} + Py = Q$ where $P = -\frac{2}{x+1}$, $Q = (x+1)^3$ I.F $= e^{-\int \frac{2}{x+1}dx} = e^{-2\log(x+1)} = e^{\log(x+1)^{-2}} = (x+1)^{-2}$	10	5	low	Easy	CO-1	Descriptive	

		Solution $y(IF) = \int Q(IF)\,dx + c$ $y\frac{1}{(x+1)^2} = \int (x+1)^3 \frac{1}{(x+1)^2}\,dx + c$ $\frac{y}{(x+1)^2} = \int (x+1)\,dx + c$ $\frac{y}{(x+1)^2} = \frac{x^2}{2} + x + c$ $\frac{y}{(x+1)^2}$						
5	Unit 1	Solve $[2x - y + 1]dx = [x - 2y + 1]dy$ Solve $[2x-y+1]dx = [x-2y+1]dy$ Solution :- We are given $[2x-y+1]dx = [x-2y+1]dy$ By comparing given D.E. with $M\,dx + N\,dy = 0$ $M = 2x - y + 1 \quad and \quad N = -(x-2y+1)$ $\frac{\partial M}{\partial y} = -1 \quad and \quad \frac{\partial N}{\partial x} = -1$ $Thus \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \qquad Hence\ the\ given\ D.E.\ is\ Exact$ It's Solution is $\int_{y=constant} M.dx + \int (Terms\ in\ N\ not\ containing\ x).dy = C$ $\int_{y=constant} [2x-y+1]dx + \int -(-2y+1.)dy = C$ $\Rightarrow 2.\frac{x^2}{2} - yx + x + 2.\frac{y^2}{2} - y = C$ $\Rightarrow x^2 + y^2 - xy + x - y = C$	10	5	low	Easy	CO-1	Descriptive

6	Unit 1	Solve $\frac{2y}{x}dx + (2logx - y)dy = 0$ **Soln.** Comparing given D.E. $\frac{2y}{x}dx + (2\log x - y)\,dy = 0$ with $Mdx + N\,dy = 0$, we get $M = \frac{2y}{x}$ & $N = 2\log x - y$ $\Rightarrow \frac{\partial M}{\partial y} = \frac{2}{x}$ and $\frac{\partial N}{\partial x} = \frac{2}{x}$ $\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\Rightarrow$ The given D.E. is exact $\therefore$ Its solution is given by $\int_{y=const} Mdx + \int(\text{Terms of N not containing x})\,dy = C$ $\Rightarrow \int_{y=const} \frac{2y}{x}dx + \int(-y)\,dy = C$ $\Rightarrow 2y\int\frac{dx}{x} - \int y\,dy = C$ $\Rightarrow 2y\log x - \frac{y^2}{2} = C$	10	5	low	Easy	CO-1	Descriptive
7	Unit 1	Solve $\frac{dy}{dx} + \left[\frac{x+ycosx}{1+sinx}\right] = 0$	10	5	low	Easy	CO-1	Descriptive

		Soln. The given equation can be written as $\frac{dy}{dx}+\frac{\cos x}{1+\sin x}y=-\frac{x}{1+\sin x}$ which is of the form $\frac{dy}{dx}+Py=Q$ where $P=\frac{\cos x}{1+\sin x}$ & $Q=-\frac{x}{1+\sin x}$ $\therefore$ I.F. $=e^{\int P dx}$ $=e^{\int \frac{\cos x}{1+\sin x}dx}$ Put $1+\sin x=t$ $\Rightarrow \cos x dx = dt$ $\therefore$ I.F. $=e^{\int \frac{1}{t}dt}=e^{\log t}$ $=e^{\log(1+\sin x)}$ $\therefore$ I.F. $=1+\sin x$ $\therefore$ The solution of (1) is given by $y\,(\text{I.F.})=\int Q\,(\text{I.F.})\,dx+C$ $\therefore\ y(1+\sin x)=\int\frac{(-x)}{1+\sin x}(1+\sin x)\,dx+C$ $=-\int x dx + C$ $\therefore$ The required solution is $\boxed{y(1+\sin x)=\frac{-x^2}{2}+C}$ **Ans.**						
8	Unit 1	Solve $\frac{dy}{dx}=-\frac{xy^2}{2+x^2y}$	10	5	low	Easy	CO-1	Descriptive

