EE5609: Matrix Theory Assignment 9

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***Abstract*—This document uses properties of vector basis and dimensions.**

Download the latex code from

https://github.com/Atul 191/EE5609/Assignment9

1 Problem

Find three vectors in R^3 which are linearly dependent and are such that any two of them are linearly independent

2 Solution

Let

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{v_3} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{2.0.3}$$

Then,

$$\mathbf{v_1} + \mathbf{v_2} - \mathbf{v_3} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$

Thus, they are linearly dependent. As We know that $\mathbf{v_1} and \mathbf{v_2}$ are linearly independent as they form the standard basis vectors.
Now, Suppose

$$\mathbf{a}\mathbf{v_1} + \mathbf{b}\mathbf{v_2} = 0 \tag{2.0.5}$$

Then

$$\begin{pmatrix} a+b \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.6}$$

Hence, the second co-ordinate implies $\mathbf{b} = 0$ and the first coordinate in turn implies $\mathbf{a} = 0$.
Thus $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent. Similarly, $\mathbf{v_2}$ and $\mathbf{v_3}$ are linearly independent