EE5609: Matrix Theory
Assignment 10

Lt Cdr Atul Mahajan
MTech Artificial Intelligence
AI20MTECH13001

***Abstract*—This document solves CSIR problem**

Download the latex code from

https://github.com/Atul 191/EE5609/Assignment10

1 Problem

Let $p(x) = \alpha x^2 + \beta x + \gamma$ be a polynomial, where $\alpha, \beta, \gamma \epsilon R$. Fix $X_0 \epsilon R$. Let $S = \{(a, b, c) \epsilon R^3 : p(x) = a(x - x_0)^2 + b(x - x_0) + c\}$ for all $x \epsilon R$. Find the number of elements in S is

1) 0
2) 1
3) Strictly greater than 1 but finite
4) Infinite

2 Solution

$$p(x) = \alpha x^2 + \beta x + \gamma \tag{2.0.1}$$

$$\implies p(x) = (\alpha \beta \gamma)\left(x^2 x 1\right)^T \tag{2.0.2}$$

$$S = \{(a, b, c) \epsilon R^3 : p(x) = a(x - x_0)^2 + b(x - x_0) + c\},$$

$$\text{for all}\, \mathbf{x} \epsilon R \, (Fix X_0) \tag{2.0.3}$$

$$p(x) = (abc)\left((x - x_0)^2 (x - x_0) 1\right)^T \tag{2.0.4}$$

$$= a\left(x^2 - 2x_0 x + x_0^2\right) + b(x - x_0) + c \tag{2.0.5}$$

$$= ax^2 + (b - 2ax_0)x + \left(a{x_0}^2 - bx_0 + c\right) \tag{2.0.6}$$

Refer (2.0.2) and (2.0.6) and comparing the co-coefficients of powers of x,

$$\alpha = a, \beta = b - 2ax_0, \gamma = a{x_0}^2 - bx_0 + c \tag{2.0.7}$$

$$a = \alpha, b = \beta + 2\alpha x_0, c = \gamma - \alpha {x_0}^2 + (\beta + 2\alpha x_0)x_0 \tag{2.0.8}$$

Here α, β, γ and x_0 are the real fixed numbers. So a, b, c have unique values.

Hence S contain only 1 element.So option 2 is correct