		Soln. The given equation can be written as $xy^2dx + (2 + x^2y)\,dy = 0$ which is of the form Mdx + Ndy = 0 $\therefore\ M = xy^2 \ \ \&\ \ N = (2 + x^2y)$ $\Rightarrow \frac{\partial M}{\partial y} = 2xy \ \&\ \frac{\partial N}{\partial x} = 2xy$ $\therefore\ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, hence given D.E. is exac $\therefore$ Its solution is given by $\int\limits_{y=\text{constant}} Mdx + \int (\text{Terms of N not containing x})\,dy = C$ $\Rightarrow \int\limits_{y=\text{const}} (xy^2)\,dx + \int 2dy = C$ $\Rightarrow y^2 \int x\,dx + 2\int dy = C$ $\Rightarrow \frac{x^2y^2}{2} + 2y = C$ $\therefore$ The required solution is $\boxed{\frac{x^2y^2}{2} + 2y = C}$ **Ans.**						
9	Unit 1	Solve $tay\ \frac{dy}{dx} + \tan x = cosy\ cos^2x$	10	5	low	Moderate	CO-1	Descriptive

Soln. Given equation is

$$\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$$

Dividing by cos y, we get

$$\frac{\tan y}{\cos y}\frac{dy}{dx} + \frac{\tan x}{\cos y} = \cos^2 x$$

$$\Rightarrow \sec y \tan y \frac{dy}{dx} + \sec y \tan x = \cos^2 x$$

which is of the form $f'(y)\frac{dy}{dx} + f(y)P = Q$

Put $\sec y = t$

$$\therefore \sec y \tan y \frac{dy}{dx} = \frac{dt}{dx}$$

$\therefore$ Equation (1) becomes

$$\frac{dt}{dx} + \tan x \,. t = \cos^2 x$$

which is of the form $\frac{dy}{dx} + Py = Q$

$\therefore$ Here $P = \tan x$ & $Q = \cos^2 x$

$$\Rightarrow \text{I.F.} = e^{\int P dx} = e^{\int \tan x \, dx} = e^{\log \sec x} = \sec x$$

Hence solution of (2) is

$$t.(\text{I.F.}) = \int Q.(\text{I.F.}).\,dx + C$$

$$\Rightarrow \sec y . \sec x = \int \cos^2 x \,. \sec x \, dx + C \quad / \because t = \sec y$$

$$\Rightarrow \sec y \sec x = \int \cos x \, dx + C$$

$\therefore$ The required solution is

$\boxed{\sec y \sec x = \sin x + C}$ **Ans.**

10	Unit 1	Solve $[y(1+\frac{1}{x})+\cos y]dx+[x+\log x-x\sin y]dy=0$ Solution :- We are given $[y(1+\frac{1}{x})+\cos y]dx+[x+\log x-x\sin y]dy=0$ By comparing given D.E. with $M\,dx+N\,dy=0$ $M=y(1+\frac{1}{x})+\cos y \quad and \quad N=x+\log x-x\sin y$ $\frac{\partial M}{\partial y}=1+\frac{1}{x}-\sin y \quad and \quad \frac{\partial N}{\partial x}=1+\frac{1}{x}-\sin y$ $Thus \quad \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x} \qquad Hence\, the\, given\, D.E.\, is\, Exact$ It's Solution is $\int_{y=constant} M.dx+\int(Terms\ in\ N\ not\ containing\ x).dy=C$ $\int_{y=constant}\left[y(1+\frac{1}{x})+\cos y\right].dx+\int(0).dy=C$ $\Rightarrow y\int(1+\frac{1}{x})dx+\cos y\int dx=C$ $\Rightarrow y(x+\log x)+x.\cos y=C$	10	3.5	low	Moderate	CO-1	Descriptive
11	Unit 1	Solve $[(1+\log xy)]dx+[1+\frac{x}{y}]dy=0$ Solution :- We are given $[(1+\log xy)]dx+[1+\frac{x}{y}]dy=0$ By comparing given D.E. with $M\,dx+N\,dy=0$ $M=1+\log xy \quad and \quad N=1+\frac{x}{y}$	10	3.5	low	Moderate	CO-1	Descriptive

