Assignment 4

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***Abstract*—This problem shows the given equation of a circle inferences that it passes through the co-ordinate axes.**

Download code from

https://github.com/Atul191/Assignment−4

1 Problem

Prove that the circle with following equation passes through the co-ordinate axes

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{a}\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} + \mathbf{a}^2 = 0 \tag{1.0.1}$$

2 Solution

Let r be the radius of the circle. Now for proving that the circle touches the co-ordinate axes we have to prove that it touches x axis and y axes at points such that:

$$\mathbf{Point_1} = \begin{pmatrix}r\\0\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{Point_2} = \begin{pmatrix}0\\r\end{pmatrix} \tag{2.0.2}$$

The general equation of a circle is given by

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{O}^T\mathbf{x} + \mathbf{f} = 0 \tag{2.0.3}$$

Where **O** is the centre and **r** is the radius of the circle.
Substituting (2.0.1) in (1.0.1), we rewrite (1.0.1)as:

$$\begin{pmatrix}r & 0\end{pmatrix}\begin{pmatrix}r\\0\end{pmatrix} - 2\begin{pmatrix}a & a\end{pmatrix}\begin{pmatrix}r\\0\end{pmatrix} + \mathbf{a}^2 = 0 \tag{2.0.4}$$

$$\implies \mathbf{r}^2 - 2(\mathbf{ar}) + \mathbf{a}^2 = 0 \tag{2.0.5}$$

$$\implies \mathbf{r}^2 - 2(\mathbf{ar}) + \mathbf{a}^2 = 0 \tag{2.0.6}$$

$$\implies (\mathbf{r} - \mathbf{a})^2 = 0 \tag{2.0.7}$$

$$\implies \mathbf{r} = \mathbf{a} \tag{2.0.8}$$

Similarly, substituting (2.0.2) in (1.0.1), we rewrite (1.0.1)as:

$$\begin{pmatrix}0 & r\end{pmatrix}\begin{pmatrix}0\\r\end{pmatrix} - 2\begin{pmatrix}a & a\end{pmatrix}\begin{pmatrix}0\\r\end{pmatrix} + \mathbf{a}^2 = 0 \tag{2.0.9}$$

$$\implies \mathbf{r}^2 - 2(\mathbf{ar}) + \mathbf{a}^2 = 0 \tag{2.0.10}$$

$$\implies \mathbf{r}^2 - 2(\mathbf{ar}) + \mathbf{a}^2 = 0 \tag{2.0.11}$$

$$\implies (\mathbf{r} - \mathbf{a})^2 = 0 \tag{2.0.12}$$

$$\implies \mathbf{r} = \mathbf{a} \tag{2.0.13}$$

Therefore, the circle touches x axis at $\mathbf{Point_1}$ i.e $\begin{pmatrix}a\\0\end{pmatrix}$ and y axis at $\mathbf{Point_2}$ i.e $\begin{pmatrix}0\\a\end{pmatrix}$
Hence,it is proved that the circle touches the co-ordinate axes

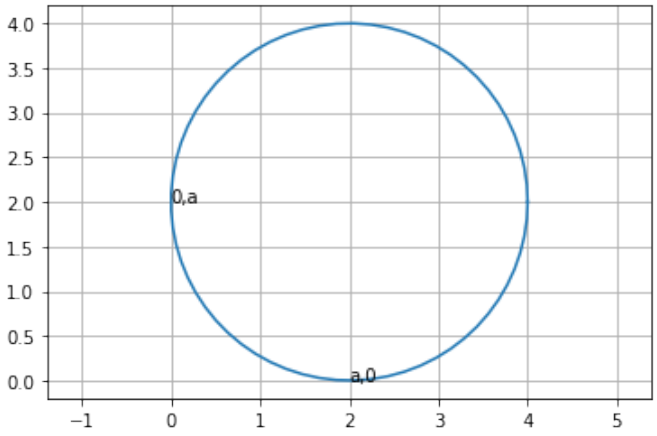

Fig. 0: Circle touching the co-ordinate axes a=2