EE5609: Matrix Theory Assignment 11

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***Abstract*—This document solves CSIR problem**

Download the latex code from

https://github.com/Atul 191/EE5609/Assignment11

1 Problem

Let $\mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = x^2, y^2 + \sin x$. Then the derivative of f at (x, y) is the linear transformation given by

1) $\begin{pmatrix} 2x & 0 \\ \cos x & 2y \end{pmatrix}$
2) $\begin{pmatrix} 2x & 0 \\ 2y & \cos x \end{pmatrix}$
3) $\begin{pmatrix} 2y & \cos x \\ 2x & 0 \end{pmatrix}$
4) $\begin{pmatrix} 2x & 2y \\ 0 & \cos x \end{pmatrix}$

2 Solution

Let $f_1 = x^2$ and $f_2 = y^2 + \sin x$.
Begin by finding the derivative of f(x,y)

$$Df(x, y) = \begin{pmatrix} Df_1x & Df_1y \\ Df_2x & Df_2y \end{pmatrix} \tag{2.0.1}$$

$$= \begin{pmatrix} 2x & 0 \\ \cos x & 2y \end{pmatrix} \tag{2.0.2}$$

So option 1 is correct.
Now to prove that Derivatives is a linear transformation we dwell on the definition of linear transformation that it satisfies two properties i.e additivity and homogeneity as $\mathbb{R}^n \to \mathbb{R}^m$

$$D(cf) = cD(f) \tag{2.0.3}$$

$$D(f + g) = D(f) + D(g) \tag{2.0.4}$$

Now refer (2.0.3) we proceed as

$$D(cf) = \begin{pmatrix} Dcf_1 & Dcf_1 \\ Dcf_2 & Dcf_2 \end{pmatrix} \tag{2.0.5}$$

$$= c\begin{pmatrix} Df_1 & Df_1 \\ Df_2 & Df_2 \end{pmatrix} \tag{2.0.6}$$

$$= cD(f) \tag{2.0.7}$$

Now refer (2.0.4) we proceed as

$$D(f + g) = \begin{pmatrix} D(f_1 + g_1) & D(f_1 + g_1) \\ D(f_2 + g_2) & D(f_2 + g_2) \end{pmatrix} \tag{2.0.8}$$

$$\begin{pmatrix} Df_1 & Df_1 \\ Df_2 & Df_2 \end{pmatrix} + \begin{pmatrix} Dg_1 & Dg_1 \\ Dg_2 & Dg_2 \end{pmatrix} \tag{2.0.9}$$

$$= D(f) + D(g) \tag{2.0.10}$$

Hence both properties are satisfied so we can say that it is a linear transformation