		$\frac{\partial M}{\partial y}=\frac{1}{y}$ and $\frac{\partial N}{\partial x}=\frac{1}{y}$ $Thus\ \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ Hence the given D.E. is Exact It's Solution is $\int_{y=constant} M.dx+\int(Terms\ in\ N\ not\ containing\ x).dy=C$ $\int_{y=constant}[1+\log xy].dx+\int(1).dy=C$ $\Rightarrow\int(1+\log x+\log y)dx+\int dx=C$ $\Rightarrow x+x.\log x-x+x.\log y+y=C$ $\Rightarrow x.\log(xy)+y=C$						
12	Unit 1	Solve $y\sin 2xdx-[1+y^2+\cos^2 x]dy=0$ Solution :- We are given $y\sin 2xdx-[1+y^2+\cos^2 x]dy=0$ By comparing given D.E. with $M\,dx+N\,dy=0$ $M=y\sin 2x$ and $N=-(1+y^2+\cos^2 x)$ $\frac{\partial M}{\partial y}=\sin 2x$ and $\frac{\partial N}{\partial x}=2\cos x.\sin x=\sin 2x$ $Thus\ \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ Hence the given D.E. is Exact It's Solution is $\int_{y=constant} M.dx+\int(Terms\ in\ N\ not\ containing\ x).dy=C$ $\int_{y=constant}[y\sin 2x\ ].dx+\int-(1+y^2).dy=C$	12	3.5	medium	moderate	CO-1	Descriptive

		$\Rightarrow \frac{-y\cos 2x}{2} - y - \frac{y^3}{3} = C$ $\Rightarrow 3y\cos 2x + 6y + 2y^3 = C$						
13	Unit 1	Solve $y\log y\frac{dx}{dy} + x - \log y = 0$ Solution : - The above equation can be written as $\frac{dx}{dy} + \frac{x}{y\log y} = \frac{1}{y}$ Which is of the form $\frac{dx}{dy} + P_1 x = Q_1$ $I.F. = e^{\int P_1 dy} = e^{\int \frac{dy}{y\log y}} = \log y$ It's Solution is $\mathrm{x\ (I.F.)} = \int \mathrm{Q\,(I.F.)dy} + C$ $x.\log y = \int \frac{1}{\mathrm{y}}(\mathrm{logy})\,\mathrm{dy} + C$ $x.\log y = \int \mathrm{t}.dt + C$ $\quad put\ \log y = t \to \frac{1}{y}dy = dt$ $x.\log y = \frac{t^2}{2} + C \quad \Rightarrow x.\log y = \frac{(\log y)^2}{2} + C$	12	3.5	medium	moderate	CO-1	Descriptive
14	Unit 1	$Solvex\ (x-1)\ \frac{dy}{dx} - y = x^2(x-1)^2$	12	3.5	medium	moderate	CO-1	Descriptive

		Soln. We have $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$ $\Rightarrow \frac{dy}{dx} - \frac{y}{x(x-1)} = x(x-1)$ which is a linear equation in y i.e. of the form $\frac{dy}{dx} + Py = Q$ where $P = \frac{-1}{x(x-1)}$ and $Q = x(x-1)$ $\therefore$ I.F. $= e^{\int P\,dx} = e^{-\int \frac{1}{x(x-1)}dx} = e^{-\int\left(\frac{1}{x-1}-\frac{1}{x}\right)dx}$ ......*(partial fraction method)* $= e^{\int\left(\frac{1}{x}-\frac{1}{x-1}\right)dx} = e^{\log x - \log(x-1)}$ $= e^{\log\left(\frac{x}{x-1}\right)} = \frac{x}{x-1}$ $\therefore$ Solution is $y(I.F.) = \int Q.(I.F.)\,dx + C$ $\Rightarrow y\left(\frac{x}{x-1}\right) = \int x(x-1)\frac{x}{x-1}dx + C = \int x^2 dx + C$ $\therefore$ The required solution is $\boxed{\frac{yx}{x-1} = \frac{x^3}{3} + C}$ **Ans.**						
15	Unit-I	Solve $x\frac{dy}{dx} + y\log y = xye^x$	12	3.5	medium	moderate	CO-1	Descriptive

