EE5609: Matrix Theory Assignment 8

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***Abstract*—This document uses properties of vector spaces and subspaces.**

Download the latex code from

https://github.com/Atul 191/EE5609/Assignment8

1 Problem

Let $\mathbf{V}$ be the (real) vector space of all functions f from $\mathbf{R}$ into $\mathbf{R}$. Is $f(-1) = 0$ a subspace of $\mathbf{V}$

2 Theorem

A non-empty subset $\mathbf{W}$ of $\mathbf{V}$ is a subspace of $\mathbf{V}$ if and only if for each pair of vectors α β in $\mathbf{W}$ and each scalar c in $\mathbb{R}$ the vector $c\alpha + \beta$ is again in $\mathbf{W}$.

3 Solution

For each of the function to be a subspace, it must be closed with respect to addition and scalar multiplication in $\mathbf{V}$ defined as, for f g ϵ $\mathbf{W}$ and c ϵ $\mathbb{R}$

Then,

$$
\begin{aligned}
(\mathbf{cf} + \mathbf{g})(-1) &= \mathbf{cf}(-1) + \mathbf{g}(-1) && (3.0.1)\\
&= \mathbf{c}(0) + 0 && (3.0.2)\\
&= 0 && (3.0.3)
\end{aligned}
$$

Thus, $(\mathbf{cf} + \mathbf{g}(-1)) = 0$. Therefore,$\mathbf{W}$ is a subset of $\mathbf{V}$ and also a vector space. Therefore $\mathbf{W}$ is a subspace of $\mathbf{V}$.
Hence, $f(-1) = 0$ is a subspace of $\mathbf{V}$.