Assignment 3

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***Abstract*—This a document that explains how to prove congruency of triangles.**

Download all latex-tikz codes from

https://github.com/Atul191/Assignment−3

1 Problem

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of triangle PQR. Show that:

$$a)\quad \triangle ABM \cong \triangle PQN \tag{1.0.1}$$

$$b)\quad \triangle ABC \cong \triangle PQR \tag{1.0.2}$$

2 Solution

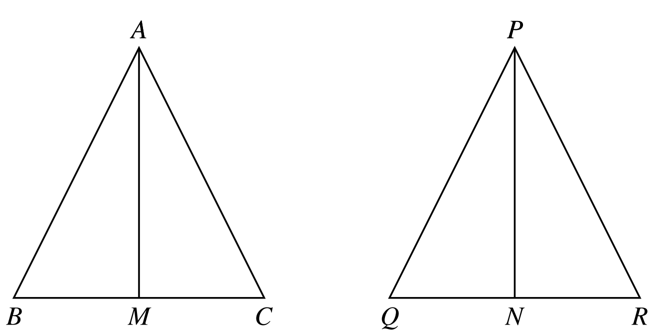

Fig. 1: $\triangle ABC and \triangle PQR$

Given Condition in the question are:

$$AB = PQ \tag{2.0.1}$$

$$BC = QR \tag{2.0.2}$$

$$AM = PN \tag{2.0.3}$$

As M and N are medians of triangle ABC and triangle PQR respectively, we deduce the following:

$$\mathbf{M} = \frac{\mathbf{B}+\mathbf{C}}{2} \tag{2.0.4}$$

$$\implies 2\mathbf{M} = (\mathbf{B}+\mathbf{C}) \tag{2.0.5}$$

$$\implies (\mathbf{B}-\mathbf{M}) = (\mathbf{M}-\mathbf{C}) \tag{2.0.6}$$

$$\implies \|\mathbf{B}-\mathbf{M}\| = \|\mathbf{M}-\mathbf{C}\| \tag{2.0.7}$$

$$\text{Also} \tag{2.0.8}$$

$$\mathbf{N} = \frac{\mathbf{Q}+\mathbf{R}}{2} \tag{2.0.9}$$

$$\implies 2\mathbf{N} = (\mathbf{Q}+\mathbf{R}) \tag{2.0.10}$$

$$\implies (\mathbf{Q}-\mathbf{N}) = (\mathbf{N}-\mathbf{R}) \tag{2.0.11}$$

$$\implies \|\mathbf{Q}-\mathbf{N}\| = \|\mathbf{N}-\mathbf{R}\| \tag{2.0.12}$$

Refer(2.0.7)and (2.0.12)

$$\|\mathbf{B}-\mathbf{M}\| = \|\mathbf{Q}-\mathbf{N}\| \tag{2.0.13}$$

Hence in triangle ABM and triangle PQN sides AB,BM and MA are equal to PQ,QN and NP so by SSS congruency criteria

$$\triangle ABM \cong \triangle PQN \tag{2.0.14}$$

Now for proving congruence of triangle ABC and triangle PQR we know that the corresponding angles of congurent triangles are equal and to prove that we make a hypothesis and proceed as follows

$$\angle ABM = \angle PQN \tag{2.0.15}$$

and one of the proved condition

$$BM = QN \tag{2.0.16}$$

Refer(2.0.15)

$$\cos\angle ABM = \cos\angle PQN \tag{2.0.17}$$

$$\frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{M})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{B}-\mathbf{M}\|} = \frac{(\mathbf{Q}-\mathbf{P})^T(\mathbf{Q}-\mathbf{N})}{\|\mathbf{Q}-\mathbf{P}\|\,\|\mathbf{Q}-\mathbf{N}\|} \tag{2.0.18}$$

Equating (2.0.16)

$$\implies \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{M})}{\|\mathbf{B}-\mathbf{A}\|} = \frac{(\mathbf{Q}-\mathbf{P})^T(\mathbf{Q}-\mathbf{N})}{\|\mathbf{Q}-\mathbf{P}\|} \tag{2.0.19}$$

It can be shown that

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{M}) = \|\mathbf{A}-\mathbf{B}\|^2 - (\mathbf{A}-\mathbf{M})^T(\mathbf{A}-\mathbf{B}) \tag{2.0.20}$$

$$(\mathbf{Q}-\mathbf{P})^T(\mathbf{Q}-\mathbf{N}) = \|\mathbf{P}-\mathbf{Q}\|^2 - (\mathbf{P}-\mathbf{N})^T(\mathbf{P}-\mathbf{Q}) \tag{2.0.21}$$

Substituting (2.0.20) and (2.0.21) in (2.0.19)

$$\|\mathbf{A}-\mathbf{B}\| - \frac{(\mathbf{A}-\mathbf{M})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{B}-\mathbf{A}\|} = \|\mathbf{P}-\mathbf{Q}\| - \frac{(\mathbf{P}-\mathbf{N})^T(\mathbf{P}-\mathbf{Q})}{\|\mathbf{Q}-\mathbf{P}\|} \tag{2.0.22}$$

$$\|\mathbf{A}-\mathbf{B}\| - \|\mathbf{A}-\mathbf{M}\|\cos\angle BAM = \|\mathbf{P}-\mathbf{Q}\| - \|\mathbf{P}-\mathbf{N}\|\cos\angle QPN \tag{2.0.23}$$

Refer (2.0.1) and (2.0.3)

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{P}-\mathbf{Q}\| \tag{2.0.24}$$

$$\|\mathbf{A}-\mathbf{M}\| = \|\mathbf{P}-\mathbf{N}\| \tag{2.0.25}$$

$$\therefore \cos\angle BAM = \cos\angle QPN \tag{2.0.26}$$

$$\implies \angle BAM = \angle QPN \tag{2.0.27}$$

Hence our hypothesis is right as we prove that corresponding angles of congurent triangles are equal. So we get

$$\angle ABM = \angle PQN \tag{2.0.28}$$

$$\therefore \angle ABC = \angle PQR \tag{2.0.29}$$

So by applying SAS criteria we conclude that

$$\triangle ABC \cong \triangle PQR \tag{2.0.30}$$