		Solve: $x\frac{dy}{dx}+y\log y=xy\ e^x$ **Solution.** $x\frac{dy}{dx}+y\log y=xye^x$ Dividing by xy, we get $\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}\log y=e^x$ ...(1) Put $\log y = z$, so that $\frac{1}{y}\frac{dy}{dx}=\frac{dz}{dx}$ Equation (1) becomes, $\frac{dz}{dx}+\frac{z}{x}=e^x$ $\text{I.F.}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$ Solution is $zx=\int xe^x\,dx+C$ $zx=xe^x-e^x+C$ $\Rightarrow$ $x\log y=xe^x-e^x+C$ **Ans.**						
16		Solve $(x^3-x)\frac{dy}{dx}-(3x^2-1)y=x^5-2x^3+x$	12	3.5	medium	difficult	CO-1	Descriptive

		Solution. We have $(x^3 - x)\frac{dy}{dx} - (3x^2 - 1)y = x^5 - 2x^3 + x$ $\Rightarrow \quad \frac{dy}{dx} - \frac{3x^2-1}{x^3-x}y = \frac{x^5-2x^3+x}{x^3-x} \quad \Rightarrow \quad \frac{dy}{dx} - \frac{3x^2-1}{x^3-x}y = x^2 - 1$ $\text{I.F.} = e^{\int -\frac{3x^2-1}{x^3-x}dx} = e^{-\log(x^3-x)} = e^{\log(x^3-x)^{-1}} = \frac{1}{x^3-x}$ Its solution is $y(\text{I.F.}) = \int Q(I.F.)\,dx + C \quad \Rightarrow y\left(\frac{1}{x^3-x}\right) = \int \frac{x^2-1}{x^3-x}dx + C$ $\Rightarrow \quad \frac{y}{x^3-x} = \int \frac{x^2-1}{x(x^2-1)}dx + C \quad \Rightarrow \quad \frac{y}{x^3-x} = \int \frac{1}{x}dx + C$ $\Rightarrow \quad \frac{y}{x^3-x} = \log x + C \quad \Rightarrow \quad y = (x^3 - x)\log x + (x^3 - x)C$						
17	Unit 1	$Solve \frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$ Solution : - The above equation can be written $\frac{1}{\cos^2 y}\frac{dy}{dx} + 2x\tan y = x^3$ $\sec^2 y\frac{dy}{dx} + 2x\tan y = x^3 \quad ....(1)$ Which is of the form $f'(y)\frac{dy}{dx} + f(y)P = Q$ $put \quad \tan y = v \Rightarrow \sec^2 y\frac{dy}{dx} = \frac{dv}{dx}$	12	3.5	medium	Difficult	CO-1	Descriptive

		$\therefore (1) \Rightarrow \frac{dv}{dx} + 2xv = x^3$ ....(2) $I.F. = e^{\int P\,dx} = e^{\int 2x.dx} = e^{x^2}$ It's Solution is $v\,(I.F.) = \int Q\,(I.F.)dx + C$ $v.e^{x^2} = \int x^3 e^{x^2}\,dx + C$ $v.e^{x^2} = \frac{1}{2}(t-1)e^t + C$ $\quad put\ x^2 = t \Rightarrow 2x\,dx = dt$ $\tan y.e^{x^2} = \frac{1}{2}(x^2-1)e^{x^2} + C$ $\tan y. = \frac{1}{2}(x^2-1) + Ce^{-x^2}$						
18	Unit 1	Solve $\tan\left[(1+y^2)\frac{dx}{dy} + x\right] = y$ Solution :-	12	3.5	medium	Difficult	CO-1	Descriptive	

		The given D.E can be written as $\frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2}$ $= \frac{\tan^{-1}y}{1+y^2} - \frac{1}{1+y^2}x$ $\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{\tan^{-1}y}{1+y^2}$ ---- (1) ch is of the form $\frac{dx}{dy} + Px =$ ere $P = \frac{1}{1+y^2}$ & $Q = \frac{\tan^{-1}y}{1+y^2}$ I.F. $= e^{\int P dy} = e^{\int \frac{dy}{1+y^2}}$ $= e^{\tan^{-1}y}$ soln of eqn (1) is given by $(I.F.) = \int Q(I.F.)\,dy + c$ $e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} . e^{\tan^{-1}y}\,dy + c$ t $\tan^{-1}y = t$ in R.H.S. of (2) $\therefore \frac{1}{1+y^2}dy = dt$ (2) $\Rightarrow x e^{\tan^{-1}y} = \int t.e^t . dt + c$ $= te^t - e^t + c$ $\therefore x e^{\tan^{-1}y} = \tan^{-1}y . e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$ $\therefore x e^{\tan^{-1}y} = (\tan^{-1}y - 1) e^{\tan^{-1}y} + c$ $\therefore x = (\tan^{-1}y - 1) + C e^{\tan^{-1}y}$						
19	Unit 1	$Solve \frac{dx}{dy} = \frac{1 + y^2 + cos^2x}{ysin2x}$ Solution :-	12	3.5	medium	Difficult	CO-1	Descriptive

		Soln. The given D.E. can be written as $y \sin 2x\, dx - (1 + y^2 + \cos^2 x)\, dy = 0$ Comparing it with $M\,dx + N\,dy = 0$, we get $M = y \sin 2x$, $\Rightarrow \frac{\partial M}{\partial y} = \sin 2x$, $N = -(1 + y^2 + \cos^2 x)$ $\Rightarrow \frac{\partial N}{\partial x} = -2\cos x\,(-\sin x) = \sin 2x$ $\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence given D.E. is exact. $\therefore$ Its solution is given by $\int_{y=const.} M dx + \int (\text{Terms of N not containing x})\, dy = C$ $\Rightarrow \int_{y=constant} y \sin 2x\, dx + \int [-(1 + y^2)]\, dy = C$ $\Rightarrow y \int \sin 2x dx - \int (1 + y^2)\, dy = C$ $\Rightarrow -\frac{y \cos 2x}{2} - y - \frac{y^3}{3} = C$ $/ \because C_1 = -$ $\Rightarrow 3y \cos 2x + 6y + 2y^3 = C_1$ $\therefore$ The required solution is $\boxed{3y \cos 2x + 6y + 2y^3 = C_1}$ Ans.						
20	Unit 1	$Solve\ (y^2 e^{xy^2} + 4x^3)\, dx + (2xye^{xy^2} - 3y^2)\, dy = 0$ Consider $(y^2 e^{xy^2} + 4x^3)\, dx + (2xye^{xy^2} - 3y^2)\, dy = 0$ Comparing with $Mdx + Ndy = 0$, we get $M = y^2 e^{xy^2} + 4x^3$, $N = 2xye^{xy^2} - 3y^2$ $\therefore \frac{\partial M}{\partial y} = 2ye^{xy^2} + y^2 2xye^{xy^2} = 2ye^{xy^2} + 2xy^3 e^{xy^2}$ & $\frac{\partial N}{\partial x} = 2ye^{xy^2} + 2xyy^2 e^{xy^2} = 2ye^{xy^2} + 2xy^3 e^{xy^2}$ Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the given equation is exact. $\therefore$ Solution is $\int_{y=constant} Mdx + \int$ [Terms of N not containing x] $dy = C$ $\Rightarrow \int_{y=const.} \left(y^2 e^{xy^2} + 4x^3\right) dx + \int -3y^2 dy = C$ $\Rightarrow \frac{y^2}{y^2} e^{xy^2} + \frac{4x^4}{4} - \frac{3y^3}{3} = C$ $\Rightarrow e^{xy^2} + x^4 - y^3 = C$ $\therefore$ The required solution is $\boxed{e^{xy^2} + x^4 - y^3 = C}$ Ans.	12	3.5	medium	Difficult	CO-1	Descriptive

21	Unit 1	$Solve[\cos x \tan y + \cos(x + y)]dx + [\sin x \sec^2 y + \cos(x + y)]dy = 0$ Solution :- We are given $[\cos x \tan y + \cos(x+y)]dx + [\sin x \sec^2 y + \cos(x+y)]dy = 0$ By comparing given D.E. with $M\,dx + N\,dy = 0$ $M = \cos x \tan y + \cos(x+y)$ *and* $N = \sin x \sec^2 y + \cos(x+y)$ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \cos x \sec^2 y - \sin(x+y)$ *Thus* $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ *Hence the given D.E. is Exact* It's Solution is $\int_{y=constant} M.dx + \int (Terms\ in\ N\ not\ containing\ x).dy = C$ $\int_{y=constant} [\cos x \tan y + \cos(x+y)].dx + \int (0).dy = C$ $\int \tan y[\cos x].dx + \int \cos(x+y).dx + 0 = C$ $\Rightarrow \tan y \sin x + \sin(x+y) = C$	15	3.5	high	Very difficult	CO-1	Descriptive
22	Unit 1	$Solve\,\frac{dy}{dx} = \frac{1}{xy(1+xy^2)}$ The above equation can be written as $\frac{dx}{dy} - xy = x^2 y^3 \quad ....(1)$	15	3.5	high	Very difficult	CO-1	Descriptive

Which is in Bernoulli' s form $\frac{dx}{dy} + P_1 x = Q_1 x^n$

to solve it, divide both side by x^2

$$\frac{1}{x^2}\frac{dx}{dy} - \frac{xy}{x^2} = \frac{x^2 y^3}{x^2} \qquad \Rightarrow x^{-2}\frac{dx}{dy} - yx^{-1} = y^3 \quad \ldots.(2)$$

$$put\ \ x^{-1} = v \Rightarrow x^{-2}\frac{dx}{dy} = -\frac{dv}{dy}$$

$$Eqn\ (2)\ reduces\ to\quad \frac{dv}{dy} + yv = -y^3 \quad \ldots\ldots..(3)$$

$$I.F. = e^{\int P dy} = e^{\int y.dy} = e^{\frac{y^2}{2}}$$

It's Solution is $\quad v\,(\text{I.F.}) = \int Q\,(\text{I.F.})\,\text{dy} + C$

$$v.e^{\frac{y^2}{2}} = \int -y^3\, e^{\frac{y^2}{2}}\,\text{dy} + C$$

$$put\ \frac{y^2}{2} = t \Rightarrow y.dy = dt$$

$$v.e^{\frac{y^2}{2}} = \int 2t.e^t\,\text{dt} + C \quad = 2.e^t(t-1) + C$$

$$v.e^{\frac{y^2}{2}} = e^{\frac{y^2}{2}}(2 - y^2) + C$$

$$\Rightarrow \frac{1}{x} = (2 - y^2) + C.e^{\frac{-y^2}{2}}$$

23	Unit-I	Solve $r\sin\theta - \frac{dr}{d\theta}\cos\theta = r^2$ The given equation can be written as $-\frac{dr}{d\theta}\cos\theta + r\sin\theta = r^2$ Dividing by $\cos\theta$, we get $-\frac{dr}{d\theta} + r\tan\theta = r^2\sec\theta$ Dividing by r^2, we get $-r^{-2}\frac{dr}{d\theta} + r^{-1}\tan\theta = \sec\theta$ which is of the form $f'(r)\frac{dr}{d\theta} + f(r)P = Q$ Put $r^{-1} = v$ $\quad \therefore -r^{-2}\frac{dr}{d\theta} = \frac{dv}{d\theta}$ $\therefore$ (1) becomes $\frac{dv}{d\theta} + v\tan\theta = \sec\theta$	15	3.5	high	Very difficult	CO-1	Descriptive

where $P = \tan\theta$ and $Q = \sec\theta$

$\therefore$ I.F. $= e^{\int \tan\theta \, d\theta} = e^{\log \sec\theta} = \sec\theta$

$\therefore$ Solution of (2) is

$$v \sec\theta = \int \sec\theta . \sec\theta \, d\theta + C$$

$$\Rightarrow \quad v \sec\theta = \tan\theta + C$$

$$\Rightarrow \quad v = \sin\theta + C\cos\theta$$

Putting $v = \frac{1}{r}$, we get

$$\frac{1}{r} = \sin\theta + C\cos\theta$$

$\therefore$ The required solution is

$$\boxed{r = \frac{1}{\sin\theta + C\cos\theta}}$$ **Ans.**